

**EVALUATION OF THE INTERWELL CONNECTIVITY OF  
LITTLE CREEK FIELD, MISSISSIPPI FROM PRODUCTION DATA**

A thesis

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# TABLE OF CONTENTS

|  |            |
|--|------------|
| <b>ACKNOWLEDGEMENTS.....</b>   | <b>ii</b>  |
| <b>LIST OF TABLES.....</b>   | <b>v</b>   |
| <b>LIST OF FIGURES.....</b>  | <b>vii</b> |
| <b>ABSTRACT.....</b>   | <b>x</b>   |
| <b>CHAPTER 1: INTRODUCTION.....</b>  | <b>1</b>   |
| 1.1 Objective.....   | 4          |
| 1.2 Literature Review.....   | 4          |
| <b>CHAPTER 2: STATISTICAL ANALYSIS PROCEDURE.....</b>  | <b>11</b>  |
| 2.1 Multivariate Linear Regression (MLR).....  | 11         |
| 2.1.1 Mathematical Development.....  | 12         |
| 2.1.2 Uses of MLR.....   | 16         |
| 2.2 Balanced Multivariate Linear Regression (BMLR).....  | 16         |
| 2.2.1 Balanced Condition.....  | 16         |
| 2.2.2 Mathematical Development.....  | 17         |
| 2.2.3 Uses of BMLR.....  | 18         |
| 2.3 Diffusivity Filters.....   | 18         |
| 2.3.1 Mathematical Development.....  | 20         |
| 2.3.2 Illustrative Examples of the Application of Diffusivity Filters.....                       | 23         |
| 2.4 Further Improvement of the MLR Model.....  | 25         |
| 2.4.1 Statistical Implications .....   | 27         |
| 2.5 Assumptions of Model and Possible Sources of Error.....                                      | 31         |
| 2.5.1 Possible Sources of Error .....  | 32         |
| <b>CHAPTER 3: APPLICATION OF MLR METHOD TO LITTLE CREEK FIELD<br/>DATA.....</b>                  | <b>34</b>  |
| 3.1 Denbury Resources Little Creek Field.....  | 34         |
| 3.1.1 Field Description.....   | 34         |
| 3.1.2 Phase 2 Area.....  | 35         |
| 3.2 Application of MLR Method.....   | 40         |
| 3.2.1 Results.....   | 40         |
| 3.2.2 Application of the SE-N and SE-P Procedure to MLR Results.....                             | 47         |
| 3.3 Application of MLR Method with Diffusivity Filters.....                                      | 51         |
| 3.3.1 MLR with 1 Month Diffusivity Filter.....   | 53         |
| 3.3.2 Application of the SE-N and SE-P Procedure to MLR with 1-month<br>Diffusivity Filters..... | 56         |

|  |  |            |
|--|--|------------|
| 3.3.3  | MLR with 6 Month Diffusivity Filters.....  | 58         |
| 3.3.4  | Application of the SE-N and SE-P procedure to MLR with 6-month<br>Diffusivity Filters..... | 61         |
| 3.4  | Chapter Summary.....   | 63         |
| <b>CHAPTER 4: SIMPLE LINEAR MODEL APPLICATION.....</b> |  | <b>66</b>  |
| 4.1  | Simple Linear Model.....   | 67         |
| 4.1.1  | Hypothesis testing.....  | 68         |
| 4.1.2  | Results.....   | 69         |
| 4.2  | MLR Approach With Significant Injector-Producer Well Pairs.....                            | 72         |
| 4.2.1  | Results.....   | 72         |
| 4.3  | Relationships Between $b$ Values and Reservoir Characteristics .....                       | 75         |
| 4.4  | MLR Approach With Application of Hypothesis Test .....                                     | 78         |
| 4.5  | Result Summary of the MLR Cases.....   | 79         |
| <b>CHAPTER 5: WELL PERFORMANCE EVALUATION.....</b>     |  | <b>82</b>  |
| 5.1  | Inflow Performance Relationships.....  | 82         |
| <b>CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS.....</b> |  | <b>89</b>  |
| <b>REFERENCES.....</b>                                 |  | <b>92</b>  |
| <b>APPENDIX A.....</b>                                 |  | <b>95</b>  |
| <b>APPENDIX B.....</b>                                 |  | <b>100</b> |
| <b>VITA.....</b>                                       |  | <b>106</b> |

## LIST OF TABLES

|  |    |
|--|----|
| <b>Table 3.1</b> List of wells in the Phase 2 Area of Little Creek Field.....  | 35 |
| <b>Table 3.2</b> Total injection amount of CO <sub>2</sub> injection in reservoir barriers during the time period analyzed.....                | 37 |
| <b>Table 3.3</b> Total production rate for each producer during the selected time period for the analysis.....                                 | 39 |
| <b>Table 3.4</b> MLR without Diffusivity filters.....  | 41 |
| <b>Table 3.5</b> MLR without Diffusivity filters, after the SE-N and SE-P procedure is applied. ....   | 48 |
| <b>Table 3.6</b> Average porosity and permeability values.....   | 52 |
| <b>Table 3.7</b> Time (days) calculation results.....  | 52 |
| <b>Table 3.8</b> MLR with 1 month diffusivity filters.....   | 53 |
| <b>Table 3.9</b> MLR with 1 month Diffusivity filters, after the SE-N and SE-P procedure.....  | 56 |
| <b>Table 3.10</b> MLR with 6 month Diffusivity filters.....  | 59 |
| <b>Table 3.11</b> MLR with 6 month Diffusivity filters, after the SE-N and SE-P procedure is applied. ....                                     | 61 |
| <b>Table 4.1a</b> SLM showing <i>b</i> values of each injector-producer well pair for Little Creek Field Phase 2 data.....                     | 68 |
| <b>Table 4.1b</b> SLM showing the <i>R</i> <sup>2</sup> value of each injector-producer well pair for the Little Creek Field Phase 2 data..... | 68 |
| <b>Table 4.2</b> Showing significant and non-significant <i>b</i> values that have been set to zero..  | 70 |
| <b>Table 4.3</b> MLR with significant well pairs.....  | 73 |
| <b>Table 4.4.</b> MLR with significant well pairs after hypothesis test.....   | 78 |
| <b>Table 4.5.</b> Result Summary of the MLR models.....  | 79 |

|   |    |
|---|----|
| <b>Table 4.6.</b> F-test Result summary of the MLR models applied to the Little Creek Field Phase data.....   | 80 |
| <b>Table 5.1</b> Productivity index for wells in the Phase 2 portion of Denbury Resources Little Creek field in Mississippi.....  | 86 |
| <b>Table A.1</b> Weighting coefficients $\beta_{ij}$ for Little Creek Field. MLR with 12 month Diffusivity filters, with $O_d = 5$ .....  | 95 |
| <b>Table A.2</b> Weighting coefficients $\beta_{ij}$ after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12 month Diffusivity filters, with $O_d = 5$ ..... | 98 |

## LIST OF FIGURES

|  |    |
|--|----|
| <b>Figure 2.1</b> Injection and equivalent production rate change in a reservoir with no dissipation.....  | 23 |
| <b>Figure 2.2</b> The injection rate change and equivalent change in production rate between an injector-producer pair in a reservoir with small dissipation.....  | 24 |
| <b>Figure 2.3</b> Corresponding filter function showing large coefficient at time zero, while the rest are much smaller.....   | 24 |
| <b>Figure 3.1</b> Location of wells in Phase 2 Area of Little Creek Field.....   | 36 |
| <b>Figure 3.2</b> Monthly injection rates for injector wells for the selected time period.....   | 38 |
| <b>Figure 3.3</b> Monthly production rates for producer wells for the selected time period.....  | 39 |
| <b>Figure 3.4</b> Total injection and production rates for the selected time period.....   | 40 |
| <b>Figure 3.5</b> MLR without Diffusivity filters, with $O_d=7.4$ . Representation of weighting coefficients $\beta_{ij}$ .....  | 41 |
| <b>Figure 3.6</b> Plot of modeled rate versus actual rate for Producer 27-14 (P3).....   | 43 |
| <b>Figure 3.7</b> MLR without Diffusivity filters, with $O_d=7.4$ . Weighting coefficients $\beta_{ij}$ vs injector-producer distance.....   | 46 |
| <b>Figure 3.8</b> MLR without Diffusivity filters, with $O_d=7.4$ . Comparison between total modeled liquid production rate and the total observed liquid production rate.....   | 46 |
| <b>Figure 3.9</b> MLR without Diffusivity filters, with $O_d=7.4$ . Representation of the positive weighting coefficients $\beta_{ij}$ after the SE-N and SE-P procedure is applied.....                                     | 49 |
| <b>Figure 3.10</b> MLR without Diffusivity filters, with $O_d=7.4$ . Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied..... | 50 |

|   |    |
|---|----|
| <b>Figure 3.11</b> MLR with 1-month diffusivity filters, with $O_d=7.2$ . Representation of the positive weighting coefficients $\beta_{ij}$ .....  | 54 |
| <b>Figure 3.12</b> MLR with 1-month diffusivity filters, with $O_d=7.4$ . Comparison between total modeled liquid production rate and the total observed liquid production rate.....  | 55 |
| <b>Figure 3.13.</b> MLR with 1-month diffusivity filters, with $O_d=7.4$ . Representation of the positive weighting coefficients $\beta_{ij}$ after the SE-N and SE-P procedure is applied.....                                   | 57 |
| <b>Figure 3.14</b> MLR with 1-month diffusivity filters, with $O_d=7.4$ . Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied..... | 58 |
| <b>Figure 3.15.</b> MLR with 6-month diffusivity filters, with $O_d=6.2$ Representation of the weighting coefficients $\beta_{ij}$ .....  | 59 |
| <b>Figure 3.16</b> MLR with 6-month diffusivity filters, with $O_d=6.2$ . Comparison between total modeled liquid production rate and the total observed liquid production rate.....  | 60 |
| <b>Figure 3.17</b> MLR with 6-month diffusivity filters, with $O_d=6.2$ . Representation of the positive weighting coefficients $\beta_{ij}$ after the SE-N and SE-P procedure is applied.....                                    | 62 |
| <b>Figure 3.18</b> MLR with 6-month diffusivity filters, with $O_d=6.2$ . Comparison between modeled liquid production rate and the observed liquid production rate after the SE-N and SE-P procedure is applied.....             | 63 |
| <b>Figure 4.1</b> Representation of the significant $b$ values.....   | 70 |
| <b>Figure 4.2</b> Representation of weighting coefficients $\beta_{ij}$ .....   | 74 |
| <b>Figure 4.3</b> Log Normal Distribution of $b$ values.....  | 76 |
| <b>Figure 4.4</b> Showing a weak relationship between the Log Normal $b$ values and distance.....   | 77 |
| <b>Figure 4.5</b> MLR with significant well pairs after hypothesis thesis test, with $O_d=7.2$ . .....  | 78 |

**Figure A.1** Weighting coefficients  $\beta_{ij}$  for Little Creek Field. MLR with 12-month diffusivity filters, with  $O_d=5$ . Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients.....95

**Figure A.2** Comparison between total modeled liquid production rate and the total observed liquid production rate for the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .....97

**Figure A.3** Representation of the positive weighting coefficients  $\beta_{ij}$  after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .....99

**Figure A.4** Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .....99

**Figure B.1** showing a weak relationship between the average oil production for each producer and summed  $b$  values for each producer.....101

**Figure B.2** showing a weak relationship between the log normal  $b$  values for each producer and the Dykstra Parsons coefficient for each injector-producer well pair.....103

**Figure B.3** showing the relationship between the log normal beta values for each producer and the Lorenz Coefficient for each injector-producer well pair.....105

## ABSTRACT

The understanding of geological characteristics and heterogeneity of a reservoir enables better decisions for reservoir development. Statistical methods use universally available injection and production rate data to help evaluate reservoir characteristics and behavior.

In this research project, statistical methods typically used to infer communication between injector-producer well pairs in a waterflood reservoir using only production and injection rate data are applied to a CO<sub>2</sub> flood. The multivariate linear regression (MLR) technique computes weighting coefficients possibly related to the fraction of the flow in a producer that comes from each of the injectors (Albertoni and Lake, 2002). MLR was applied to the Phase 2 portion of the Little Creek field, Mississippi CO<sub>2</sub> flood. Albertoni and Lake use “diffusivity filters” to model the time lag and attenuation between the stimulus (injection) and the response (production), and further modify the model by successive elimination of negative weighting coefficients (SEN) and successive elimination of positive coefficients larger than 1 (SEP). Diffusivity filters do not improve the results for the Little Creek Field. The statistical implications of the SEN and SEP procedures were compared with a less complex simple linear model (SLM) which eliminates the need to make *ad hoc* assumptions.

A statistical hypothesis test (*P*-Value test) was carried out to determine the significance of each injector-producer well pair relationship. Well pairs with non-significant relationships are then eliminated from the model. This avoids making statistically questionable assumptions to eliminate injector-producer well pairs with connection strengths (i.e., connections not in the range [0,1]). Recommendations to

improve sweep were made using results from the Simple Linear Model with the application of the statistical significance test. Suggestions for future work are also presented.

## CHAPTER 1: INTRODUCTION

Predicting the amount of oil and gas that will be recovered from a reservoir is an important task to solve as a petroleum engineer. The process of reservoir characterization is critical to this process and requires information from various data sources such as core analysis, construction of reservoir models, 3D seismic interpretation, geology, well logging, well and fluid testing, identification of reserve growth potential and many others to improve strategies for the development of early and mature fields.

For an effective reservoir development plan, a sound reservoir description and a better knowledge and understanding of how the field was and is still being operated is essential. The acquisition and processing of some of this information throughout the life of the reservoir is expensive and, in many cases the information required is unavailable. The resources for building and using various modeling methods such as numerical simulation and the lack of important information make the process of reservoir characterization difficult.

Methods using production data have been developed to determine the recovery of a field undergoing CO<sub>2</sub> or waterflooding. Recorded monthly production and injection rates are the most accessible data in any field. The analysis of production data is being used to determine reservoir characteristics, completion effectiveness and hydrocarbons-in-place. More frequently, injection and production rates along with reservoir description and characterization are used to qualitatively determine the influence of injectors on producers in a field. Plots showing cumulative injection rates, cumulative production rates, water-oil ratios, gas-oil ratios, total production and oil recoveries, all usually as a function of time are a few of the plots used to better understand waterflood or CO<sub>2</sub> flood

performance.

In an enhanced oil recovery or secondary recovery system where the production rates of individual wells are affected by injection rates in that system, an understanding of the interwell communication would maximize the performance of an existing flood. Various production and recovery analysis methods have been used to better understand flood performance, but these techniques do not use the production and injection rate data to quantitatively determine injector-producer well pair connectivity. In recent years, the quest to quantitatively assess the relationship between injectors and producers to better understand sweep efficiency has seen increasing interest. Large sets of production and injection rate data of the various wells are required to evaluate the influence each injector has on each producer. Various statistical approaches have been used to ascertain the physical relationship between injector-producer well pairs. These methods are not nearly as costly as the sophisticated models that are typically used for reservoir engineering in the oil industry. The knowledge gained from these statistical approaches can yield improved operating practices to improve oil recovery in active CO<sub>2</sub> floods, and to form strategies for implementing new CO<sub>2</sub> floods. Although statistical approaches have been used to infer the relationships between injector-producer well pairs, there is nothing that says these statistical approaches are the most accurate way to do so. The intent in these cases is to apply statistical approaches which are generally used to infer relationships between dependent and independent variables to a reservoir system where the available injection rate data is the dependent variable and the production rate data is the independent variable. Therefore, the use of these statistical approaches to attempt to infer possible relationships between injector-producer well pairs would be a much faster and

less costly method to help in the evaluation of the effectiveness of the displacement process of a system since no additional testing or lengthy characterization process would be necessary.

In this research project, the application of statistical methods to infer communication between injector-producer well pairs in a CO<sub>2</sub> flood using only production and injection rate data is presented. The remainder of this first chapter presents the main objectives of this research project and provides a review of previous work related to this topic. The second chapter presents a mathematical review of the two approaches presented by Albertoni (2002) for use in waterfloods. Chapter 2 also describes the use of diffusivity filters, the assumptions used when applying this method and the possible sources of error that may occur. The concept and effects of the overdetermination coefficient, which indicates how the number of effective data points included in the linear regression calculation process affects the quality of the results along with the statistical implications associated with the implementation of the successive elimination of negative weighting coefficients (SEN) and the successive elimination of positive coefficients larger than 1 (SEP) procedures to the multivariate linear regression (MLR) method with and without diffusivity filters are also discussed in Chapter 2. Chapter 3 shows the application of these approaches to the Phase 2 portion of the Little Creek Field Mississippi. Chapter 4 introduces an alternative statistical method with fewer ad hoc assumptions that can be used to infer the connectivity between injector-producer pairs. Chapter 5 discusses the inflow performance relationships for the selected wells in Phase 2. Finally, Chapter 6 presents conclusions and recommendations for future work.

## **1.1 OBJECTIVE**

The objective of the project is to present and apply a method for evaluating the interwell connectivity of Denbury Resources Little Creek Field in Mississippi using linear regression analysis. Results will provide a better understanding of the flow of fluid in the reservoir, which may in turn lead to recommendations to improve operating practices. These insights may help improve oil recovery in other active CO<sub>2</sub> floods and form strategies for monitoring new CO<sub>2</sub> floods.

## **1.2 LITERATURE REVIEW**

Several methods have been developed to evaluate the rate performance of an existing well with that of the surrounding injectors. The Spearman rank correlation method tests the relationship between variables regardless of the shape of the populations from which the samples are drawn has been used in recent years to infer relationships between injector-producer wellpairs. Heffer et al. (1995) showed that these correlations are somewhat related to the local orientation of horizontal stresses in a waterflood. According to Heffer et al. (1995), the signals passed between wells which were measured by the rank correlations had at least some component coupled to geomechanics. Heffer et al (1995) used coupled geomechanical-fluid flow numerical modeling to simulate reservoir behavior and suggested that it could be used as a predictive tool for planning and managing waterfloods and determining optimal locations for infill drilling.

Refunjol (1996) utilized the Spearman rank correlation analysis to determine preferential flow trends in a reservoir. Refunjol (1996) correlated the ranks of time series of injection and production rates from pairs composed of each injection well and all adjacent producers. Different time lags were used to find the extreme rank correlation

coefficient values of all injectors and then utilized these values to infer preferential flow directions. This was done by grouping the injector-producer well pairs based on their Spearman rank correlation, then constructing a histogram based on the orientation of well pairs with maximum correlation. The technique was applied to a field and the Spearman rank correlation coefficient, tracer response and reservoir geology were used to determine preferential flow trends in a reservoir. Refunjol (1996) concluded that the Spearman rank correlation coefficient technique was successfully applied but recommended that for future work, a linear model that related each production well to all the other injectors and the other producers be implemented.

Panda and Chopra (1998) introduced an integrated approach to determine injector-producer interaction. In their work a multi-variate data set consisting of production, injection, sand/shale, and well location information was first generated, then an artificial neural network (ANN) was trained to estimate the interaction between injector-producer well pairs. The method was successfully applied to numerical simulations of a waterflood and then the well interactions between different injection and production wells in a heterogeneous permeability field was computed. Panda and Chopra (1998) suggest that the application of their method to a stochastically generated synthetic permeability field, “which is a realistic replication of an actual field” indicates that true field data can be used to determine interaction between wells which in turn can indicate the presence of sealing faults, pinchouts and their zones assisting operators to decide the placement of infill wells, redesign fluid injection schemes and targeting remaining floodable oil.

DeSant’ Anna Pizarro (1998) used the Spearman rank coefficient to estimate

autocorrelation using production and injection rates and validated this technique with numerical simulations. The results attained from this analysis provided more insight to the advantages of the Spearman rank technique as well as its limitations such as how dependent the results are on reservoir parameters.

Soeriawinata and Kelkar (1999) utilized the Spearman rank to relate injectors to their adjacent producers. According to Soeriawinata and Kelkar (1999), the basic assumption in this analysis is that if good communication exists between two injectors and a producer, the cross correlation of the water injection rates to the liquid production rates is higher than the cross correlation of each single injector. Soeriawinata and Kelkar (1999) also accounted for the superposition effect in the reservoir caused by the influence of multiple injection wells on a producing well. According to Soeriawinata and Kelkar (1999), the fact that there are a number of injectors and producers operating simultaneously causes both superposition and noise, which must be accounted for in analyzing the data. The drawback to this method is that “it is practically impossible to distinguish whether a slight increase in cross correlation is due to superposition effect or due to noise in field rate data” (Soeriawinata and Kelkar, 1999). However, “if there is a significant jump in cross correlation value when the rate from an injector is added, it is most probably caused by constructive interference in injection rates rather than caused by noise” (Soeriawinata and Kelkar, 1999). This method was applied to a mature waterflood and strong connectivities and potential barriers in the field were identified.

A method that calculates the fraction of the flow in a producer was developed by Albertoni (2002). Albertoni (2002) presented two different statistical methods used to evaluate the connectivity between injectors and producers in a waterflood. These were

called the Multivariate Linear Regression (MLR) and the Balanced Multivariate Linear Regression (BMLR) method. Albertoni (2002) views the reservoir as a system that processes a stimulus (injection) and returns a response (production). The methods presented by Albertoni use liquid (water and oil) production and injection rates in reservoir volumes of every well in a waterflood system. Albertoni (2002) suggests that gas rates should be disregarded in the analysis; periods with no significant free gas production must be selected for the analysis. The reservoir effect on the input signal (injection) and the output signal (production) is dependent on the location and the orientation of each injector-producer pair in that system. This technique uses different statistical methods based on constrained multivariate linear regression to quantitatively determine the communication between wells in the system. Diffusivity filters are also used in this method to account for the time lag and attenuation that occurs between the injector and producer. The methods were applied to a synthetic field generated by numerical simulation with five-spot injection patterns and also to a waterflood in Argentina. Results were very difficult to validate, but seemed to agree with the presence of known geological features (Albertoni, 2002).

Dinh (2003) used the multivariate linear regression techniques along with the diffusivity filter concept suggested by Albertoni (2002) to quantify communication between well pairs in a reservoir using injection and production rate data. The method was tested on a synthetic reservoir model using the BOAST98 numerical simulator and then was applied to a waterflood field in northeastern Oklahoma. The results obtained by Dinh (2003) from the tested reservoir models reflected the characteristics of anisotropy, vertical heterogeneity, sealing faults and flow channels. Dinh (2003) applied the

MLR approach with and without diffusivity filters to the field data and found that the application of the MLR approach without diffusivity filters yielded better results than the other cases. Dinh (2003) concluded that for a media with small dissipation, a short diffusivity filter function should be considered and further analysis to determine how frequently the flowrates being used in these type of models should be measured.

Dinh and Tiab (2007) used the MLR approach introduced by (Albertoni and Lake, 2002) to determine the interwell connectivity between injector and producer well-pairs in a waterflood system using bottom-hole pressures of injectors and producers. They suggested that the use of bottom-hole pressures eliminated the need to apply diffusivity filters to flowrate data to account for the time lag and attenuation that occurred between injector-producer well pairs, making this approach a much simpler method to infer injector-producer connectivity. Dinh and Tiab (2007) also stated that for their method, an overdetermination factor (which indicates how the number of effective data points included in the linear calculation process affects the quality of the results) greater than 1 can be used to obtain good results. This suggested that in comparison to past studies conducted by Albertoni (2002) and Dinh (2003) with numerous flowrate data, minimal data can be used to achieve good results for this method. This also eliminated the need to include shut-in-periods which are usually “unavoidable when a large number of data points are used, creating significant errors in the analysis” (Dinh and Tiab, 2007). This new approach was compared to the same case studied by Dinh (2003) applying the MLR approach without diffusivity filters to flowrate data for the same field. Dinh and Tiab (2007) concluded from their results that better results were obtained with the implementation of the new approach in comparison to the previous case.

Dinh and Tiab (2008) conducted an extended study of the new approach they introduced in 2007 using bottom hole pressure data of injectors and producers to determine interwell connectivity. The new method introduced by Dinh and Tiab (2007) included constraints such as constant production rates and constant injection rates. Dinh and Tiab (2008) noted that the constraints placed on the new approach made it difficult to apply the technique to a real field study where production rates vary and are hardly kept constant. Dinh and Tiab (2008) analyzed systems with constant injection rates and varying production rates. According to Dinh and Tiab (2008), results from this analysis proved to be almost exactly similar to the case with constant production rates and varying injection rates. Dinh and Tiab (2008) concluded “signal wells could either be producers or injectors.” Dinh and Tiab (2008) also concluded that the response wells could either be flowing or shut-in”. Dinh and Tiab (2008) suggested that further investigations on characteristics of interwell relative permeability and the effect of interwell flow on the interwell permeability should be conducted. Also, the extension of the study to include wells with different wellbore conditions such as horizontal wells and hydraulic fractured wells was suggested.

Sayarpour et al. (2008) used a capacitance resistive model (CRM) to provide further knowledge about waterflood performance. According to Sayarpour et al. (2008), “unlike conventional analytical tools, the CRM can rapidly attain a performance match without having to build an independent geologic model”. The “estimation of the fraction of injected water directed from an injector to various producers and the time taken for an injection signal to reach a producer are the key elements in performance assessment” (Sayarpour, et al. 2008). Sayarpour, et al (2008) conducted four case studies using CRMs

in complex reservoirs to show CRMs ability to “determine connectivity between injector-producer well pairs and to understand flood efficiencies for the entire or a portion of a field.” Sayarpour, et al. (2008) concluded that “the rapid history matching capability of the capacitance resistive model would serve as a great tool for any grid based modeling study.”

The goal of this study is to determine the connectivity between injector-producer well pairs in the Phase 2 portion of Denbury Resources Little Creek Field CO<sub>2</sub> flood. The field data provided by Denbury Resources is limited to injection and production rates. The application of statistical approaches to the Little Creek Field data would enable us to quantitatively determine the connectivity between injector-producer well pairs in the system without having to use simulation models which require information from other data sources which are unavailable at Little Creek. As discussed earlier, the Spearman rank correlation method has been used to relate pairs of wells, each pair consisting of an injector and a producer. In comparison to the Spearman rank correlation, the MLR and BMLR statistical approaches introduced by Albertoni (2002) allow for the quantitative determination of the communication between wells in a waterflood in a single step. This should be a faster way to determine interwell connectivity especially in a large field with numerous injectors and producers such as the Little Creek Field in comparison to the Spearman rank model which would require analyzing the data for each well pair. Also, the application of diffusivity filters may be able to account for any time lag and attenuation that occurs between injector and producer well pairs in the system. With that being said, the statistical approach introduced by Albertoni (2002) will be applied to the Little Creek Field.

## **CHAPTER 2: STATISTICAL ANALYSIS PROCEDURE**

Knowledge of the relationship between injector-producer well pairs and an estimate of the fraction of flow caused by each injector in a producer would enable a better understanding of the sweep efficiency of a field undergoing waterflood or CO<sub>2</sub> injection. This information could allow for suggestions to be made for operational changes that might be made to optimize recovery such as flood pattern changes, recompletion of wells and drilling of infill wells.

Various statistical approaches have been used to infer a physical relationship between injector-producer well pairs in a waterflood or CO<sub>2</sub> flood as discussed earlier. Two statistical techniques presented by Albertoni (2002) which were used to assess the connectivity between injectors and producers in a waterflood are reviewed in this chapter. These techniques were called the Multivariate Linear Regression (MLR) and the Balanced Multivariate Linear Regression (BMLR) method. Albertoni (2002) also defined diffusivity filters which were used to account for the potential time lag and attenuation of changes that occur between the injectors and producers. These will also be discussed as will the statistical implications of the suggestions made by Albertoni and Lake (2002) to improve the results obtained from the application of the MLR method with and without diffusivity filters.

### **2.1. MULTIVARIATE LINEAR REGRESSION (MLR)**

According to Sachs (1984) multivariate analysis is the development of general mathematical models for analyzing multiple dependent variables. The MLR method relates the variations in a response variable to the variations of hypothesized predictors.

Parameters in the model are estimated and relations among the variables are determined. An important assumption of the MLR method is that the predictor variables are linearly independent, i.e. no linear relationship exists between the predictor variables. If the predictor variables covary, the model may produce spurious results.

A field undergoing a flood has multiple injectors and producers acting at the same time. Applying the multivariate linear regression model to a flood, the liquid production rate of a well will be the dependent variable while the injection rates for every injector in the field are modeled as independent variables. Because the model assumes that no linear relationship exists between active injectors in the system, injector rate variations should only influence the production rates values in the system and not the other injection rates. For fields where the injection and production rates are balanced (total injection and production rates are equal), this assumption seems reasonable. For unbalanced systems, this assumption is questionable. Once all the parameters are determined, the model quantifies how each injector influences each producer.

### 2.1.1. Mathematical Development

Albertoni (2002), suggests that a linear model can be expressed as follows

$$y = \beta_o + \sum_{K=1}^K \beta_k x_k + \varepsilon \quad (2.1)$$

where  $y$  is the dependent variable,  $x_k$  are the independent variables, and  $\varepsilon$  is a random error term used to account for imbalances, measurement and fitting errors in the model.

According to Lake et al. (1997), the error term is assumed to be normally distributed with a zero mean ( $E(\varepsilon) = 0$ ). The  $\beta_o$  and the  $\beta_k$  terms are the coefficients to be determined

by regression.

To use the MLR method to estimate the production rate of a producer well  $j$ , Equation 2.1 can be written as

$$\hat{q}_j(t) = \beta_{oj} + \sum_{i=1}^I \beta_{ij} i_i(t) \quad (j = 1, 2 \dots N) \quad (2.2)$$

where  $N$  is the total number of producers and  $I$  is the total number of injectors. This equation states that for any given time period, the total production rate of well  $j$  ( $\hat{q}_j(t)$ ) is a linear combination of the rates of every injector in the field ( $\hat{i}_i(t)$ ) plus a constant  $\beta_{oj}$  term. The  $\beta_{oj}$  term is a constant that tries to account for the unbalance in the field. This unbalance will include liquid production not associated with injected fluid (primary production), as well as injection losses (injection not affecting producers). If  $\sum_{j=1}^N \beta_{oj} = 0$  then the total field is balanced. Using field data, Equation 2.2 suggests that injection rate changes in the model cause instantaneous production rate changes which would imply steady state flow in the reservoir. In many cases, there are time lags and attenuation that occur as fluid flows from injectors to producers. Diffusivity filters which are further discussed in Section 2.3 were proposed to account for the time lag and attenuation in the system. The  $\beta_{ij}$ 's are the weighting coefficient terms. The  $\beta_{ij}$  terms represent the effect each injector  $i$  has on each producer  $j$ . Thus, the larger  $\beta_{ij}$ , the greater the effect. If the injection rate and production rates are given, the constant term  $\beta_{oj}$  and the weighting factors  $\beta_{ij}$  can be estimated. Using the given production and injection rates, the variance in the production rate can be determined as the difference between the observed production rate and the modeled production rate calculated from

$$\sigma_{MLRj}^2 = \text{Var} \left( \hat{q}_{j-} - q_j \right) = E \left[ \left( \hat{q}_{j-} - q_j \right)^2 \right] \quad (2.3)$$

The constant term  $\beta_{oj}$  and the weighting parameter  $\beta_{ij}$  can be determined by minimizing this variance. This will lead to the following set of linear equations

$$\begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1I}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2I}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{I,1}^2 & \sigma_{I2}^2 & \cdots & \sigma_{II}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_{1j} \\ \beta_{2j} \\ \vdots \\ \beta_{Ij} \end{pmatrix} = \begin{pmatrix} \sigma_{1j}^2 \\ \sigma_{2j}^2 \\ \vdots \\ \sigma_{Ij}^2 \end{pmatrix} \quad (2.4)$$

The left hand side square matrix in Equation 2.4, is the injector-injector covariance matrix and the right hand side vector terms are the covariance values between the injectors and each producer. There are then  $N$  equations of the form in Eqn. 2.4; one for each producer in the system. The weighting coefficients  $\beta_{ij}$  can be determined by standard linear solution methods. After the matrix is solved for the  $\beta_{ij}$  terms, the constant term  $\beta_{oj}$  can be determined by

$$\beta_{oj} = \bar{q}_j - \sum_{i=1}^I \beta_{ij} \bar{i}_i \quad (2.5)$$

The over bar symbol in Equation 2.5 represent mean values. A set of  $I+1$  equations and  $I+1$  unknowns must be solved for each producer in the system.

After the parameters in the model are estimated and relations among the variables are determined, the modeled production rate can be compared to the actual production rate using a coefficient of determination value ( $R^2$ ). The coefficient of determination

represents how accurately two data populations are correlated (Albertoni, 2002).

Therefore, this measures the quality of the correlation between modeled and observed production for the case under study. It is defined as

$$R^2 = 1 - \frac{\sum_{m=1}^M \left( \hat{q}_j^{(m)} - q_j^{(m)} \right)^2}{\sum_{m=1}^M \left( q_j^{(m)} - \bar{q}_j \right)^2} \quad (2.6)$$

where  $m$  is the number of data values. For this thesis,  $m$  is the number of months of data.

The overdetermination coefficient ( $O_d$ ) introduced by Albertoni (2002) is defined as “the number of effective data points divided by the number of unknowns” (Albertoni and Lake, 2002), or

$$O_d = \frac{M_e}{(I+1)} \quad (2.7)$$

where  $I$  represents the number of injectors and  $M_e$  is the total number of effective data points. The  $O_d$  coefficient indicates how the number of effective data points included in the linear regression calculation process affects the quality of the results. For this case, the use of more data points (more rate data) and fewer injectors in the analysis represents a larger overdetermination of the problem. According to Albertoni (2002), greater overdetermination leads to more reliable results. Albertoni (2002) suggests that good results can be obtained with overdetermination coefficients larger than 6. An overdetermination coefficient smaller than one indicates that the system is underdetermined, with the number of observations less than the number of unknown coefficients.

### 2.1.2 Uses of MLR

A field which has undergone a flood is considered to be unbalanced when the total fluid injection rate in the field is significantly different from fieldwide liquid production rate (at reservoir conditions). The possible unbalance in the field is accounted for by the  $\beta_{oj}$  term in the MLR model so Albertoni (2002) suggests that the MLR approach be used to quantitatively estimate the communication between injector-producer well pairs. Albertoni (2002) also suggests that when analyzing a section of a flood, the MLR method must be used without making any changes to the injection rates to account for the imbalance caused by flow across the open boundaries of the section of the flood being analyzed; but, “since the water rate crossing the boundaries may not be actually constant, the production wells close to the boundaries may suffer some boundary effects” (Albertoni, 2002).

## 2.2. BALANCED MULTIVARIATE LINEAR REGRESSION (BMLR)

### 2.2.1 Balanced Condition

If the sum of the injection rates in a field is approximately equal to the sum of the production rates in the field, the flood is said to be balanced. In this case, Albertoni (2002) suggests that the BMLR approach should be used. Therefore the constant term  $\beta_{oj}$  in Eq 2.2 will be zero. The production rate for a well  $j$  in this model can be defined as

$$\hat{q}_j(t) = \sum_{i=1}^I \lambda_{ij} i_i(t) \quad (j= 1, 2 \dots N) \quad (2.8)$$

This equation states that the production rate can be modeled as a linear combination of

the injection rate values. The weighting coefficient terms  $\lambda_{ij}$  account for the imbalance within the system. In the BMLR model, the coefficient terms  $\lambda_{ij}$  replace the coefficient terms  $\beta_{ij}$  used in the MLR model to differentiate between the balanced and the unbalanced models. The  $\lambda_{ij}$  terms and the  $\beta_{ij}$  terms mean the same thing with the exception of the  $\beta_{oj}$  term.

The balanced condition is given by

$$\sum_{j=1}^j q_j = \sum_{j=1}^j \sum_{i=1}^I \lambda_{ij} \bar{i}_i = \sum_{i=1}^I i_i \quad (2.9)$$

This condition suggests that the average liquid production rate is a linear combination of the average injection rates. Therefore liquid production rates are a result of the average fluid rates injected at the injectors. If the  $\beta_{oj}$  term is zero, Equation 2.5 is identical to Equation 2.9

### 2.2.2 Mathematical Development

To determine the weighting coefficients in the Balanced Multivariate Linear Regression model, the squared error of the predicted value must be minimized.

$$\sigma_{AMLRj}^2 = E \left[ \left( \hat{q}_j - q_j \right)^2 \right] \quad (2.10)$$

The combination of Equations 2.8 and 2.9 with the minimization of the variance in Equation 2.10 leads to the matrix system of linear equations for the BMLR method shown below

$$\begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1I}^2 & \bar{i}_1 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2I}^2 & \bar{i}_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{I,1}^2 & \sigma_{I2}^2 & \cdots & \sigma_{II}^2 & \bar{i}_1 \\ \bar{i}_1 & \bar{i}_2 & \cdots & \bar{i}_1 & 0 \end{pmatrix} \bullet \begin{pmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \vdots \\ \lambda_{Ij} \\ \mu_j \end{pmatrix} = \begin{pmatrix} \sigma_{1j}^2 \\ \sigma_{2j}^2 \\ \vdots \\ \sigma_{Ij}^2 \\ \bar{q}_j \end{pmatrix} \quad (2.11)$$

The  $\mu_j$  term in the matrix is the Lagrange multiplier used in the derivation process to account for the predicted rate for each producer which must be equal to the average production rate. The  $\lambda_{ij}$  and  $\mu_j$  can be determined from the set of  $I + I$  linear equations and  $I+I$  unknowns in Equation (2.11) using any matrix solving method. The weighting coefficients  $\lambda_{ij}$  obtained from Equation 2.11 account for the effect of each injector  $i$  on each producer  $j$ .

### 2.2.3 Uses of BMLR

The Balanced Multivariate Linear Regression model is used when the total amount of field-wide injection is about equal to the total amount of production. Thus, the flood is balanced. According to Albertoni (2002), the field production and injection data may show occasional differences between these rates but they must be balanced most times. This model assumes that fluid injected into the field is produced by the producers and there is no fluid flow across boundaries.

## 2.3 DIFFUSIVITY FILTERS

In the case of a field undergoing a flood, the flow regimes change continuously from transient flow to flow patterns that approximate steady state flow. So, the assumed steady state flow model from the application of the MLR statistical approach to the

provided production and injection rate data is used for simplicity and diffusivity filters were proposed to correct for the effect of transient flow.

Diffusivity filters are used to account for the time lag and attenuation that occurs between the stimulus (injection) and the response (production) (Albertoni, 2002). The filters transform the injection rates of an injector  $i$  affecting a producer  $j$  for an injector-producer pair so that they take the form of an equivalent injection rate acting in an incompressible medium, which results in an effective injection rate at a certain time. In cases where there are large distances between injector and producer pairs and large dissipation in the medium, the use of diffusivity filters becomes very important.

The diffusivity constant is defined by

$$\eta = \frac{k}{\phi\mu c_t} \quad (2.12)$$

Dissipation is the reciprocal of the diffusivity constant

$$d = \frac{1}{\eta} = \frac{\phi\mu c_t}{k} \quad (2.13)$$

The dissipation definition above suggests that a large dissipation would exist in a system with a small permeability, a large porosity, viscosity, and total compressibility. If dissipation did not exist in the reservoir, a change in the rate of injection for an injector  $i$ , would cause a corresponding and immediate change in the rate of production for a producer  $j$ . That change would be independent of the distances between injectors and producers. A time lag and attenuation of the signal between the injector and producer exists in most reservoirs. In comparison to a less dissipated reservoir, a dissipated reservoir should experience a more significant time lag between the time at which a change in injection rate occurs and the time at which a corresponding change in the

production rate is observed. According to Albertoni (2002), when diffusivity filters are applied to the MLR and BMLR procedures, the diffusivity filters are applied only to injection rates.

### 2.3.1 Mathematical Development

Albertoni (2002) states that “the pressure change ( $\Delta P$ ) at any point of an infinite, homogeneous reservoir with constant thickness, caused by a change in an injection rate ( $\Delta i$ ), can be expressed as” (Albertoni, 2002):

$$\Delta P = C_1 \times E_i \left( -d \frac{r^2}{t} \right) \quad (2.14)$$

The  $C_1$  term in Equation 2.14 is a constant,  $E_i$  is the exponential integral function,  $t$  is time,  $r$  is the distance from the point to the injection well and  $d$  which was defined previously in Equation 2.13 is the dissipation of the medium. Albertoni (2002) states that when the superposition principle is applied, the change in pressure caused by an impulse over a unit time can be expressed as:

$$\Delta P = \begin{cases} C_1 \times E_i \left( -d \frac{r^2}{t} \right) & t \leq 1 \\ C_1 \times \left[ E_i \left( -d \frac{r^2}{t} \right) - E_i \left( -d \frac{r^2}{(t-1)} \right) \right] & t > 1 \end{cases} \quad (2.15)$$

Consider a linear model of the form

$$q = J \times \left( \bar{P} - P_{wf} \right) \quad (2.16)$$

for the production rate of a well at a distance  $r$  from an injector. A combination of

Equations 2.15 and 2.16 results in a change in production rate caused by a unit injection impulse which can be written as

$$\Delta q = \begin{cases} C_2 \times E_i \left( -d \frac{r^2}{t} \right) & t \leq 1 \\ C_2 \times \left[ E_i \left( -d \frac{r^2}{t} \right) - E_i \left( -d \frac{r^2}{(t-1)} \right) \right] & t > 1 \end{cases} \quad (2.17)$$

The  $C_2$  term in Equation 2.17 is a new proportionality constant, which is unknown. Equation 2.17 is referred to as a continuous filter function. The injection rate of an injector  $i$  is the sum of different injection impulses caused by previous rates. According to Albertoni (2002) “This equation is only used to correct the proposed steady state models for transient effects” (Albertoni, 2002). Equation 2.17 can be used to generate a filter function that can be used to determine the production rate of a producer  $j$  at any given distance  $r$  and time  $t$  given an injection history. For an injector-producer pair, the filter function can be used to change the injection rate of an injector  $i$  so that the response at the producer  $j$  is equal to the response of that same producer  $j$  in an incompressible medium. The linear regression model approaches presented previously can then be applied using the new filtered injection rates and the original production rates to determine the weighting coefficients. In order to eliminate the unknown proportionality constant  $C_2$  from the equation, a normalized filter function is defined as:

$$F_n = \frac{\Delta q}{\int_{t=0}^{t=\infty} \Delta q dt} \quad (2.18)$$

The  $C_2$  term then cancels because of the  $\Delta q$  term is in the numerator and denominator.

Albertoni (2002) proposed a discrete filter function of one to twelve months. For a 12 month effect, the 12 normalized filter coefficients of the discrete filter function are :

$$\alpha^{(n)} = \int_{t=n}^{t=n+1} F_n dt = \frac{\int_{t=12}^{t=n+1} \Delta q dt}{\int_{t=0}^{t=12} \Delta q dt} \quad (n=1, 2, \dots, 12) \quad (2.19)$$

The number of filter coefficients can be more or less than the proposed 12 months depending on how large the dissipation is. The normalized diffusivity coefficients  $\alpha^{(n)}$  calculated using Eq. 2.19 does not depend on the  $C_2$  term . According to Albertoni (2002) they are less than or equal to one, and they should sum to a value of one. In a dissipative medium, the sum of the normalized diffusivity coefficients should equal 1. Equation 2.17 suggests that the discrete filter function depends on the distance from the injector,  $r$ , the time,  $t$  and the dissipation,  $d$ .

The convoluted or effective injection rate expressed in terms of diffusivity filters of an injector  $i$  influencing a producer  $j$  at a time  $t$  is expressed as:

$$i_{ij}^c(t) = \sum_{n=0}^{11} \alpha_{ij}^{(n)} i_i(t-n) \quad (2.20)$$

Equation 2.20 implies that the effect injector  $i$  has on producer  $j$  can be determined not only by the current injection rate of injector  $i$  at time  $t$  ( $n=0$ ), but also by the rates of the previous 11 months. For example incorporating diffusivity filters for the MLR method (Eq. 2.2), the modeled production rate in a well  $j$  is given as:

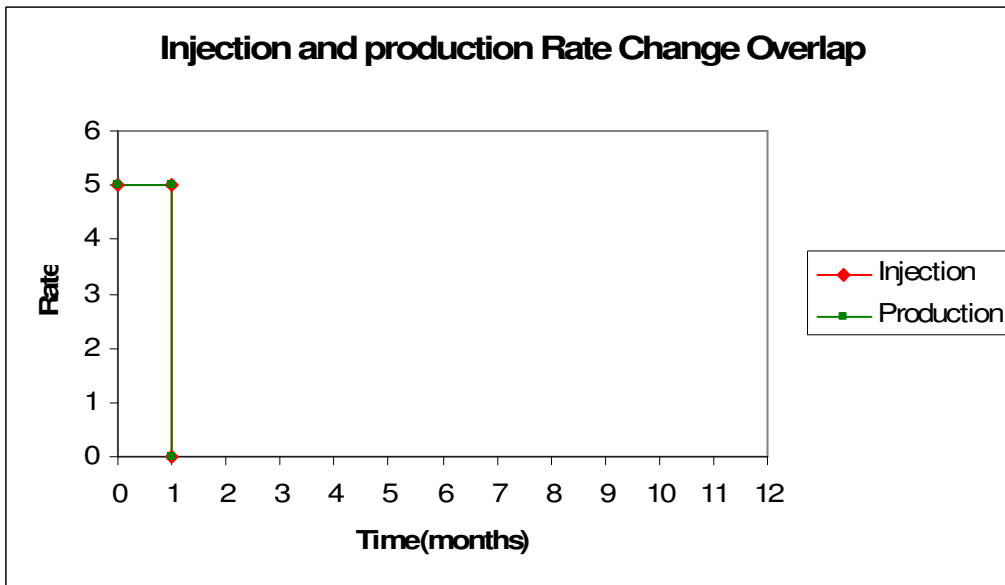
$$\hat{q}_j(t) = \beta_{oj} + \sum_{i=1}^I \beta_{ij} i_{ij}^c(t) \quad (2.21)$$

Also, for the BMLR method (Eq. 2.9), the modeled production rate for a producer  $j$  using a diffusivity filter would be given by:

$$\hat{q}_j(t) = \sum_{i=1}^I \lambda_{ij} i_{ij}^c(t) \quad (2.22)$$

### 2.3.2 Illustrative Examples of the Application of Diffusivity Filters

In the case of a reservoir with no dissipation, a change in injection rate from an injector  $i$  would lead to an instantaneous production rate change from a producer  $j$ . An illustrative example of this is shown in Figure 2.1

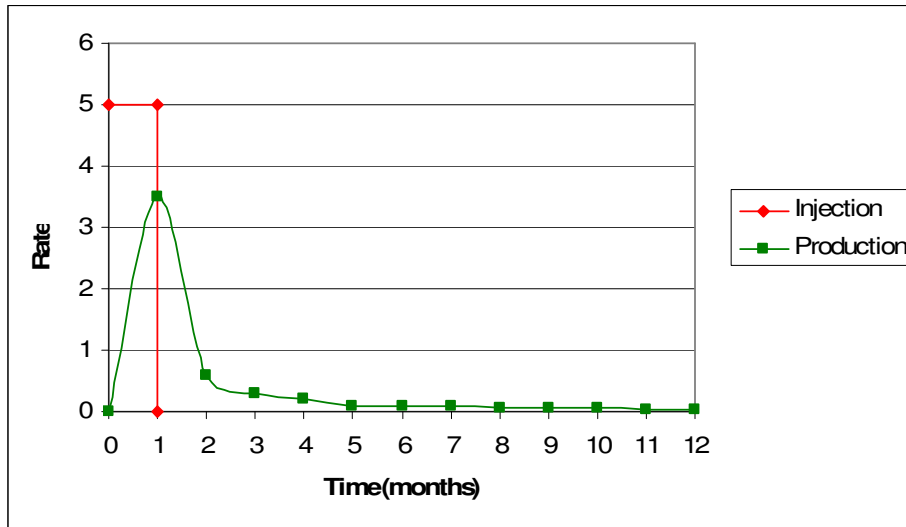


**Figure 2.1** Injection and equivalent production rate change in a reservoir with no dissipation (after Albertoni, 2002).

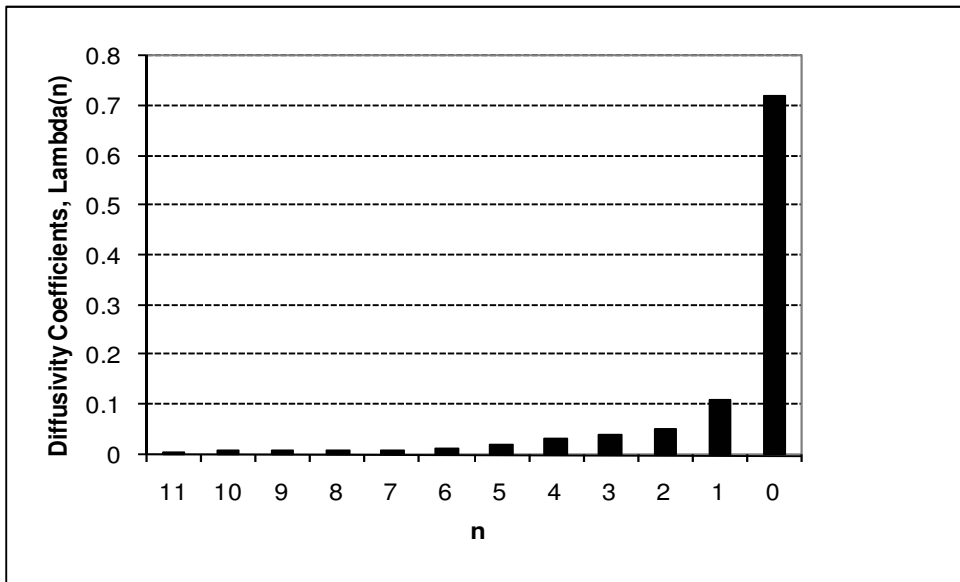
The case where there is some dissipation in a reservoir is shown in Figure 2.2, where a change in injection rate at time zero causes a slight change in the production rate instantaneously. As shown, the effect of this change in injection rate is attenuated over the next 11 months. In the span of 12 months, the areas under the production and injection rate curve are equal. The diffusivity filter functions indicate the fractions of the injection rate change that affect the production rate at time zero.

The diffusivity filter function at time zero  $\alpha^{(0)}$  is not equal to one and the other 11

coefficients are greater than zero. This implies that a change in production is not only influenced by the injection impulse at time zero but also by impulses from the previous 12 months.



**Figure 2.2** The injection rate change and equivalent change in production rate between an injector-producer pair in a reservoir with small dissipation (after Albertoni, 2002).



**Figure 2.3** Corresponding filter function showing large coefficient at time zero, while the rest are much smaller (after Albertoni, 2002).

## 2.4 FURTHER IMPROVEMENT OF THE MLR MODEL

Albertoni (2002) applied the MLR and BMLR approach with and without diffusivity filters to numerically simulated fields. For the first case, a single layered homogeneous reservoir was used. The waterflood was balanced so ideally, the BMLR model was the appropriate approach to be used; but for the sake of comparison, Albertoni applied both methods with and without diffusivity filters to the model. Results from the application of these approaches suggest that only positive relationships existed between injector-producer well pairs in the system. Both approaches generated similar conclusions. The weighting coefficients were larger for near well pairs and smaller for well pairs with more separation.

Albertoni (2002) ran two cases of reservoirs with sealing faults in the simulated model. In the first case, the sealing fault divided the entire reservoir into two separate regions. The BMLR approach with diffusivity filters was used for this model. The results obtained indicated that very small negative relationships existed between well pairs on opposite sides of the fault. According to Albertoni (2002) these results were expected. He suggested that the negative coefficients that were obtained between well pairs on each side of the fault represented the presence of a transmissibility barrier. Albertoni (2002) states that these negative coefficients have no physical interpretation and were considered as zeros, indicating that no communication exists between those well pairs. In the second case, a sealing fault partially crosses the reservoir and did not divide the field into two parts as in the previous model. Negative weighting coefficients were not obtained in this case. However, the magnitude of the weighting coefficients for injector-producer well pairs separated by the fault were very small. Albertoni (2002)

suggested that this implied that there was virtually no connectivity between those wells and inferred that the results represented the fault that was present in the model.

Albertoni then applied the statistical approaches to two waterflooded fields in Argentina: The Chihuido de la Sierra Negra (ChSN) field and the Bloque I field. Albertoni, applied MLR with diffusivity filters since only a portion of the ChSN field was being analyzed, rates were not balanced and open boundaries existed in the model. There were many negative weighting coefficients for this model. Weighting coefficients greater than one were also shown in the results. Albertoni stated that these negative coefficients and coefficients greater than one were to be expected because of the small overdetermination, open boundaries and the change in production/injection conditions present in this case which did not satisfy the assumptions on which the method was based. Albertoni (2002) stated that the negative weighting coefficients “are just statistical results that minimize the error but they have no meaning” and the weighting coefficients greater than one are also unrealistic. The results suggested that three injectors in the model showed very little connectivity with inner producers which Albertoni (2003) found to be in agreement with the presence of an inferred fault. However, results also showed connectivity between these same injectors and other producers in the model. Albertoni suggested that either some error existed in the model due to the boundary effects or due to the small overdetermination factor, or, the fault is not completely sealing.

Excluding the injector-producer well pairs with the  $\beta_{ij}$  coefficients less than 0 and greater than 1 would imply that there is no relationship between those injector-producer well pairs. Therefore, the injector has no effect on the producer. This is a surprising methodology since the elimination of the largest coefficients in a typical

multivariate linear regression model would be eliminating the variables that have the largest effect on the dependent variable (Edwards, 1984).

The results obtained by Albertoni (2002) from the implementation of the MLR approach with and without diffusivity filters to the field data that were analyzed, motivated a new approach by Albertoni to improve models. Albertoni (2002) introduced an approach for eliminating the negative weighting coefficients called the successive elimination of negative weighting coefficients (SEN). First, the most negative  $\beta_{ij}$  weighting coefficient is set to zero which eliminates that well pair from consideration. Next, the regression method is performed again recalculating the entire set of weighting coefficients with one fewer injector-producer well pairs. If there are additional negative weighting coefficients, the new most negative weighting coefficient is set to zero, and the  $\beta_{ij}$  weighting coefficients are again recalculated. This procedure is repeated until no negative coefficients remain (Albertoni, 2002). The SEN procedure was extended by Albertoni (2002). Large positive weighting coefficients (those greater than 1) were also eliminated following the same type of elimination procedure. This process is called the successive elimination of physically non-significant weighting coefficients (SEP).

#### **2.4.1 Statistical Implications associated with the application of SEN and SEP procedure to the MLR model with and without diffusivity filters.**

Albertoni introduced the SEN and SEP procedure to improve the results obtained from the implementation of the MLR model to field data because physical explanations could not be inferred from the negative weighting coefficients and the positive weighting coefficients that were greater than one. It is important to note that the MLR method

is essentially a statistical approach that is being used to infer a physical relationship between injector-producer well pairs. For most practical cases injection and production rates are not completely independent therefore, attributing physical meaning to negative  $\beta_{ij}$  coefficients and positive coefficients greater than one obtained from the MLR approach may not be appropriate. Traditional MLR techniques provide tests to indicate the significance of the weighting factors. In this work these significance tests will be used to evaluate the weighting coefficients and a comparison will be made to the SEN and SEP techniques.

According to Edwards (1984) “if the relationship between two variables  $X$  and  $Y$  is linear, then when the value of  $b$  (the slope) is positive, the relationship is also described as positive; that is, an increase in  $X$  is accompanied by an increase in  $Y$  and a decrease in  $X$  is accompanied by a decrease in  $Y$ ” (Edwards, 1984). When the value of  $b$  is negative, the relationship is also described as negative. Also, according to Edwards (1984) “a negative relationship means that an increase in  $X$  is accompanied by a decrease in  $Y$ , and a decrease in  $X$  is accompanied by an increase in  $Y$ ” (Edwards, 1984). For the MLR method, using similar arguments implies that negative weighting coefficients  $\beta_{ij}$  represent a negative linear relationship that exists between an injector-producer well pair; that is, an increase in injection rate is accompanied by a decrease in production rate and a decrease in injection rate is accompanied by an increase in production rate. Positive weighting coefficients represent a positive linear relationship that exists between an injector-producer well pair; that is, an increase in injection rate is accompanied by an increase in production rate and a decrease in injection rate is accompanied by a decrease in injection rate. In the SEN and SEP procedures, the weighting coefficients with the

largest magnitudes are eliminated first from the model. This method is performed again, eliminating the coefficients with the largest magnitude from the model if the weighting coefficients are either negative or positive and larger than one. Application of the SEN and SEP procedure to the MLR model with and without diffusivity filter would therefore eliminate the well pairs with the largest influence in the model and account only for the well pairs with lower influence.

The primary objective of using the regression techniques is to evaluate the interwell connectivity between injector-producer well pairs which would in turn provide a better understanding of sweep in the reservoir and serve as a guide for suggestions for operational changes which could be made for improving recovery in a waterflood or CO<sub>2</sub> flood. The elimination of injector-producer well pairs with the largest influence on the predicted production rate is counter-intuitive to this objective.

According to Albertoni (2002), results obtained after the SEN procedure showed that 45% of the weighting coefficients had been set to zero. After the SEN procedure, not only were the negative coefficients set to zero but some of the large positive weighting coefficients became smaller. Even though these large coefficients had become smaller, they were still larger than one and Albertoni considered this to be incorrect. The  $R^2$  value for this model also decreased after this application.

Next, Albertoni applied the MLR technique with diffusivity filters to the Bloque I field located in southern Argentina. The results obtained from this analysis suggested that 47% of the weights were negative. There were also large positive weighting coefficients greater than one. Results obtained from the application of the SEP procedure showed a decrease in the  $R^2$  value after the application of the SEP procedure. Albertoni (2002)

concluded that “this procedure is non-unique but physically more significant.” He also concludes, “better physical results were obtained at the expense of poorer statistical results.” It is debatable whether, this procedure is “physically more significant” rather than conceptually more satisfying.

Dinh (2003) applied the MLR and ABMLR approach with and without diffusivity filters to a synthetic reservoir using the Boast98 numerical simulator. Several cases were studied by Dinh (2003) using this model. For the first case study, the MLR and ABMLR techniques were applied to an ideal homogeneous reservoir with zero compressibility. Results showed both cases to have an  $R^2$  value of one. For the next case, compressibility factors were added to the homogeneous reservoir. Since the compressibility was no longer zero, this indicates that some dissipation between well pairs should be expected and the application of diffusivity filters to the model was needed. The results for the MLR case with and without diffusivity filters looked similar but the coefficients were generally larger than those obtained without filtering. Here the  $R^2$  value increased when the diffusivity filters were applied. The BMLR approach was then applied to the same data from the simulation. Here also the results were better for the case with diffusivity filters than the case without filters. The  $R^2$  value increased after the application on the diffusivity filters. Only positive weighting coefficients were obtained in this work.

Similar to Albertoni (2002), Dinh (2003) also analyzed two cases of reservoirs with transmissibility barriers simulated in the synthetic field. In the first case the transmissibility barrier divided the entire reservoir into two separate regions. In the second case, the transmissibility barrier only partially divided the field. The MLR

approach with diffusivity filters was used for this analysis. Dinh (2003) concluded that the results for the first case showed the presence of a transmissibility barrier. According to Dinh (2003) negative weighting coefficients between injector-producer well pairs indicated the non-connectivity across the transmissibility barrier. Dinh (2003) also concluded that the results obtained for the second case also indicated the presence of the transmissibility barrier. Similar to Albertoni's (2002) results for a similar case, the results obtained did not show any negative relationships between the injector-producer well pairs located across the boundary but the weighting coefficients of those well pairs were very small.

Dinh (2003) used actual field data to analyze the Delaware-Childers field. The MLR approach with 12-month, 6-month, 3-month, 2-month and no diffusivity filters and the SEP method was used in this analysis. Dinh (2003) concluded that applying the SEP procedure to every case eliminated the negative coefficients and positive coefficients larger than one which were considered physically meaningless. Dinh (2003) concluded that the results of the application of the MLR approach with and without diffusivity filters showed that better results were obtained for the MLR case without filters. The MLR case without filters had a higher  $O_d$  and  $R^2$  value in comparison to the other models.

## **2.5 ASSUMPTIONS OF MODEL AND POSSIBLE SOURCES OF ERROR**

This section describes the general assumptions and possible sources of error for the presented regression techniques. According to Albertoni (2002), the general assumption for this model includes constant injection and production conditions and also, constant reservoir conditions. To maintain constant injection and production conditions,

new wells cannot be drilled during the time period selected for the analysis. Note that if new wells are drilled, they would have to be included into the regression analysis resulting in the application of a completely new analysis, which in turn results in a new set of weighting coefficients for the MLR and BMLR methods. Also, since the regression methods assume that changes occurring in production rates are solely due to injection rate changes, the production bottomhole pressure must be constant for the selected time period for the analysis. The regression methods assume constant reservoir conditions for the selected time period for the analysis. This suggests that the reservoir has constant reservoir and fluid properties such as permeability's, porosity, fluid viscosity and total compressibility.

### **2.5.1 Possible Sources of Error**

Errors may occur if the assumptions of the model stated above are violated or too few data are available. The model assumes constant total compressibility. To maintain total compressibility, the rock, oil, water and gas compressibilities, along with the oil water and gas saturations have to be constant. Gas is highly compressible while water and oil have lower compressibility's; so a change in gas compressibility would have a greater impact on the total compressibility in comparison to changes in oil and water compressibility. Changes in gas saturation can be identified looking at the gas oil ratio (GOR). In waterfloods, "the GOR is constant when all the free gas has been produced or redissolved in the oil, so that the gas saturation is equal to the residual gas saturation" (Albertoni, 2002). If we are producing with a high GOR, this would indicate that we have not reached solution gas and our gas saturation is not constant. Albertoni (2002) suggests that for a waterflood, selecting a time period with constant and minimum GOR (equal to

dissolve gas-oil ratio,) would be equivalent to periods where the total compressibility is approximately constant (periods where the gas saturation is relatively small and constant). The Little Creek field is undergoing a CO<sub>2</sub> flood, is producing gas and does not have a constant GOR. This means that the gas compressibility and total compressibility are not constant, possibly causing errors.

As mentioned earlier, Albertoni (2002) suggests that better results can be expected from the statistical models if the overdetermination coefficient is greater than or equal to six. The regression process is very iterative and can be time consuming. Also, in the case of an open boundary, the  $\beta_{oj}$  term which accounts for the possible unbalance in the field is constant in the MLR model for each injector-producer well pair. This could lead to possible error in the analysis because the change in flowrates of the injectors outside the boundary are not being accounted for.

## **CHAPTER 3: APPLICATION OF MLR METHOD TO LITTLE CREEK FIELD DATA**

The Little Creek Field in Mississippi currently being operated by Denbury Resources has been under CO<sub>2</sub> flood since 1974. It is a heterogeneous reservoir which violates the assumptions made by Albertoni (2002) for the MLR model. Nonetheless, MLR could help us quantitatively describe continuity at Little Creek without having to use other costly sophisticated models that are typically used for reservoir engineering analysis in the oil industry. This chapter presents a brief field description and the results of MLR analysis with and without diffusivity filters applied to Little Creek data. It examines the SEN and SEP procedures. The results shown in this chapter suggests that diffusivity filters are not needed for Little Creek models.

### **3.1. DENBURY RESOURCES LITTLE CREEK FIELD**

#### **3.1.1. Field Description**

The Little Creek field in Mississippi was discovered by Shell in January, 1958. The original oil in place in the reservoir was estimated to be 101.9 million barrels (Cronquist, 1968, Hanson, 1977b) of which approximately 25 million barrels of oil (MMBO) was recovered from primary production. According to Cronquist (1965), a line-drive waterflood was started in 1962 in the Little Creek Field and an additional 21.7 MMBO was produced. A pilot CO<sub>2</sub> flood was conducted between February, 1974 and February, 1977. According to Hanson (1977) about 120,000 barrels of oil were produced from the pilot area. The field was purchased by J.P. Oil Company from Shell in 1996 and then acquired by Denbury Resources in September, 1999 (Senocak, 2008).

### 3.1.2. Phase 2 Area

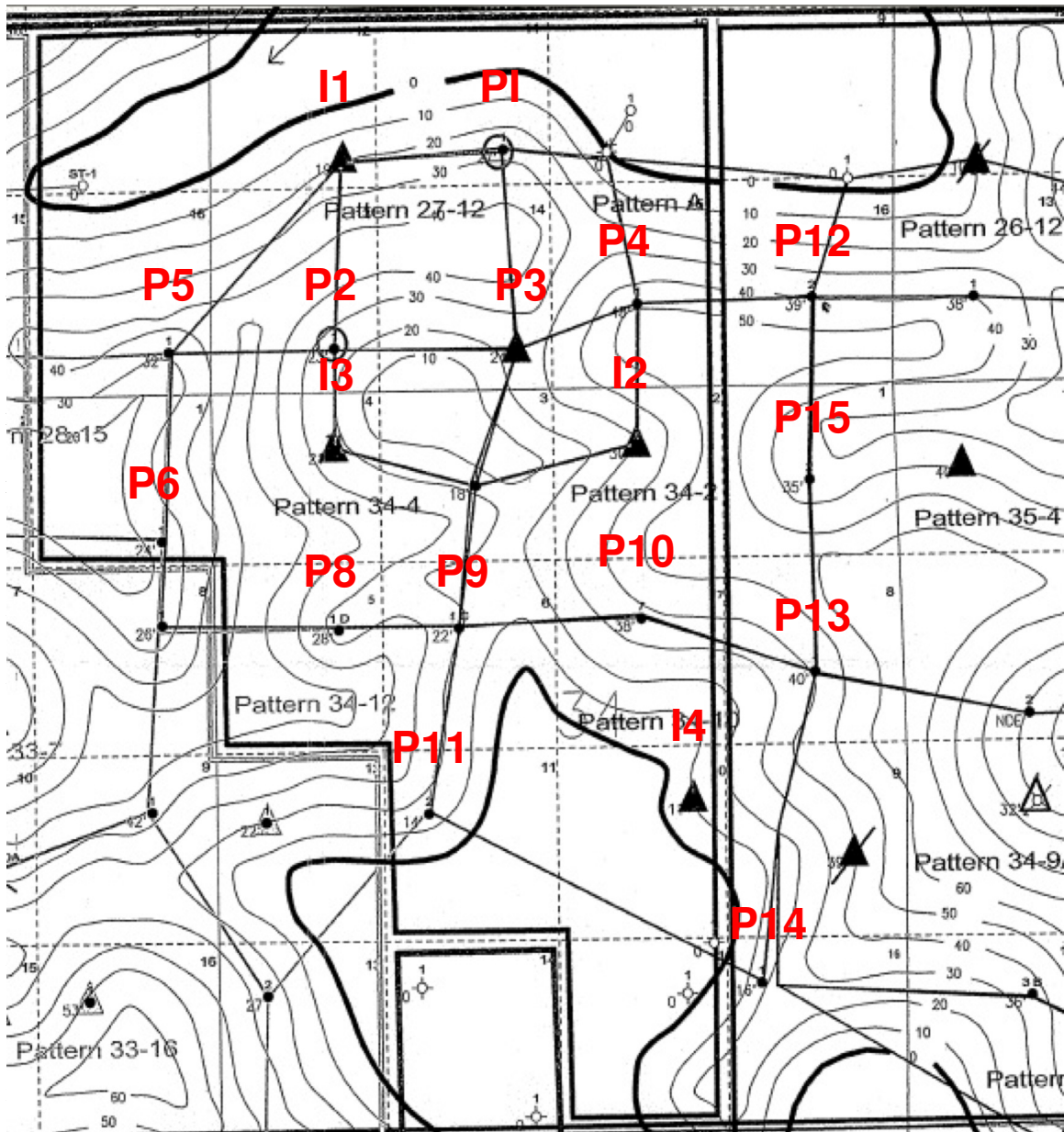
One of the assumptions of the MLR models is that the injection and production conditions are constant. Periods where the producers are shut-in should be excluded from the analysis according to Albertoni (2002). The Phase 2 portion of the field shown in Figure 3.2 includes the producers and injectors shown in Table 3.1

| Well Type | Well Name | Well Number |
|-----------|-----------|-------------|
| Producers | 27-11     | P1          |
|           | 27-13     | P2          |
|           | 27-14     | P3          |
|           | 27-15     | P4          |
|           | 28-16     | P5          |
|           | 33-01     | P6          |
|           | 34-03     | P7          |
|           | 34-05     | P8          |
|           | 34-06     | P9          |
|           | 34-07     | P10         |
|           | 34-11     | P11         |
|           | 27-16     | P12         |
|           | 34-08     | P13         |
|           | 34-16     | P14         |
|           | 34-01     | P15         |
|           | 33-08     | P16         |
| Injectors | 27-12     | I1          |
|           | 34-02     | I2          |
|           | 34-04     | I3          |
|           | 34-10     | I4          |

**Table 3.1** List of wells in the Phase 2 Area of Little Creek Field

The selected time period for the analysis was between January, 1989 to December 31, 1991. For this period, there are injection and production rate data for all producer wells except producer wells P8, P10 and P16. Well P8 does not have rates for December, 1989; producer well P10 does not have rate values reported for September, 1990; producer well P16 has no recorded production rate data for the entire selected time period for the analysis. When there is no recorded production rate data the assumption that the well was shut-in during that time period is made. Since there is no recorded production rate during

the selected time frame for producer well P16, it was not included in the analysis.



**Figure 3.1** Shows location of wells in Phase 2 Area of Little Creek Field (from Denbury Resources Inc., 2007)

Producer wells P5, P6, P8, P9, P11 and P16 are also included in patterns 28-15, 33-7 and 34-12 in the Phase 3 area of the field. Producer wells P12, P13, P14 and P15

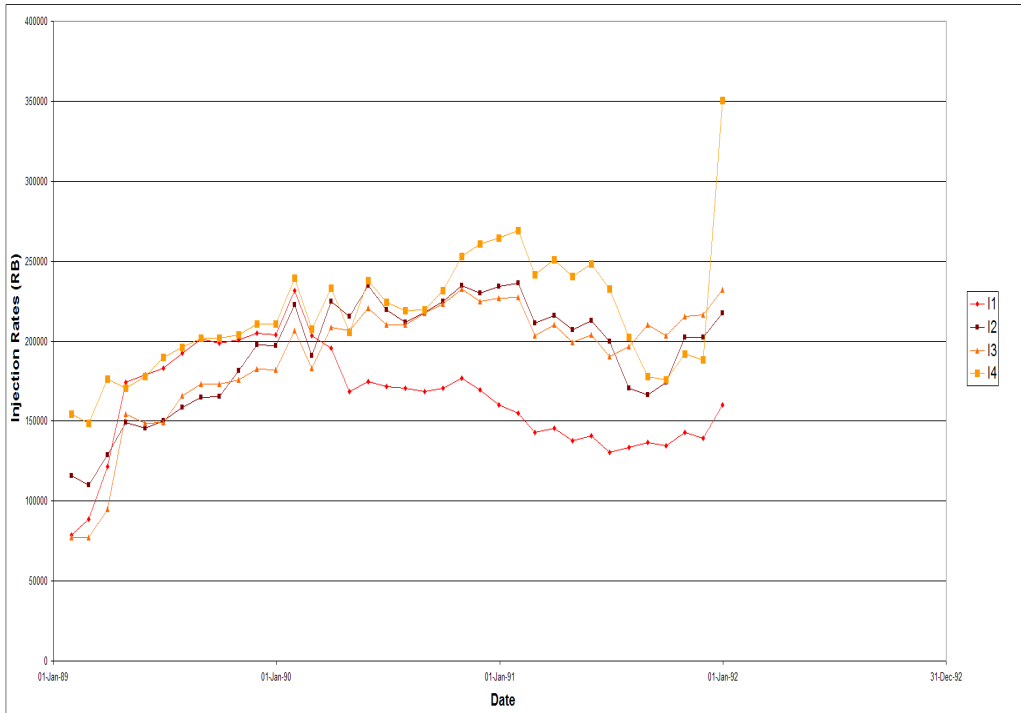
are also included in Patterns 34-9/35-12, 35-4 and 26-12 in the Phase 1 area. Adjacent injectors and producers in those patterns were intermittently on or off production during the time selected for the analysis and so may influence the production rates of the wells included in both the Phase 1 and 2 area. This may lead to errors in modeling. Due to this, the producer wells P12, P13, P14 and P15 were not included in the analysis. Therefore, 11 producers and 4 injectors in Phase 2 were analyzed for the selected time period.

The total injection rate in reservoir barrels (RB) for each injector during the selected time period for the analysis (Table 3.2) indicates a small variation in the total injection between the wells. The injection rate data was provided in surface units (MSCF). A constant carbon dioxide formation volume factor of 1.8 Mscf/rb was used to compute the reservoir injection rates.

| DATE             | I1 (RB)   | I2 (RB)   | I3 (RB)   | I4 (RB)   |
|------------------|-----------|-----------|-----------|-----------|
| 01/1/89-01/31/91 | 5,887,666 | 6,942,968 | 6,833,725 | 7,807,380 |

**Table 3.2** Total injection amount of CO<sub>2</sub> injection in reservoir barrels during the time period analyzed.

The total monthly injection rates in reservoir barrels for injector wells I1, I2, I3, I4 for the selected time period (Figure 3.2) shows that all of the wells have an initial period where the monthly injection increases. The injection rates for three of the wells plateau while one declines.



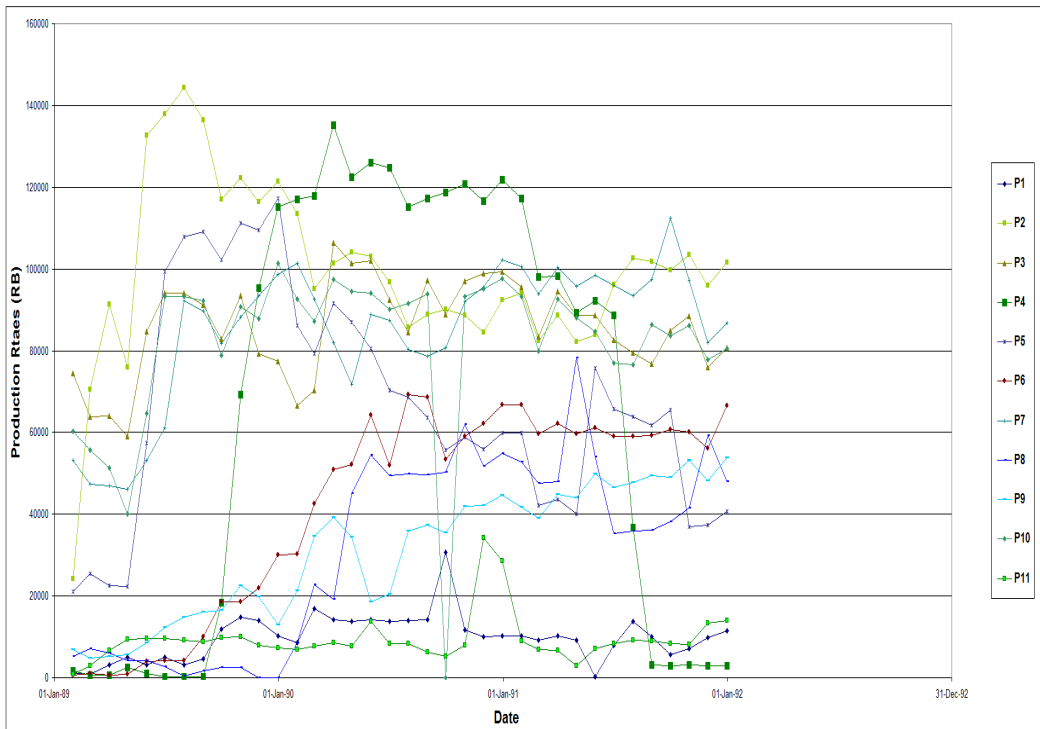
**Figure 3.2** Monthly injection rates for injector wells for the selected time period.

The surface rates for the oil production were provided in stock tank barrels (STB), gas production rates (assumed to be all CO<sub>2</sub>) were in MSCF and water production rates were in STB. The production rates were converted to reservoir barrels using an oil formation volume factor of 1.32 rb/stb, a gas formation factor of 0.556 bbl/MSCF and a water formation volume factor of 1.1 rb/stb. Table 3.3 shows the total production for each producer during the selected time period. As shown in Table 3.3, producers P1 and P11 are an order of magnitude smaller than all of the other wells in the system for the selected time period in this analysis. Even before applying the MLR approach to this system, the production rates of producers P1 and P11 shown in Table 3.3 suggests that in comparison to the other wells in this system, producers P1 and P11 are not being influenced much by the injectors in the system.

| Well Type | Well Name | Well Number | Total Production Rate (RB) |
|-----------|-----------|-------------|----------------------------|
| Producers | 27-11     | P1          | 353,140                    |
|           | 27-13     | P2          | 3,567,339                  |
|           | 27-14     | P3          | 3,082,363                  |
|           | 27-15     | P4          | 2,394,668                  |
|           | 28-16     | P5          | 2,396,054                  |
|           | 33-01     | P6          | 1,515,952                  |
|           | 34-03     | P7          | 3,060,219                  |
|           | 34-05     | P8          | 1,129,449                  |
|           | 34-06     | P9          | 1,119,470                  |
|           | 34-07     | P10         | 2,944,077                  |
|           | 34-11     | P11         | 338,465                    |

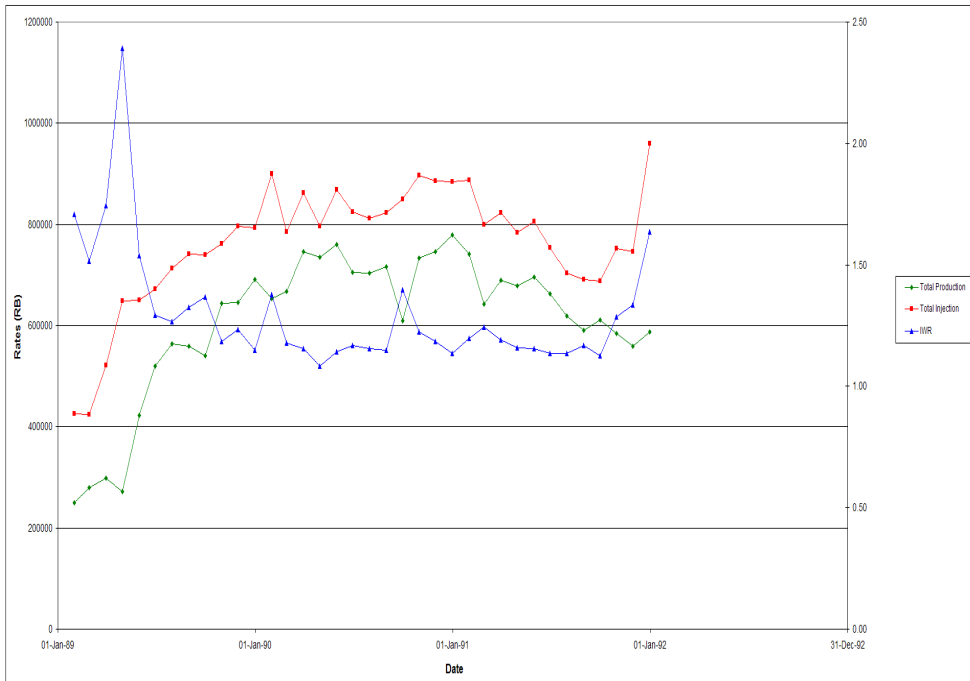
**Table 3.3** Total production rate for each producer during the selected time period for the analysis

Figure 3.3 shows the total monthly production rates for the production wells for the selected time period.



**Figure 3.3** Monthly production rates for producer wells for the selected time period.

The total injection and production values for the system during the period of analysis is 27,471,739 RB and 21,901,196 RB respectively. The cumulative injection to withdrawal ratio (IWR) is then 1.25. Figure 3.4 shows the total injection and production rates for the selected time period being analyzed.



**Figure 3.4** Total injection and production rates for the selected time period

### 3.2 Application of MLR Method

Since only a portion of the field is being analyzed, the boundaries are open and the injection and production rates for the selected period of analysis are not balanced. The multivariate linear regression (MLR) method is then the suggested method for this analysis (Albertoni, 2002).

#### 3.2.1 Results

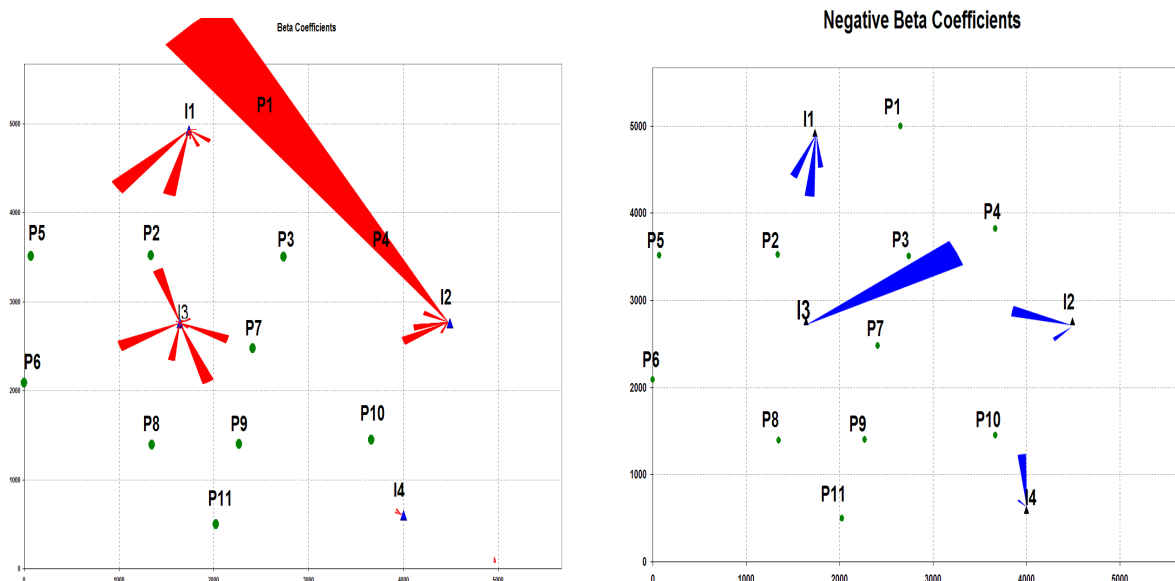
The application of the MLR approach without diffusivity filters to the Phase 2 portion of Denbury's Little Creek Field gives the weighting coefficients shown in Table

3.4. The overdetermination coefficient is 7.4 which is greater than the value of 6 suggested by Albertoni (2002) to ensure good results. The weighting coefficients indicate that there are both positive and negative linear relationships between the injectors and producers in the system.

|              | P1    | P2    | P3    | P4      | P5     | P6     | P7    | P8     | P9    | P10   | P11   |
|--------------|-------|-------|-------|---------|--------|--------|-------|--------|-------|-------|-------|
| $R^2$        | 0.40  | 0.64  | 0.35  | 0.85    | 0.60   | 0.93   | 0.56  | 0.88   | 0.84  | 0.20  | 0.10  |
| $\beta_{ci}$ | -9392 | 47522 | 48618 | -181762 | -19857 | -28419 | 24580 | -10063 | 2119  | 26243 | 165   |
| I1           | 0.03  | 0.51  | 0.01  | 0.17    | 0.67   | -0.36  | 0.00  | -0.46  | -0.25 | 0.14  | 0.03  |
| I2           | 0.09  | -0.65 | 0.18  | 2.93    | -0.04  | 0.24   | -0.06 | 0.35   | -0.18 | 0.09  | 0.02  |
| I3           | 0.02  | 0.42  | 0.03  | -1.37   | 0.03   | 0.46   | 0.36  | 0.29   | 0.49  | 0.07  | -0.01 |
| I4           | -0.04 | 0.06  | -0.02 | -0.39   | -0.09  | -0.03  | 0.01  | -0.02  | 0.05  | 0.01  | 0.01  |

**Table 3.4** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$

One way to show the weighting coefficients in Table 3.4 is as a rose diagram (Figure 3.5). The weighting coefficients  $\beta_{ij}$  are represented by inverted arrows

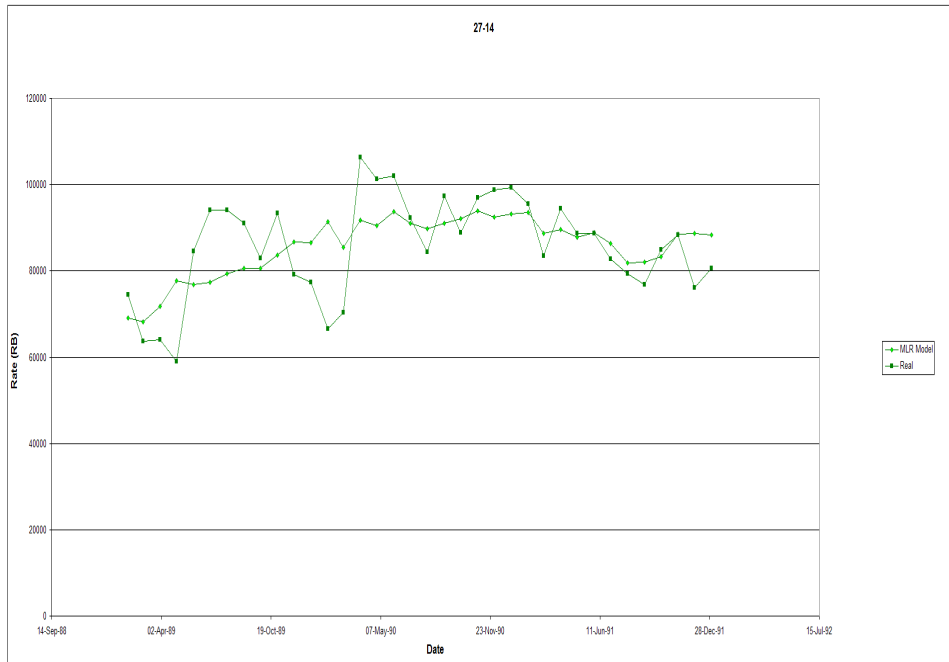


**Figure 3.5** Representation of the weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$ . Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients

that start from an injector  $i$  and point to a producer  $j$  where the size of the arrow is proportional to the value of  $\beta_{ij}$ . The positive and negative values for  $\beta_{ij}$  are shown in Figure 3.5. If it is assumed that the negative weighting coefficients have no meaning, then what the left graph in Figure 3.5 shows is that, injector I1 has the strongest influence on producers P2 and P5 as opposed to the other producers. Injector well I1 has a larger positive effect on producer wells P2 and P5 in comparison to the other producer wells in the system. Thus, as the injection rate for injector well I1 increases, a larger effect from this injector is mostly seen in producers P2 and P5. Considering the location of injector I1 relative to the sand boundary, there may be some of the injection fluid that flows out of the boundary which reduces the effect it is likely to have on the rest of the producers. Injector well I1 shows one of the smallest weighting coefficient values with producer well P1 which has a much shorter injector-producer well pair distance in comparison to the other wells in the system. Injector I1 and producer P1 are located on the boundary so there is a possibility that some of the injected fluid from I1 is being lost to non-productive layers in the fields; thereby reducing the effect injector I1 has on producer P1. Producer P1 is the second lowest producer in the system which verifies that there is little or no connectivity between P1 and the injectors in the system.

Injector well I1 also shows very little connectivity with producer well P3, considering P3 is approximately the same distance from I1 as the P2 and P5 wells which show a high degree of connectivity with I1. Results shown in Table 3.4 and Figure 3.5 suggest that P3 in general is not being influenced much by injectors in the system. What is interesting about this though is that P3 is the second largest producer in the

system. This raises the question whether this MLR model resolves the true connectivity between P3 and injectors in the system. In comparison to the other producers in the system, producer P3 has a low  $R^2$  which suggests that the MLR model and the actual rates for P3 are not strongly correlated. Figure 3.6 shows a plot of the modeled rate versus actual rate for producer P3 for the time period selected for this analysis.



**Figure 3.6.** Plot of modeled rate versus actual rate for Producer 27-14 (P3)

The plot shows a weak correlation between the modeled and actual rates for the time period of January 1989 to February 1990 when actual rates vary considerably. A better correlation is seen between rates from the time period of March 1990 to October 1991 when rates are more stable.

Producer P11 also has a small positive coefficient with injector I1 which could be due to the fact that it is of the furthest distance from injector I1 in comparison to the other producers in the system and so may take a longer time for the effect of an injection rate

change in I1 to show in producer P11. Interestingly, injector well I1 has negative coefficients with producer wells P6, P8 and P9 which are all south of injector well I3 in the field. Paring these results with those seen in the work of Albertoni and Dinh suggests that injector I2 may be acting as a barrier, thereby preventing the flow of fluid from Injector I1 in the northern part of Phase 2 to the producers P6, P8 and P9. In contrast, producer P11 (also located south of injector I3) shows a positive relationship with injector I1. This contradicts the above discussion; this inconsistency raises questions as to the true meaning of the weighting coefficients under these circumstances.

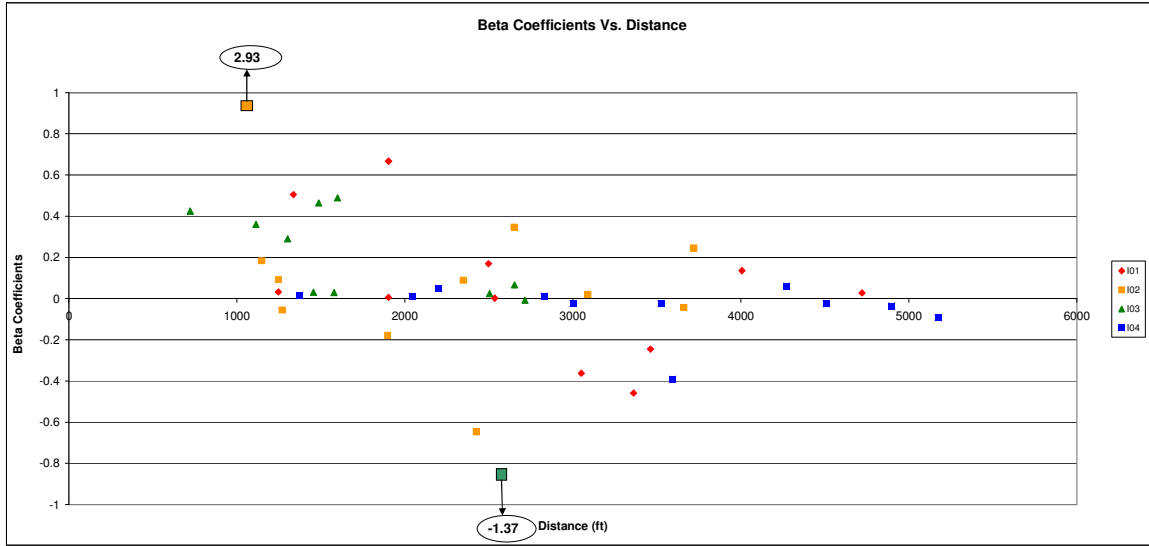
The injector-producer well pair I2 and P4 has a positive weighting coefficient that is greater than one. The coefficient is much larger than the other weighting coefficients. This implies that injector I2 has a greater influence on producer P4 in comparison to the other producers in the model. Injector well I2 has a negative relationship with producer wells P2, P5, P7 and P9. Similar to the results for injector I1, negative weighting coefficients are obtained for injector I2 between producers P2, P5 which are either close to or past nearby injectors with the exception of P7 and P9. Producer wells P1 and P10 have the same magnitude of influence from injector well I2.

Injector well I3 has positive relationships with all producers in the system except wells P4 and P11. This indicates greater connectivity between I3 and the producers in the system which suggests I3 has a more effective sweep than the other injectors. Again, I3, shows negative weighting coefficients for producers close to or past another injector except for producer P11 which is adjacent to the reservoir boundary and has extremely low production relative to the other wells.

Finally, injector well I4 has the highest injection rate for the selected time period

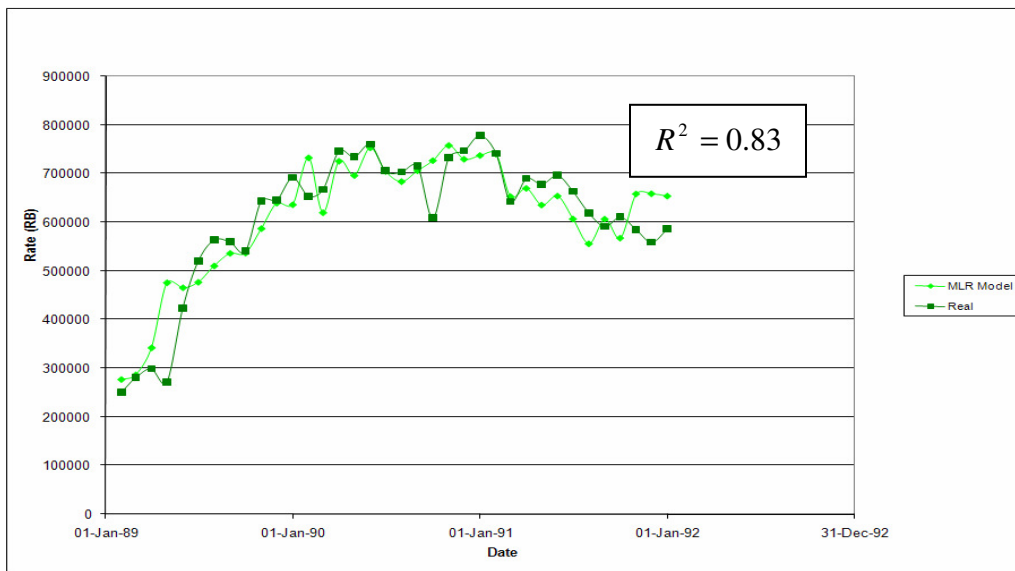
and has the lowest weighting coefficient for many of the producers indicating that it has the least influence on the producers in the system. This indicates that MLR cannot discern where the injection fluid is going. Nearby producers P12 and P13 are included in both patterns 34-10 and 34-9 and were producing for this time period. Due to this creating a possibility of error, they were not included in the analysis. Possibly, injector I4 has a stronger effect on producers P12 and P13 and therefore has a weaker displacement effect on the nearby producers. Also, similar to I1 and P1, injector I4 and producer P11 are located close to a reservoir boundary. Some of the injection fluid may be flowing out of the pattern, which would reduce the effect of I4 on the rest of the producers. I4 has a negative relationship with producer wells P1, P3, P4, P5, P6 and P8. Producers P9, P10 and P11 are located southeast of injector I3 and are closer in location to injector I4 in comparison to producer P8 which could explain why they have a positive relationship with injector I4. These wells could be collecting the fluid from I4 and preventing flow to P8 resulting in the small negative coefficient.

Generally, closer well pairs would be expected to have larger coefficients than well pairs that are further apart. As shown in Figure 3.7, several of the largest values for the beta coefficients are at the lower values for the separation distance. However, there are also a number of very low beta coefficient values at these smaller well pair distances as well and most of the negative values are in the middle to large distance values. This shows that the general assumption made relating the high connectivity of an injector-producer pair to the short distance between them is not necessarily always true.



**Figure 3.7** Weighting coefficients  $\beta_{ij}$  vs injector-producer distance for Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$ .

Figure 3.8 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate. The coefficient of determination  $R^2$



**Figure 3.8** Comparison between total modeled liquid production rate and the total observed liquid production rate for Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$ .

value which represents how accurate the MLR model and the real production rate data are correlated is 0.83. An  $R^2$  value of one would suggest that there is a perfect correlation between the total modeled and total actual production rates for the system which would indicate that the MLR model has been able to accurately capture rate fluctuations in the reservoir. The  $R^2$  value of 0.83 would imply that the model results have not perfectly captured what is going on in the reservoir but the correlation is reasonably good. These results are also lower than those seen in Albertoni (2002) and Dinh (2003) for their field cases. Figure 3.8 shows a closer correlation between the total modeled liquid production rate and the total observed liquid production rate for the time period of May, 1989 to August, 1991. Earlier data fluctuate considerably. The data suggest that there maybe a one to four month lag period which cannot be included in the MLR model without diffusivity filters.

### **3.2.2 Application of the SE-N and SE-P procedure to MLR results**

Albertoni (2002) introduced the successive elimination of negative weighting coefficients (SEN) and the successive elimination of positive weighting coefficients greater than one (SEP) procedure to improve the results obtained from the implementation of the MLR model to field data. Excluding these injector-producer well pairs implies that there is no relationship between those injector-producer well pairs. As stated earlier in Section 2.4.1, there is no statistical justification for eliminating well pairs with negative coefficients or positive weighting coefficients that are greater than one but, the SEN and SEP procedure will be applied to the MLR results to evaluate the impact of the application of this procedure.

Table 3.5 shows the results obtained after the application of the SEN and SEP

procedure to the MLR results shown in Table 3.3. Comparing the results shown in Table

|              | P1       | P2       | P3       | P4       | P5        | P6       | P7       | P8        | P9        | P10      | P11    |
|--------------|----------|----------|----------|----------|-----------|----------|----------|-----------|-----------|----------|--------|
| $R^2$        | 0.38     | 0.48     | 0.35     | 0.12     | 0.58      | 0.74     | 0.55     | 0.56      | 0.60      | 0.20     | 0.11   |
| $\beta_{ij}$ | -11422.9 | 23921.14 | 48618.19 | 47289.94 | -37834.81 | -66567.1 | 23595.78 | -58310.16 | -27977.27 | 26243.49 | 259.41 |
| I1           | 0.03     | 0.46     | 0.01     | 0.58     | 0.64      | 0.00     | 0.00     | 0.00      | 0.00      | 0.14     | 0.03   |
| I2           | 0.06     | 0.00     | 0.16     | 0.00     | 0.00      | 0.15     | 0.00     | 0.23      | 0.00      | 0.09     | 0.01   |
| I3           | 0.03     | 0.00     | 0.03     | 0.00     | 0.00      | 0.41     | 0.32     | 0.22      | 0.30      | 0.07     | 0.00   |
| I4           | 0.00     | 0.00     | 0.00     | 0.00     | 0.00      | 0.01     | 0.00     | 0.01      | 0.01      | 0.01     | 0.01   |

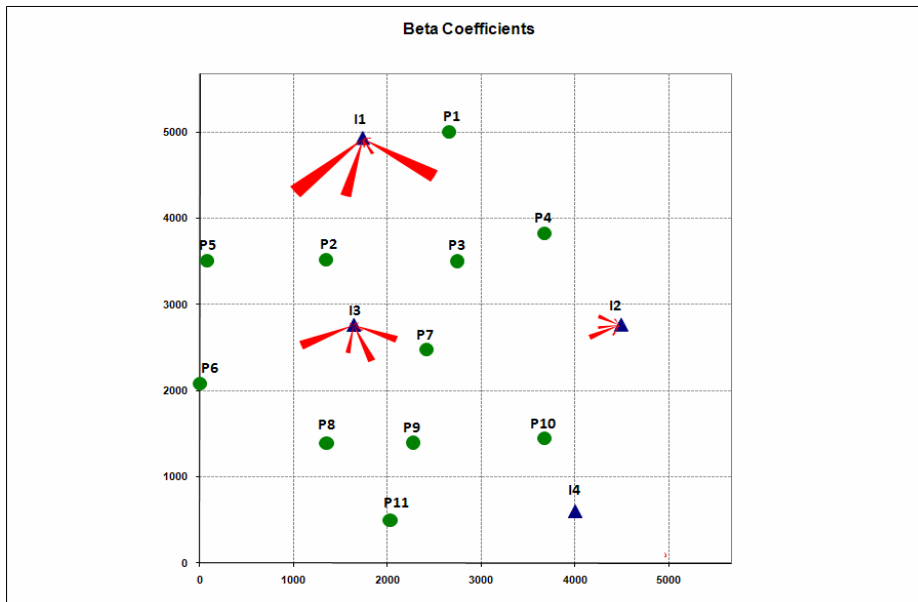
**Table 3.5** Weighting coefficients  $\beta_{ij}$  after application of SEN and SEP procedures for Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$

3.5 to those shown previously in Table 3.4 shows that after the successive elimination of both negative weighting coefficients and positive weighting coefficients greater than one, there has been an overall reduction in the evaluated connectivity between injector-producer pairs. In general, the minimum weighting coefficient converges to zero as more weighting coefficients are eliminated. Results show that after the application of the SEN and SEP procedure 13 negative weighting coefficients were eliminated and 6 positive weighting coefficients were eliminated. Out of 44 injector-producer well pairs coefficients in the original model, only 25 of them will be accounted for with this new model. Only two of these 25 coefficients increased, while 9 remained the same and the remaining 14 decreased in magnitude.

As shown in Table 3.5, the  $R^2$  values for producers P3 and P10 were unchanged and the  $R^2$  value for P11 increased slightly after the application of the SEN and SEP procedure. The rest of the wells had lower  $R^2$  values after applying the SEN and SEP procedures.

In Figure 3.9, the rose diagram of the weighting coefficients  $\beta_{ij}$  determined after

the application of the SEN and SEP procedure indicate that injector well I1 has a strong



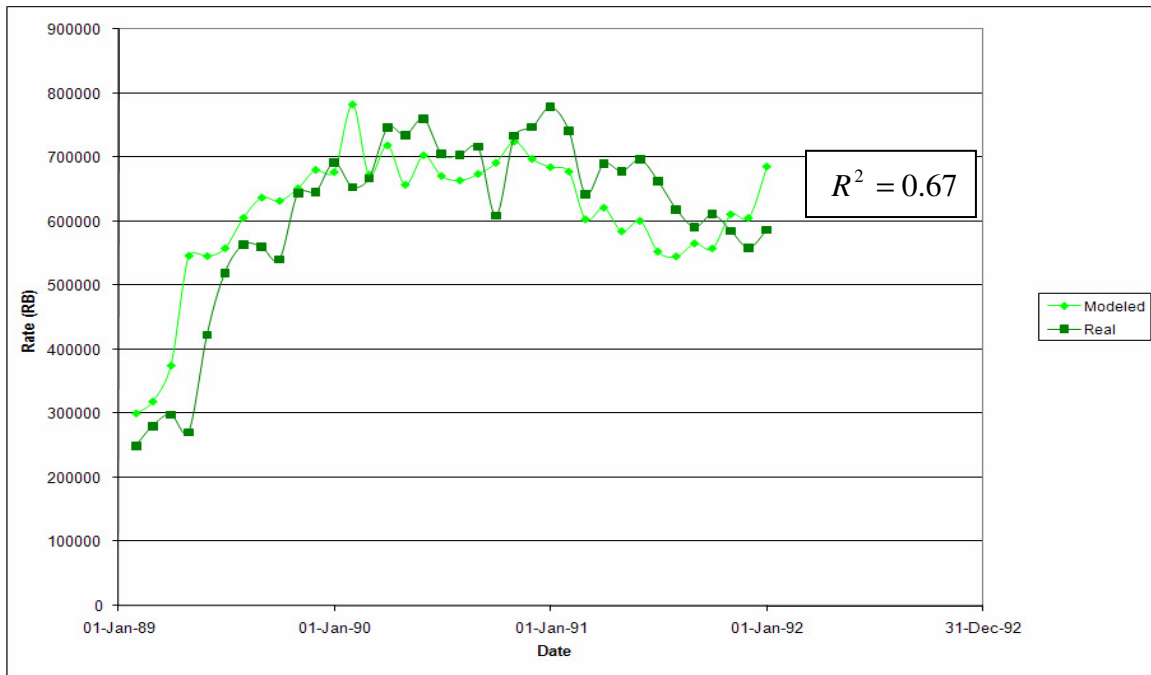
**Figure 3.9** Representation of the positive weighting coefficients  $\beta_{ij}$  after the SEN and SEP procedure is applied to the Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$ .

connectivity with wells P2, P4 and P5 whereas in the case without the application of the SEN and SEP procedure, well I1 showed a strong connectivity with wells P2 and P5 but the connectivity to P4 was much weaker. Similar to the previous case, I4 still has the least influence on the producers in the system. I3 has the strongest influence on producers to the south of the injector unlike the previous case where a strong connectivity was also seen between injector I3 and producer P2. Similar to the previous case, wells P1, P3, P10, P11 are not being influenced much by the four injectors in the system. After the application of the SEN and SEP procedure, producer wells P2, P4 and P5 only show connectivity with injector well I1. Using the SEN and SEP results to evaluate sweep improvement options would likely be only slightly different than using the unmodified MLR results. For example, the SEN and SEP results would suggest that increasing the

influence of I3 on P2 should increase recovery while the MLR results suggest that this connection has already been established. In addition, the SEN and SEP results indicate that I2 has a minor influence on recovery in this area while the MLR results show that, at least for several of the wells this influence is present especially toward the P4 well.

A plot showing the weighting coefficients  $\beta_{ij}$  versus distance for the MLR model after the application of the SEN and SEP procedure again showed that the general assumption made relating the high connectivity of an injector-producer pair to the short distance between them is not necessarily true.

Figure 3.10 shows a comparison between total modeled liquid production rate and



**Figure 3.10** Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR without Diffusivity filters, with  $O_d=7.4$ .

the total observed liquid production rate after the application of the SEN and SEP procedure. The effect of the application of the SEN and SEP procedure can be seen by

comparing Figure 3.8 to Figure 3.10. The comparison of the plots suggest that there is a noticeable decrease in the total modeled production rate in comparison to the total observed liquid production rate after the application of the SEN and SEP procedures to the system. As shown, the  $R^2$  value decreased from 0.83 to 0.67 after the SEN and SEP procedure was applied. This implies that the application of the SEN and SEP procedures did not improve the prediction of production well flow rates as would be expected with fewer degrees of freedom. A closer correlation between the total modeled liquid production rate and the total observed liquid production rate for the time period of April, 1989 to December, 1991 is shown in Figure 3.10. This suggests that there may be a four month lag period which is not accounted for by the MLR model with the application of the SEN and SEP procedures without diffusivity filters is present in this model.

### 3.3 APPLICATION OF MLR METHOD WITH DIFFUSIVITY FILTERS

As noted in Section 2.3, the production rate value at a particular time is the response to injection rate changes over the time of the filter. As presented in Albertoni (2002), this was 12 months. The actual time it takes for the production rate value of a producer  $j$  to respond to an injection rate change of an injector  $i$  will be applied to the MLR model. 6-month and 12-month diffusivity filters suggested by Albertoni (2002) and Dinh (2003) will also be applied to the MLR model to compare results. A quick calculation using a radius-of-investigation-type time of the form (Lee, 1997)

$$t = \frac{948\phi\mu c_i r_i^2}{k} \quad (3.1)$$

can be used to calculate an approximate time it takes for the production rate value of a

producer  $j$  to respond to an injection rate change of an injector  $i$ . A total compressibility value ( $C_t$ ) value of 0.00012687 and a viscosity ( $\mu$ ) value of 0.035 (CO<sub>2</sub> values) were used in Equation 3.1 to calculate time. The  $r_i^2$  value in Equation 3.1 represents the distance between the injector-producer well pair. The  $k$  value represents the permeability and  $\phi$  represents the porosity of the well. Table 3.6 below shows the average porosity and permeability values from core data provided by Denbury Resources for the wells in Phase 2. The average porosity and permeability data was not provided for producers P6, P7, P8 and injectors I3 and I4. Field average reservoir permeability of 33 md and porosity of 0.234 provided by Denbury Resources was used for those wells.

|     |       | AVG POROSITY | AVG PERMEABILITY |
|-----|-------|--------------|------------------|
| P1  | 27-11 | 27.36        | 36.98            |
| P2  | 27-13 | 26.41        | 296.04           |
| P3  | 27-14 | 27.02        | 631.67           |
| P4  | 27-15 | 28.95        | 34.60            |
| P5  | 28-16 | 28.96        | 87.25            |
| P9  | 34-06 | 24.26        | 50.06            |
| P10 | 34-07 | 19.13        | 118.7            |
| P11 | 34-11 | 29.75        | 177.13           |
| I01 | 27-12 | 25.28        | 39.02            |
| I02 | 34-02 | 17.54        | 8.04             |

**Table 3.6** shows the average porosity and permeability values

Table 3.7 shows the time it takes in days for the production rate value of a producer  $j$  to respond to an injection rate change of an injector  $i$ . The result of this

|    | P1(t,days) | P2(t,days) | P3(t,days) | P4(t,days) | P5(t,days) | P6(t,days) | P7(t,days) | P8(t,days) | P9(t,days) | P10(t,days) | P11(t,days) |
|----|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|-------------|
| i1 | 2.0        | 0.0        | 1.1        | 8.6        | 3.2        | 10.6       | 7.4        | 12.9       | 11.5       | 7.9         | 14.0        |
| i2 | 15.3       | 0.1        | 0.9        | 3.1        | 25.7       | 34.9       | 4.1        | 17.7       | 7.6        | 1.7         | 13.2        |
| i3 | 8.6        | 0.0        | 0.8        | 9.8        | 2.0        | 2.8        | 1.5        | 2.1        | 2.7        | 3.8         | 5.0         |
| i4 | 33.0       | 0.2        | 4.1        | 19.4       | 25.4       | 25.3       | 9.9        | 11.2       | 5.1        | 1.0         | 2.9         |

**Table 3.7** Time (days) calculation for injector-producer well pairs in Phase 2 of the Little Creek Field

analysis suggests that apart from producer P1 which takes approximately 33 days to respond to an injection rate change in injector I4, the rest of the producers in the system take less than a month to respond to the injection rate changes of surrounding injectors in the system. This suggests that this is a low dissipation system so a 1 month diffusivity filter would be the most that should be applied to the MLR model for the Phase 2 portion of the Little Creek Field.

### 3.3.1 MLR with 1 Month Diffusivity Filter

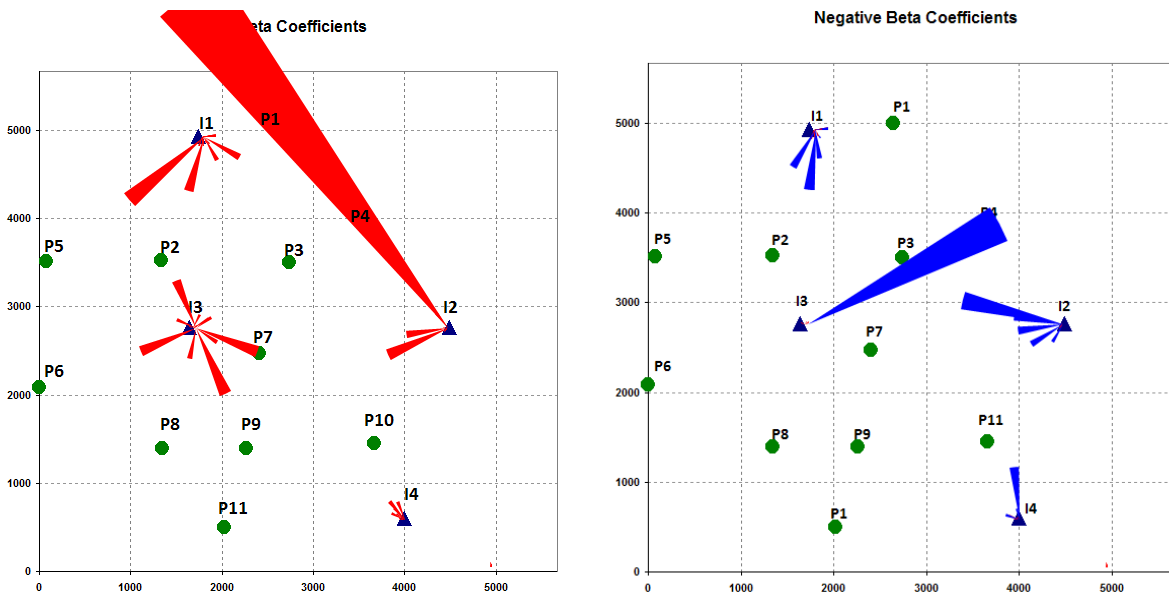
Rather than having 37 months of data to work with in this case, the number of data points is reduced to 36 with 1-month diffusivity filters. The over determination factor  $O_d$  for this case is then 7.2. As stated earlier Albertoni (2002) suggests that very good results can be obtained with an overdetermination coefficient larger than 6.

The application of MLR with 1-month diffusivity filters to the Phase 2 portion of Denbury’s Little Creek Field gives the weighting coefficients shown in Table 3.8. Similar to the case without diffusivity filters, the results shown in Table 3.8 have both positive and negative linear relationships between the injector-producer well pairs.

|              | P1    | P2    | P3    | P4      | P5     | P6     | P7    | P8     | P9    | P10   | P11   |
|--------------|-------|-------|-------|---------|--------|--------|-------|--------|-------|-------|-------|
| $R^2$        | 0.41  | 0.68  | 0.35  | 0.86    | 0.66   | 0.93   | 0.60  | 0.88   | 0.83  | 0.23  | 0.07  |
| $\beta_{ij}$ | -7731 | 78981 | 41236 | -205806 | -27771 | -30285 | 10203 | -10197 | -4491 | 11546 | 2103  |
| I1           | 0.04  | 0.43  | 0.06  | 0.30    | 0.74   | -0.33  | 0.05  | -0.45  | -0.22 | 0.20  | 0.02  |
| I2           | 0.14  | -0.77 | 0.02  | 3.17    | -0.37  | 0.32   | -0.34 | 0.50   | -0.28 | -0.16 | -0.01 |
| I3           | 0.00  | 0.39  | 0.14  | -1.60   | 0.15   | 0.44   | 0.49  | 0.23   | 0.54  | 0.19  | 0.00  |
| I4           | -0.08 | 0.12  | 0.02  | -0.39   | 0.07   | -0.08  | 0.18  | -0.11  | 0.11  | 0.14  | 0.02  |

**Table 3.8** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with 1 month diffusivity filters, with  $O_d=7.2$

The weighting coefficients shown in Table 3.8 are represented in Figure 3.11.



**Figure 3.11** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with 1-month diffusivity filters, with  $O_d=7.2$ . Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients

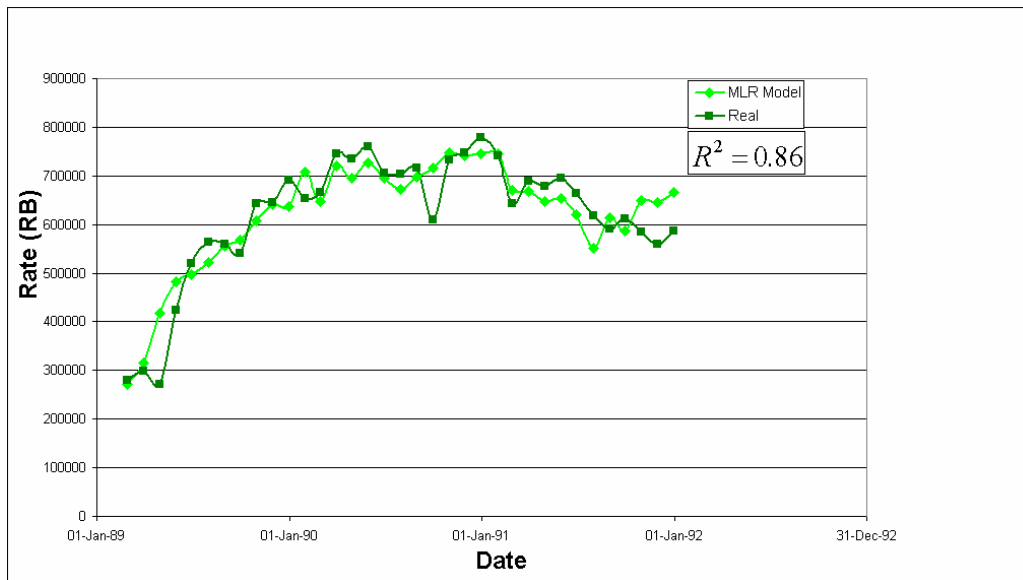
Similar to the MLR case without Diffusivity Filters, Figure 3.11 shows that injector I1 has the strongest influence on producers P2 and P5 as opposed to other producers in the system. Injector I1 still shows very little connectivity with producer P1. This model also suggests that producer P3 in general is not being influenced much by injectors in the system. Producer P4 still shows a high connectivity with injector I2.

Figure 3.11 shows that contrary to the MLR case without diffusivity filters, injector I3 shows a positive relationship with all producers in the model except producer P4. This suggests greater connectivity between I3 and the producers in the system. Injector I4 shows an increase in connectivity with nearby producers in the model. In general, application of the 1 month diffusivity filters results in a slight decrease in the number of negative weighting coefficients between injector producer well pairs from 15 (for the case without diffusivity filters) to 14. Also, the magnitude of connectivity between injector producer well pairs increased as did the  $R^2$  values of all but two

producers with two remaining constant.

A plot showing the values of the weighting coefficients versus distance was generated and results show that the general assumption made relating the high connectivity of an injector producer pair to the short distance between them is not always true.

Figure 3.12 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate. In comparison to the MLR case without



**Figure 3.12** Denbury Resources Little Creek Field. MLR with 1-month diffusivity filters, with  $O_d = 7.2$ . Comparison between total modeled liquid production rate and the total observed liquid production rate.

diffusivity filters, Figure 3.12 shows that the coefficient of determination  $R^2$  value increased from 0.83 to 0.86 after the application of the 1-month diffusivity filter. This suggests that a closer correlation between the total modeled liquid production rate and total observed liquid production rate has been achieved with the application of the 1-

month diffusivity filter to the MLR model. This  $R^2$  value is also higher than the  $R^2$  value of 0.67 for the MLR case after the application of the SEN and SEP procedure.

### 3.3.2 Application of the SE-N and SE-P procedure to MLR with 1-month Diffusivity Filters

Table 3.9 shows the results obtained after the application of the SEN and SEP procedure to the MLR model with 1 month diffusivity filters. Comparing the results

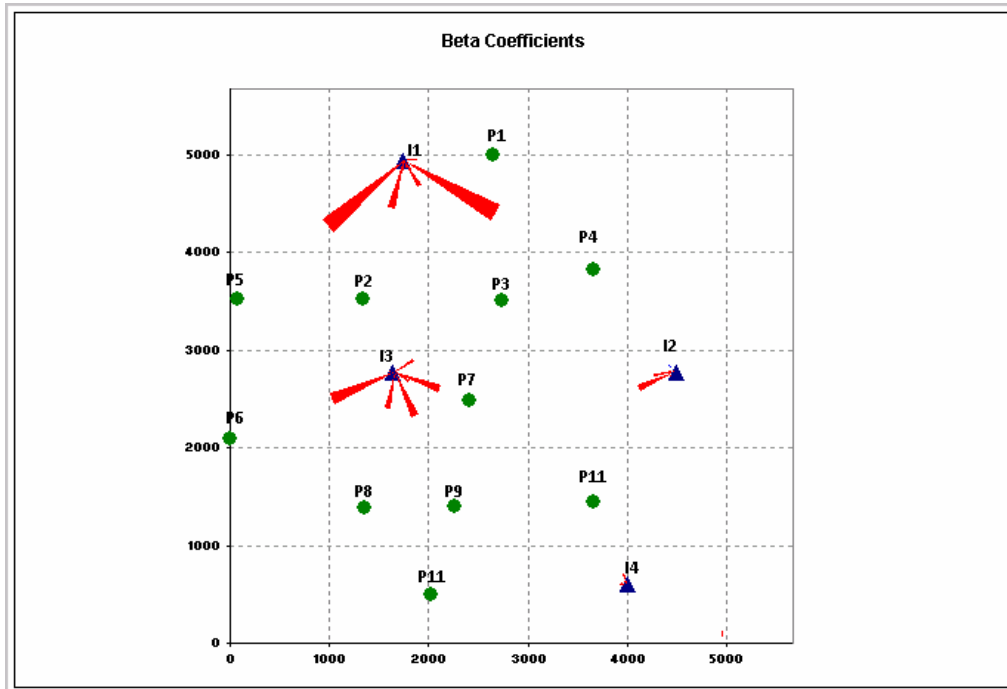
|          | P1     | P2    | P3    | P4     | P5     | P6     | P7    | P8     | P9     | P10   | P11   |
|----------|--------|-------|-------|--------|--------|--------|-------|--------|--------|-------|-------|
| $R^2$    | 0.37   | 0.33  | 0.35  | 0.17   | 0.62   | 0.78   | 0.56  | 0.58   | 0.60   | 0.23  | 0.13  |
| $B_{0j}$ | -12985 | 45993 | 41236 | -48746 | -45096 | -72681 | 15178 | -67686 | -31087 | 13833 | -5357 |
| 11       | 0.04   | 0.34  | 0.06  | 0.71   | 0.69   | 0.00   | 0.03  | 0.00   | 0.00   | 0.19  | 0.01  |
| 12       | 0.07   | 0.00  | 0.02  | 0.00   | 0.00   | 0.15   | 0.00  | 0.27   | 0.00   | 0.00  | 0.00  |
| 13       | 0.02   | 0.00  | 0.14  | 0.00   | 0.00   | 0.46   | 0.31  | 0.25   | 0.33   | 0.11  | 0.02  |
| 14       | 0.00   | 0.00  | 0.02  | 0.00   | 0.00   | 0.00   | 0.03  | 0.00   | 0.00   | 0.07  | 0.04  |

**Table 3.9** Weighting coefficients  $\beta_{ij}$  after the SEN and SEP procedure for the Little Creek Field. MLR with 1 month Diffusivity filters, with  $O_d=7.2$

shown in Table 3.9 to those shown previously in Table 3.8 shows that after the successive elimination of both negative weighting coefficients and positive weighting coefficients greater than one, 14 negative weighting coefficients and 5 positive weighting coefficients were eliminated. Therefore, out of 44 injector-producer wellpairs in the original model, only 25 have positive weights after the application of the SEN and SEP procedure. In addition, the  $R^2$  values for 10 producers decreased after the implementation of the SEN and SEP procedure. Producer 11 shows a slight increase in  $R^2$  value from 0.07 to 0.13. This implies that the SEN and SEP procedure did not help improve the prediction of flow rates for almost all the wells.

The weighting coefficients  $\beta_{ij}$  determined from the application of the SEN and

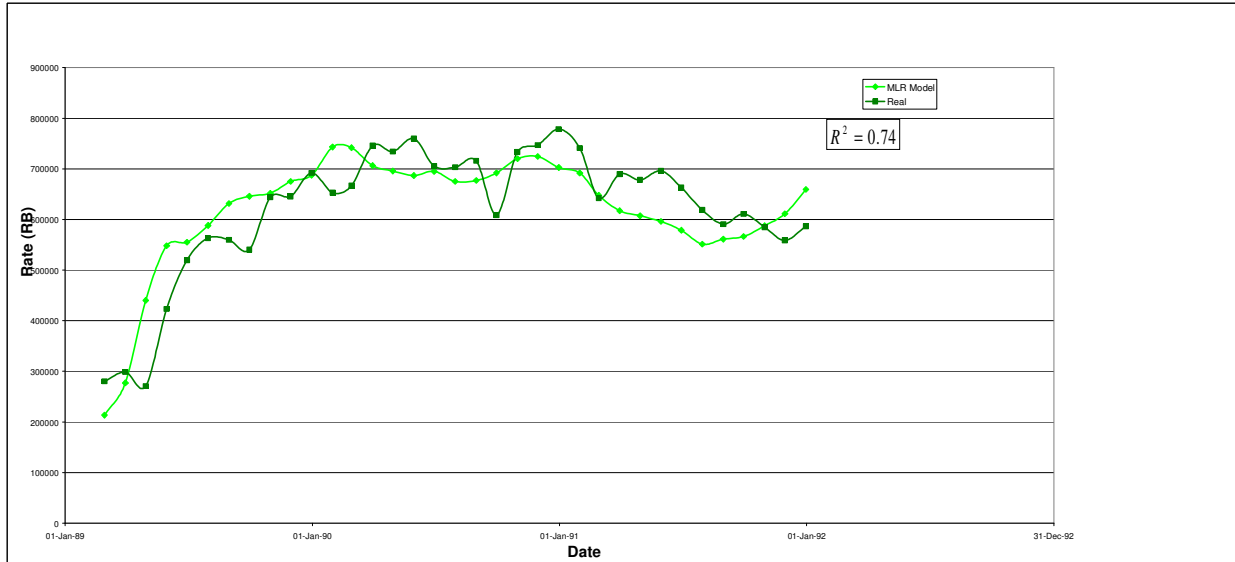
SEP procedure are shown in Figure 3.13. Figure 3.13 shows that the application



**Figure 3.13.** Positive weighting coefficients  $\beta_{ij}$  after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 1-month diffusivity filters, with  $O_d=7.2$

of the SEN and SEP procedures to the MLR model reduced the connectivity between injector I4 and producers in the system. Injector I3 shows a reduced connectivity with producers in the system and injector I1 shows an increase in the magnitude of connectivity with producer P4. In general, Figure 3.13 suggests that the sweep of fluid in the reservoir is not that efficient because the individual injectors are not influencing the producers in the area much.

Figure 3.14 shows a comparison between total modeled liquid production rate and



**Figure 3.14** Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to Little Creek Field Phase 2 data. MLR with 1-month diffusivity filters, with  $O_d = 7.2$ .

the total observed liquid production rate after the application of the SEN and SEP procedure. The  $R^2$  value decreased from 0.86 to 0.74 after the SEN and SEP procedure was applied.

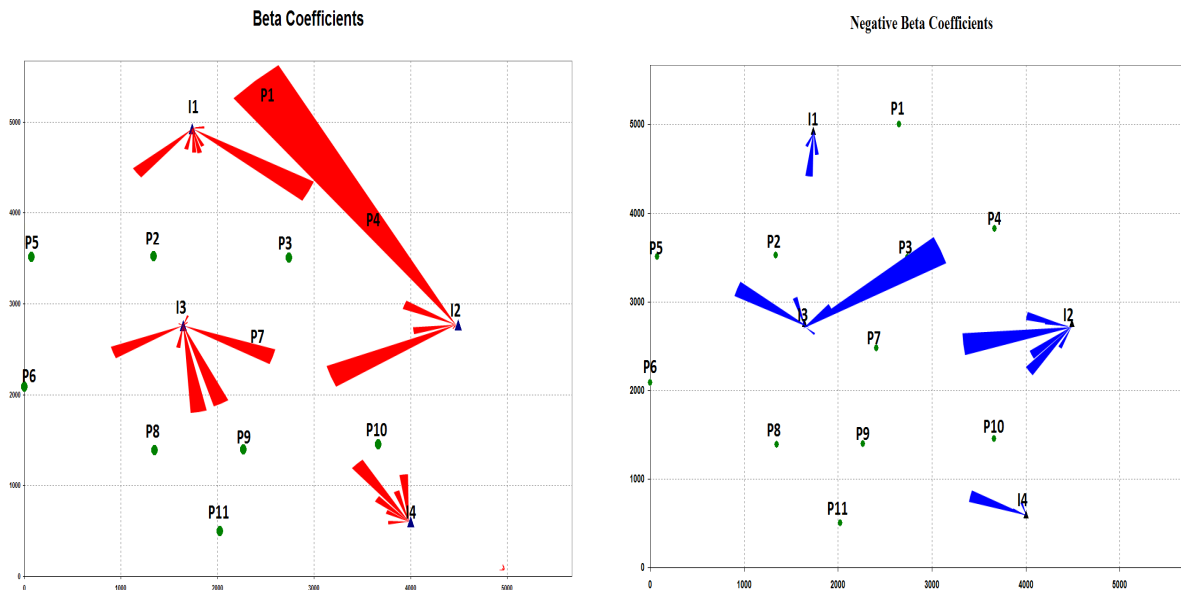
### 3.3.3 MLR with 6 Month Diffusivity Filter

The number of data points in the case of 6 month filters is reduced to 31 and the  $O_d$  is 6.2. Although the  $O_d$  factor is lesser than the case with 1-month filters, Albertoni (2002) suggests that very good results can be obtained with an  $O_d$  factor larger than 6. The application of MLR with 6-month diffusivity filters gives the weighting coefficients shown in Table 3.10. Results show both positive and negative linear relationships between the injector-producer well pairs.

|              | P1     | P2     | P3     | P4      | P5    | P6     | P7     | P8     | P9     | P10   | P11    |
|--------------|--------|--------|--------|---------|-------|--------|--------|--------|--------|-------|--------|
| $R^2$        | 0.33   | 0.83   | 0.10   | 0.94    | 0.86  | 0.91   | 0.50   | 0.87   | 0.69   | 0.09  | 0.19   |
| $\beta_{oi}$ | -11352 | 193633 | 100505 | -371302 | 96604 | -64864 | -10939 | -19647 | -20852 | 61814 | -85291 |
| I01          | 0.08   | 0.16   | -0.07  | 0.93    | 0.51  | -0.13  | 0.19   | -0.34  | -0.18  | 0.15  | 0.18   |
| I02          | 0.08   | -0.36  | 0.39   | 2.26    | -0.2  | 0.3    | -0.92  | 0.95   | -0.38  | -0.17 | -0.47  |
| I03          | 0.07   | -0.23  | -0.27  | -1.22   | -0.66 | 0.53   | 0.66   | 0.17   | 0.63   | -0.09 | 0.64   |
| I04          | -0.1   | 0.01   | -0.1   | 0.35    | 0.28  | -0.12  | 0.56   | -0.49  | 0.17   | 0.24  | 0.15   |

**Table 3.10** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with 6 month diffusivity filters, with  $O_d=6.2$

The weighting coefficients shown in Table 3.10 are represented in Figure 3.15.



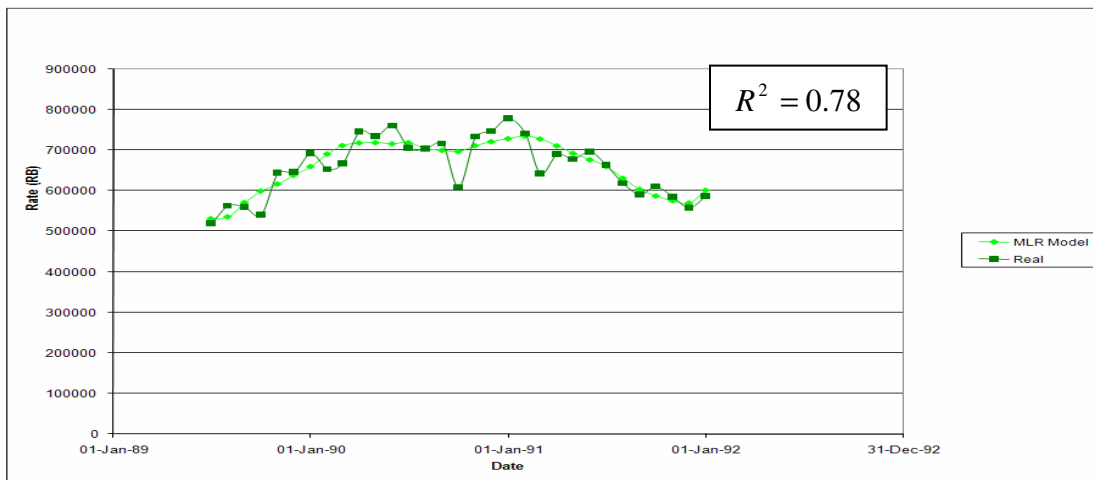
**Figure 3.15** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with 6-month diffusivity filters, with  $O_d=6.2$ . Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients

Figure 3.15 shows that in contrast to the northern part of Phase 2, fluid flow in the southern part of the Phase 2 area now appears to be uniform. Figure 3.15 also shows that in contrast to the MLR case with 1-month filters, fluid from injector I3 appears to be moving away from the line of producers, P2, P3, P4 and P5, just to the north of I3. Injector I3 strongly influences producers P6, P7, P9 and P11 which are all south of its

well location. In contrast to the MLR case with 1-month filters, injector I1 shows very little connectivity with producers directly to the south of its location. This suggests the possibility of a barrier preventing flow from injector I1 to producers to the south of its location. Injector I4 shows an increase in magnitude of connectivity with producers. Injector I1 and producer P4 also show an increase in magnitude of connectivity. Similar to the previous MLR cases with 1-month filters and without filters, injector I1 shows strong connectivity with producer P5. Coefficients for injector I2 are similar to the previous MLR models with and without filters.

Application of the 6 month diffusivity filters results in an increase in the number of negative weighting coefficients between injector producer well pairs from 15 (for the case without diffusivity filters) and 14 (for the case with 1 month filters) to 19.

Figure 3.16 shows a comparison between the total modeled liquid production



**Figure 3.16** Denbury Resources Little Creek Field. MLR with 6-month diffusivity filters, with  $O_d=6.2$ . Comparison between total modeled liquid production rate and the total observed liquid production rate .

rate and the total observed liquid production rate. In comparison to the MLR case

without diffusivity filters, the coefficient of determination  $R^2$  value decreased from 0.83 to 0.78. This  $R^2$  value is also lower than the case with 1 month filters which suggests that the application of the 6 month filter did not help improve the rate predictions of the MLR model.

### 3.3.4 Application of the SE-N and SE-P procedure to MLR with 6-month Diffusivity Filters

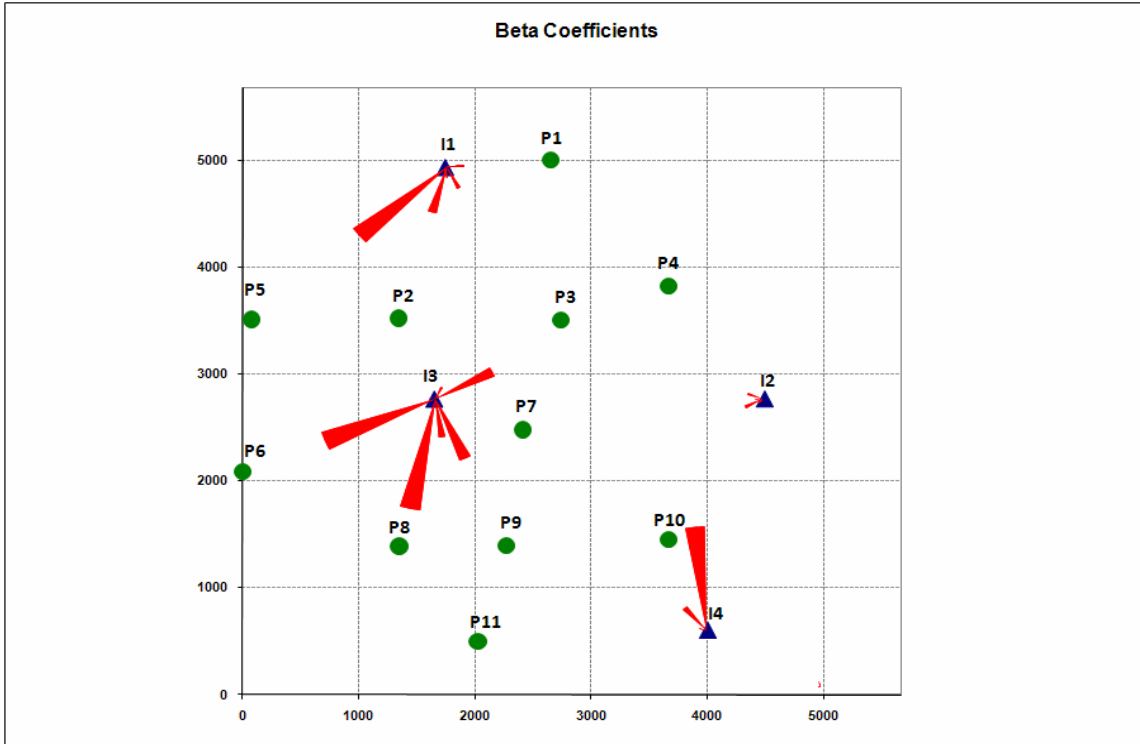
Table 3.11 shows the results obtained after the application of the SEN and SEP procedure to the MLR with 6-month diffusivity filters results shown in Table 3.11.

|              | P1     | P2    | P3    | P4      | P5     | P6      | P7    | P8      | P9     | P10   | P11    |
|--------------|--------|-------|-------|---------|--------|---------|-------|---------|--------|-------|--------|
| $R^2$        | 0.29   | 0.20  | 0.06  | 0.49    | 0.45   | 0.90    | 0.20  | 0.79    | 0.30   | 0.05  | 0.07   |
| $\beta_{ij}$ | -23572 | 52781 | 65390 | -138393 | -38971 | -104559 | 49761 | -123560 | -44543 | 61626 | -44500 |
| I1           | 0.11   | 0.29  | 0.02  | 0.00    | 0.66   | 0.00    | 0.00  | 0.00    | 0.00   | 0.14  | 0.04   |
| I2           | 0.00   | 0.00  | 0.09  | 0.00    | 0.00   | 0.05    | 0.00  | 0.11    | 0.00   | 0.00  | 0.00   |
| I3           | 0.08   | 0.00  | 0.00  | 0.38    | 0.00   | 0.68    | 0.00  | 0.69    | 0.40   | 0.00  | 0.24   |
| I4           | 0.00   | 0.00  | 0.00  | 0.65    | 0.00   | 0.04    | 0.19  | 0.00    | 0.00   | 0.00  | 0.00   |

**Table 3.11** Weighting coefficients  $\beta_{ij}$  after the SEN and SEP procedure for the Little Creek Field. MLR with 6 month Diffusivity filters, with  $O_d=6.2$

Comparing the results shown in Table 3.11 to those shown previously in Table 3.10 shows that after the successive elimination of both negative weighting coefficients and positive weighting coefficients greater than one, 17 negative weighting coefficients and 8 positive weighting coefficients were eliminated. Therefore out of 44 injector-producer wellpairs in the original model, only 19 of them have positive weights after the application of the SEN and SEP procedure. In addition, the  $R^2$  values for all 11 producers decreased. The SEN and SEP procedure did not help improve the prediction of flow rates. This is expected because, a less saturated model will have higher prediction errors.

The weighting coefficients  $\beta_{ij}$  determined from the application of the SEN and SEP procedure is shown in Figure 3.17.

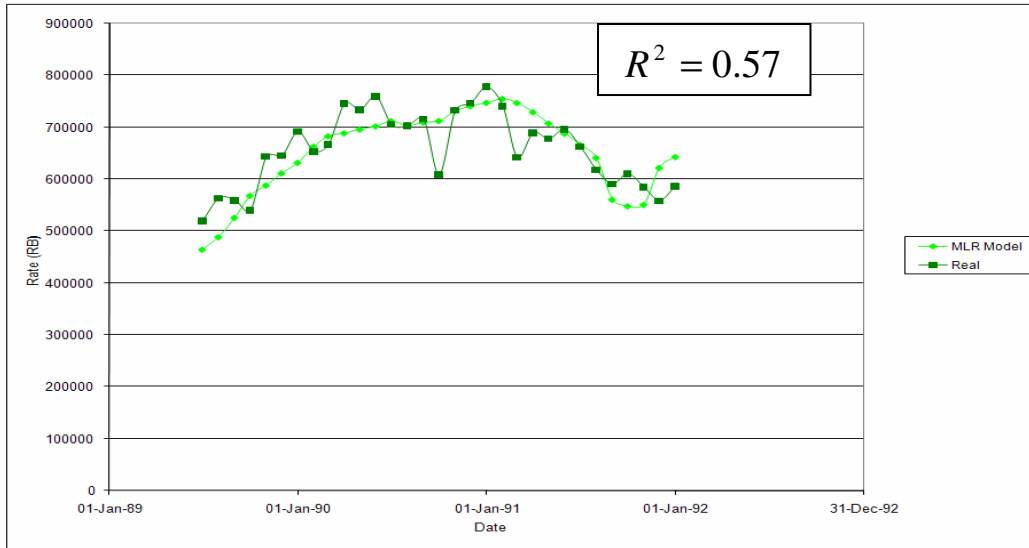


**Figure 3.17.** Positive weighting coefficients  $\beta_{ij}$  after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 6-month diffusivity filters, with  $O_d=6.2$ .

Figure 3.17 shows that the application of the SEN and SEP procedures changed the estimated connectivity between injector I2 and producers in the system. In comparison to the MLR model with 6 month diffusivity filters, injector wells I2 and I4 show the least connectivity with producers in the system. Injector well I1 has a strong connectivity with producer well P5 whereas in the case without the application of the SEN and SEP procedure, injector well I1 showed a strong connectivity with both producers P4 and P5. Injector I3 has stronger influences on producers P4, P6, P8, P9 and P11. In general, Figure 3.17 shows that the sweep of fluid in the reservoir is not that

efficient because the individual injectors are not influencing the producers in the area much as shown in Table 3.11.

Figure 3.20 shows a comparison between total modeled liquid production rate and



**Figure 3.18** Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to Little Creek Field Phase 2 data. MLR with 6-month diffusivity filters, with  $O_d=6.2$ .

the total observed liquid production rate after the application of the SEN and SEP procedure. The  $R^2$  value decreased from 0.78 to 0.57 after the SEN and SEP procedure was applied. The largest differences appear to be at the early and late times.

### 3.4 Chapter Summary

The application of the multiple linear regression techniques proposed by Albertoni (2002) to data from the Phase 2 Area of the Little Creek Field, Mississippi for the time period 1/1/1989 to 1/31/1991 yielded much more scattered results than those shown in Albertoni (2002) and in Dinh (2003). For this particular case there is a significant decrease in correlation coefficients for the longer diffusivity coefficients suggested by both Albertoni (2002) and Dinh (2003). A simple radius of investigation

calculation similar to that found in well testing indicated that the diffusivity filter should be no more than one month and appears to provide a good estimate of the value to use with the highest  $R^2$  value for the total production rate match and having fewer negative weighting coefficients. 6-month and 12-month (shown in Appendix A) diffusivity filters provide significantly worse results than either the no diffusivity filter or the one month diffusivity filter cases. Interpreted fluid flow in the reservoir varies significantly depending on whether the unrestricted weighting coefficient method is used or whether the SEN and SEP procedures are used. Considering that all of the producers and injectors except P1 and P11 show high flow rates with reasonably good pressure support throughout the area, it would appear that the unrestricted weighting coefficient method provides the better result with all of the injectors broadcasting their fluid out with fewer interpreted barriers or boundaries.

An important assumption of the MLR method is that the predictor variables are linearly independent, that is, no linear relationship exists between the predictor variables. If the predictor variables carry common information, problems could occur in the model causing spurious results. A statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated is called multicollinearity which could cause the negative weighting coefficients seen in the results. If multicollinearity does exist in these models, then the results obtained from the MLR model with and without diffusivity filters may be providing spurious results which should not be used for further interpretation of the sweep of fluid in the reservoir. For future work, a more detailed statistical analysis to determine if multicollinearity exists in the system can be conducted for better understanding of the MLR results.

As discussed earlier, Albertoni (2002) suggests that these negative weighting coefficients should be set to zero thereby eliminating those injector-producer well pairs from the analysis. Since there is no statistical justification behind Albertoni's recommendation, to avoid the need for making *ad hoc* assumptions, an alternative method called the Simple Linear Model will be used in Chapter 4 to evaluate injector-producer well pair connectivity.

## CHAPTER 4: SIMPLE LINEAR MODEL APPLICATION

Inferring the inter-well connectivity between injector and producer well pairs to evaluate the effectiveness of a displacement process when reservoir information is limited to just injection and production rate data has been shown to be a difficult task. The limited data provided for the 11 wells being analyzed in the Phase 2 area of Denbury's Little Creek Field limits the various methods that can be used to determine the effectiveness of the displacement process and how it might be improved. Using injection and production rate data only, the MLR method with and without diffusivity filters along with the implementation of the SEN and SEP procedure was applied to the Little Creek Field phase 2 data. The results in Chapter 3 show that the implementation of diffusivity filters did not significantly improve results for this system. The implementation of the SEN and SEP procedure to the MLR model showed weaker correlations between the modeled and actual production rates

The Simple Linear Model, which is a much less complicated method in comparison to the MLR model with the application of the SEN and SEP procedure, is used in this chapter to infer injector-producer well pair relationships.

The significance of the relationship between each injector-producer well pair can be determined by a statistical significance hypothesis test. It is important to note that although the Simple Linear Model is to be used to infer a physical relationship between each injector-producer well pair, like the MLR approach, there is no physical reason for actually using this type of statistical model to propose such a relationship other than to imply that there is some correlation in the data. But, with provided

data being limited to just injection and production rates, the use of a statistical approach to attempt to infer the physical relationships between injector-producer well pairs could help in evaluating the effectiveness of the displacement process of the system.

#### 4.1 Simple Linear Model

The simple linear model for a production well  $j$  is

$$q_j = a + bi_i \tag{4.1}$$

where the liquid production rate of a well  $j$ , is  $q_j$ ,  $i_i$  is the injection rate of an injector  $i$ . The constant term  $b$  is simply the rate at which  $q_j$  changes with a change in  $i_i$  (Edwards, 1984) and  $a$  is a constant that is added to the product between the constant  $b$  and  $i_i$ . When the value of  $b$  is positive, the relationship between the two variables is positive; that is, an increase in  $i_i$  is accompanied by an increase in  $q_j$  and a decrease in  $i_i$  is accompanied by a decrease in  $q_j$  (Edwards, 1984).

A negative relationship means that an increase in  $i_i$  is accompanied by a decrease in  $q_j$ , and a decrease in  $i_i$  is accompanied by an increase in  $q_j$  (Edwards, 1984).

The application of the simple linear model to the 11 producers and 4 injectors in the Phase 2 portion of Denbury Little Creek's Field gives the  $b$  values shown in Table 4.1a. Interestingly, similar to the MLR case without diffusivity filters, results shown in Table 4.1a indicate a negative relationship between injector I1 and producers P8 and P9. Also, similar to the MLR case without diffusivity filters, injector I2 has a positive weighting coefficient greater than one for this system. However, in comparison to the

MLR case, which had 15 negative weighting coefficients, only 2 negative weighting coefficients are obtained with the application of the SLM method to the Little Creek Phase 2 data. Table 4.1.b shows the  $R^2$  value of each injector-producer well pair for the Little Creek Field Phase 2 data.

|    | P1   | P2   | P3   | P4   | P5   | P6   | P7   | P8    | P9    | P10  | P11  |
|----|------|------|------|------|------|------|------|-------|-------|------|------|
| I1 | 0.07 | 0.46 | 0.08 | 0.58 | 0.64 | 0.08 | 0.14 | -0.18 | -0.09 | 0.22 | 0.03 |
| I2 | 0.09 | 0.01 | 0.20 | 1.24 | 0.13 | 0.58 | 0.33 | 0.47  | 0.29  | 0.29 | 0.06 |
| I3 | 0.08 | 0.09 | 0.16 | 0.77 | 0.13 | 0.54 | 0.32 | 0.41  | 0.31  | 0.25 | 0.06 |
| I4 | 0.05 | 0.01 | 0.13 | 0.74 | 0.03 | 0.41 | 0.23 | 0.33  | 0.23  | 0.18 | 0.06 |

**Table 4.1a** SLM showing  $b$  values of each injector-producer well pair for the Little Creek Field Phase 2. data.

|    | P1   | P2     | P3   | P4   | P5      | P6   | P7   | P8   | P9   | P10  | P11  |
|----|------|--------|------|------|---------|------|------|------|------|------|------|
| I1 | 0.14 | 0.48   | 0.05 | 0.12 | 0.58    | 0.01 | 0.07 | 0.06 | 0.03 | 0.11 | 0.03 |
| I2 | 0.33 | 0.0003 | 0.35 | 0.65 | 0.03    | 0.67 | 0.44 | 0.42 | 0.39 | 0.13 | 0.13 |
| I3 | 0.32 | 0.03   | 0.31 | 0.32 | 0.04    | 0.74 | 0.55 | 0.50 | 0.58 | 0.13 | 0.15 |
| I4 | 0.12 | 0.0002 | 0.18 | 0.27 | 0.00204 | 0.40 | 0.27 | 0.31 | 0.29 | 0.09 | 0.15 |

**Table 4.1b** SLM showing the  $R^2$  value of each injector-producer well pair for the Little Creek Field Phase 2 data.

#### 4.1.1 Hypothesis testing

A hypothesis results from speculation concerning an observed behavior.

As stated earlier, the significance of the relationship between each injector-producer well pair can be determined by a statistical significance hypothesis test.

The types of statistical hypotheses are the null hypothesis  $H_0$  and the alternative hypothesis  $H_1$ . The null hypothesis is the hypothesis which requires no action to be taken. That is, no changes need be made. When trying to determine the significance between each injector-producer well pair, the null hypotheses for the statistical model is that there is no significant relationship between each injector- producer well pairs. The

alternative hypothesis  $H_1$ , is a statement that does not agree with the null hypothesis.

The alternative hypothesis is accepted if the null hypothesis is to be rejected. In the SLM model, the alternative hypothesis states that the injector-producer well pair relationship is significant.

The choices of decisions to be made for the statistical significance hypothesis test will be to reject the null hypothesis  $H_0$  (and conclude  $H_1$ ) or to not reject the null hypothesis. The types of errors of a hypothesis test are the type I and type II errors. The  $\alpha$  ( $P$ -value) term accounts for the probability of making a type I error which occurs when  $H_0$  is incorrectly rejected. Historically, the most frequently used  $\alpha$  value has been 0.05. The  $P$ -value can be used to test the significance level of the relationship between each injector-producer well pair. A  $P$ -value less than 0.05 would then indicate that the relationship between the injector-producer well pair is significant and a  $P$ -value greater than 0.05 indicates that the relationship between the injector producer well pair is non-significant. After determining the significant  $b$  value these values can be checked to see if they provide further information about the effectiveness of the displacement which should help in the provision of recommendations for operational changes that might improve the displacement.

#### **4.1.2 Results**

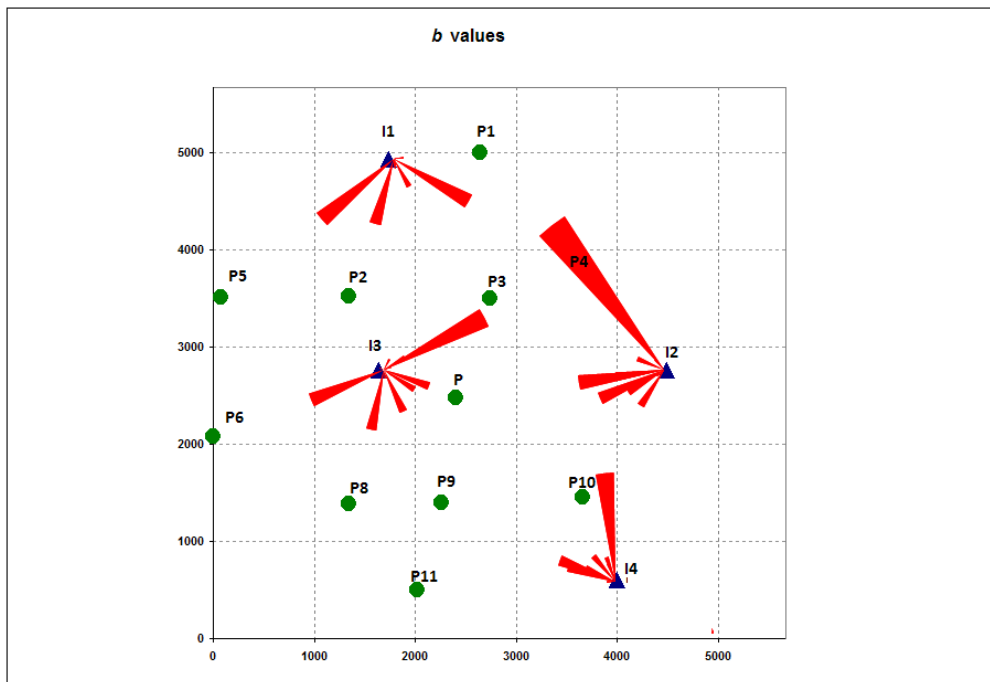
Table 4.2 shows the results of the hypothesis test. Well pairs with significant relationships are represented with  $b$  values greater than 0 and the non-significant injector-producer relationships are represented with  $b$  values equal to 0. There are 11 well pairs with insignificant relationships. Two wells are only correlated with injector I1 and

injector I1 has no correlation with 6 producers (P3, P6, P7, P8, P9, and P11). Similar to the cases with and without diffusivity filters, well pair I2 and P4 have a weighting coefficient greater than 1 and this relationship is statistically significant.

|    | P1   | P2   | P3   | P4   | P5   | P6   | P7   | P8   | P9   | P10  | P11  |
|----|------|------|------|------|------|------|------|------|------|------|------|
| I1 | 0.07 | 0.46 | 0.00 | 0.58 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| I2 | 0.09 | 0.00 | 0.20 | 1.24 | 0.00 | 0.58 | 0.33 | 0.47 | 0.29 | 0.29 | 0.06 |
| I3 | 0.08 | 0.00 | 0.16 | 0.77 | 0.00 | 0.54 | 0.32 | 0.41 | 0.31 | 0.25 | 0.06 |
| I4 | 0.05 | 0.00 | 0.13 | 0.74 | 0.00 | 0.41 | 0.23 | 0.33 | 0.23 | 0.18 | 0.06 |

**Table 4.2** Shows significant  $b$  values and non-significant  $b$  values that have been set to zero.

Figure 4.1 is a representation of the data shown in Table 4.2. Figure 4.1 shows the magnitude of the relationship between each injector-producer well pair represented with the significant  $b$  values. As shown in Figure 4.1, injector I1 appears to have a stronger



**Figure 4.1** Representation of the significant  $b$  values for the Little Creek Field Phase 2 data. connectivity with producers P2, P4 and P5 in comparison to the other producers in the model. Similar to the previous MLR models with and without diffusivity filters, injector

I1 shows a smaller magnitude of connectivity with producers. Again, looking at the relative rates in P1, all of the injectors appear to be influencing P1 at about the same level. Considering the proximity of P1 to the injectors, one would expect that I1 and I2 would have much stronger influence over P1.

Unlike the previous MLR models with and without diffusivity filters, in this SLM model, injector I2 shows connectivity with all but two producers in the system. In the previous cases, injector I2 showed little to no connectivity with most of the producers in the system. This would imply that the area around I2 has a more effective sweep than would be suggested by the MLR models.

Producer wells P2 and P5 are only being influenced by injector I1. Producer P3 is not being influenced by injector I1 and the magnitude of connectivity between producer P3 and the surrounding injectors is low. Similar to the previous MLR models (except the case with 1-month filters) the results of this SLM model also suggests that there is something preventing flow from injector I3 to the north. One possibility is a barrier or fault in the portion of the field where producers P2, P3 and P5 are located. The suggested barrier or fault seems to be preventing the injection fluid of I1 from sweeping south of these well locations and preventing the injection fluid of I3 from sweeping upwards to the north towards these well locations. No barrier or fault appears on any of the geologic maps provided by Denbury.

Injector I4 seems to be having the highest influence on producer P4 which is much further away from it in comparison to nearby producers of which it is barely influencing such as producers P10 and P11. Considering the location of injector I4 and producer P11 relative to the sand boundary, there is a possibility that some of the

injection fluid is flowing out of boundary which reduces the effect I4 is likely to have on P11 and the rest of the producers in the system. But that still does not explain why there seems to be a stronger connectivity with producer P4 and injector I4 in comparison to the other producers in the system especially P10 which is much closer to I4, but has a lower  $b$  value.

## **4.2 THE MLR APPROACH WITH SIGNIFICANT INJECTOR-PRODUCER WELL PAIRS**

Now that the injector-producer well pairs with significant relationships have been determined without making any ad hoc assumptions for the elimination of negative weighting coefficients and positive weighting coefficients greater than one, the MLR approach can be used to determine inter-well connectivity between injector-producer well pairs. The results from the application of the significance hypothesis test using  $P$ -values to the SLM method suggests that the injector-producer well pairs with significant relationships have positive relationships with each other. That is, an increase in the injection rate of an injector  $i$  results in an increase in the production rate of a producer  $j$ . Hopefully, the application of the MLR approach to the Little Creek Phase 2 data using these significant well pairs will show similar positive relationships between injector-producer well pairs.

### **4.2.1 Results**

Table 4.3 shows the results of the application of the MLR model to Little Creek Phase 2 data after the application of the significance hypothesis test using  $P$ -values to the SLM method.

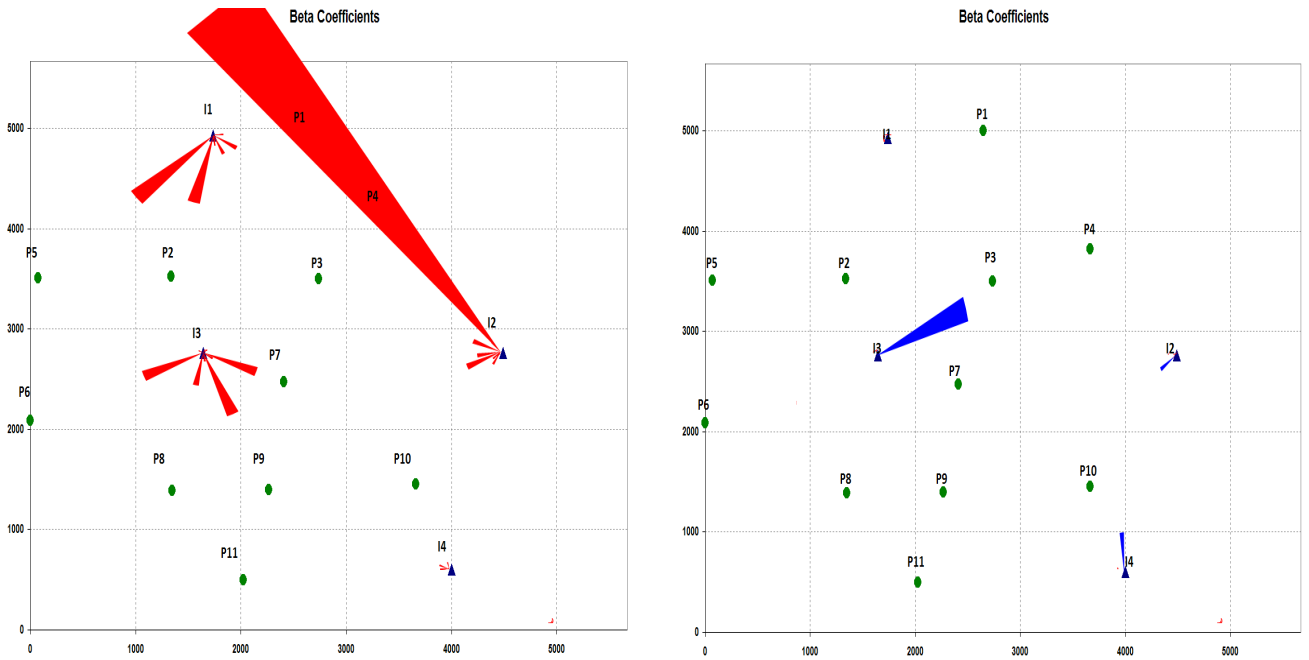
|              | P1    | P2    | P3    | P4      | P5     | P6     | P7    | P8     | P9     | P10   | p11    |
|--------------|-------|-------|-------|---------|--------|--------|-------|--------|--------|-------|--------|
| $R^2$        | 0.40  | 0.48  | 0.35  | 0.85    | 0.58   | 0.74   | 0.56  | 0.56   | 0.62   | 0.20  | 0.10   |
| $\beta_{ij}$ | -9392 | 23921 | 49235 | -181762 | -37835 | -66567 | 24808 | -58310 | -23718 | 26243 | 3042   |
| I1           | 0.03  | 0.46  | 0.00  | 0.17    | 0.64   | 0.00   | 0.00  | 0.00   | 0.00   | 0.14  | 0.00   |
| I2           | 0.09  | 0.00  | 0.19  | 2.93    | 0.00   | 0.15   | -0.06 | 0.23   | -0.24  | 0.09  | -0.003 |
| I3           | 0.02  | 0.00  | 0.03  | -1.37   | 0.00   | 0.41   | 0.36  | 0.22   | 0.45   | 0.07  | 0.00   |
| I4           | -0.04 | 0.00  | -0.02 | -0.39   | 0.00   | 0.01   | 0.01  | 0.01   | 0.07   | 0.01  | 0.01   |

**Table 4.3.** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with significant well pairs, and  $O_d=7.2$

The SLM model suggested that producer P1 had a small positive relationship with the injectors in the system but when input in the MLR model, the results suggests that there is a small negative connectivity between injector I4 and producer P1. After the elimination of the non-significant injector-producer well pairs, the SLM model suggested that, producer P3 had a positive relationship with injectors I2, I3, and I4 but the results from the MLR model suggest that producer P3 has a negative relationship with injector I4.

Also, the SLM model suggested that producers P7 and P9 had positive relationships with injector I2 but results from the MLR model suggest that this relationship is negative. The MLR model also suggests a negative relationship between injector I3 and producer P11 while the SLM model suggested that this relationship is positive. The SLM model suggests that a significant positive relationship exists between producer P4 and the surrounding injectors in the system but the MLR results shown in Table 4.3 suggest otherwise. The results suggest that Producer P4 has a negative relationship with injectors I3 and I4. Also the magnitude of the weighting coefficients increases for the injector I2 and producer P4 well pair in comparison to the SLM results.

Figure 4.2 shows the representation of the positive weighting coefficients shown in Table 4.3.



**Figure 4.2.** Weighting coefficients  $\beta_{ij}$  for Little Creek Field MLR with significant well pairs, and  $O_d=7.2$ . Left graph shows positive weighting coefficient; right graph shows negative weighting coefficients.

Similar to the MLR case without diffusivity filters shown in Figure 3.6, Results shown in Figure 4.2 suggest that injector I1 has very little connectivity with producer P1. Also, Injector I1 shows a higher connectivity with producers P2 and P5 in comparison with the other producers in the system. Again, this could be because of the loss of injection fluid from I1 out of boundary to non-productive layers in the reservoir; thereby reducing the effect I1 has on producers in the system. With the exception of Producer P11 which injector I2 shows little connectivity with, similar to the MLR case without diffusivity filters, Injector I2 shows connectivity with producers P1, P3, P4, P6, P8 and P10 with the exception of P2 and P5 whose relationship was determined as insignificant. Similar to the MLR case without diffusivity filters, Injector I3 shows connectivity with producers P1, P3, P6, P7, P8, P9 and P10. Injector I4 shows very little connectivity with

producers in the system which is also similar to the MLR case without diffusivity filters.

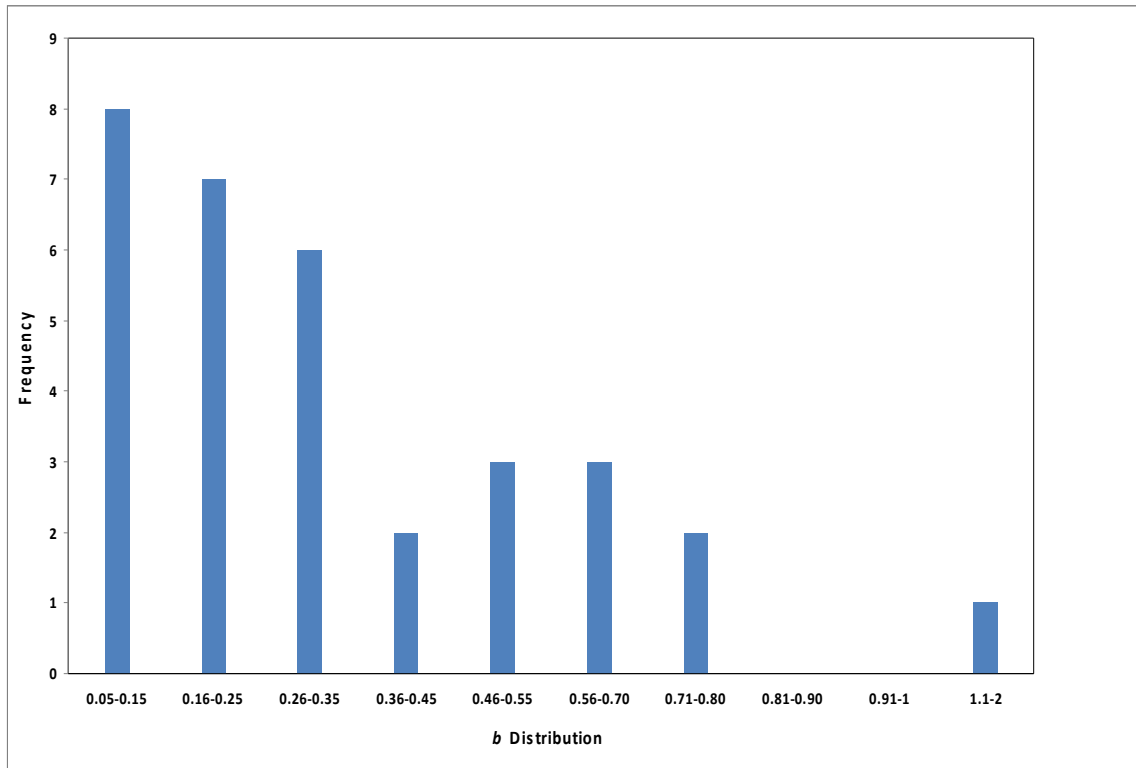
In general, evaluation of the movement of fluids in the reservoir from the results of the MLR model without diffusivity filters and the MLR approach with significant injector-producer well pairs seem to be similar. Even after the elimination of the nonsignificant injector-producer well pairs, the results from the MLR model suggest that there are still 7 negative weighting coefficients. However, of the 7 negative coefficients only 3 are larger than  $-0.06$ . The original MLR technique had 8 negative coefficients larger than  $-0.06$ . This significant reduction in the number of large negative coefficients is encouraging in that small negative coefficients may be indicators of barriers (as shown by both Albertoni (2002) and Dinh (2003)), but there is no clear physical explanation for large negative coefficients. This shows that it may be possible to use the combination of the SLM and MLR methods to obtain some physical understanding of fluid movement.

#### **4.3 RELATIONSHIPS BETWEEN $b$ VALUES AND RESERVOIR CHARACTERISTICS**

When the significant relationships between injector-producer well pairs have been determined, the  $b$  values can be used in an attempt to infer further information about the effectiveness of the displacement process which should help to provide recommendations for operational changes that might improve the displacement. Various plots showing the relationship between the  $b$  values for each injector-producer well pair and their cumulative recovery, average production, and other reservoir characteristics such as permeability and porosity for each producer can be used to evaluate those relationships.

As shown in Figure 4.3, the distribution of the  $b$  values is approximately Log-

Normal. So plots of the natural log of the  $b$  values for each producer and their cumulative



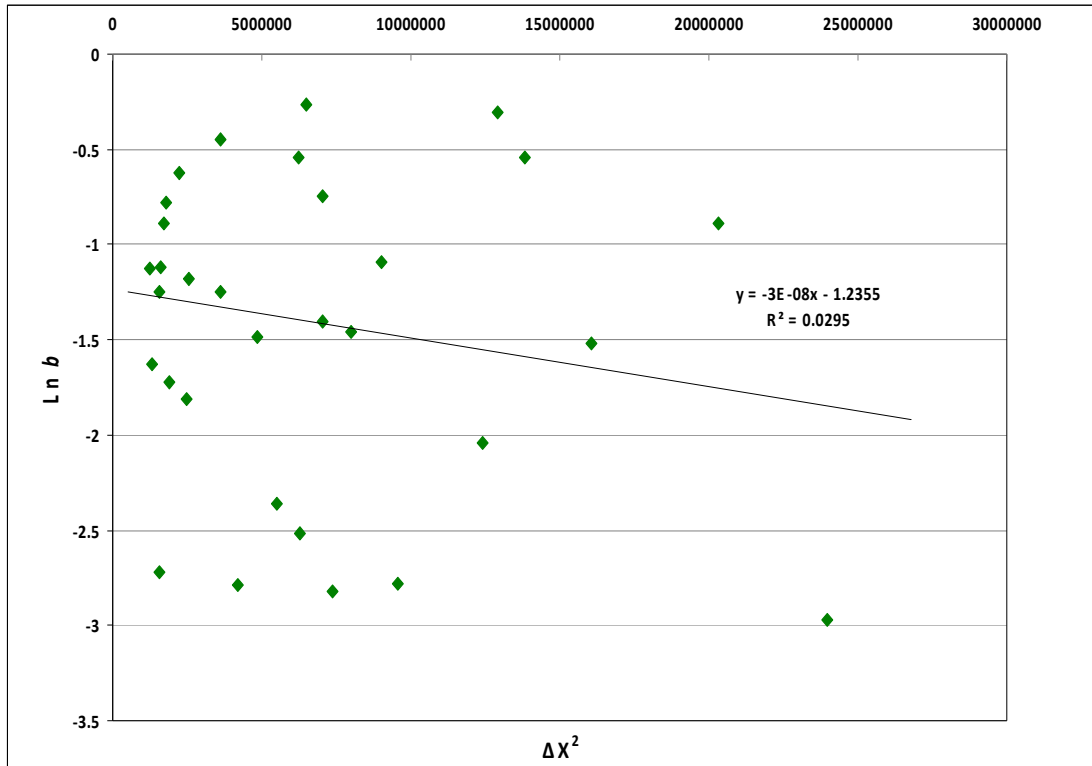
**Figure 4.3** Log Normal Distribution of  $b$  values

recovery, average production, and other reservoir characteristics should better show the desired relationships. While the correlations were better for the natural log of the  $b$  values than using the values themselves, the correlation coefficients were very small.

Significance tests were performed and the slopes were found not to be significant.

Figure 4.4 shows an example plot showing the relationship between  $\ln(b)$  values for each producer and the square of the distance between each injector-producer well pair. As shown in Figure 4.4 the relationship is negative which is expected because, at least in a homogeneous environment, as the distance between each injector-producer well pair increases, there should be a decrease in the effect an injector has on a producer. The

$R^2$  value for this relationship is very low which indicates a weak correlation. The significance hypothesis test on the slope of this relationship was found to be 0.44 which lies between the interval of -1.65 and 1.65 necessary to reject the hypothesis that the slope is significant.



**Figure 4.4** showing a weak relationship between the Log Normal  $b$  values and distance.

Plots of  $\sum b$  for each producer and the average oil rate and the cumulative recovery as well as plots of the  $\ln(b)$  versus the permeability, Dykstra-Parsons coefficient and Lorenz coefficients also yielded low correlation coefficients and insignificant slopes. These results are shown in Appendix B.

#### 4.4 THE MLR APPROACH WITH APPLICATION OF HYPOTHESIS TEST

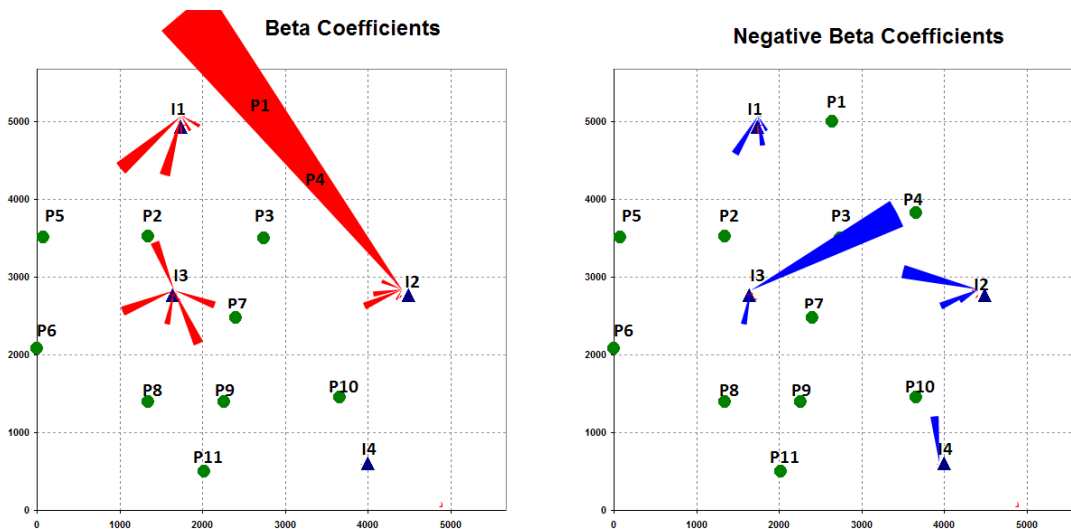
The hypothesis test (described in Appendix B) was next applied to the MLR model to

determine the significance of each injector-producer well pair and the results are shown in Table 4.4. The non-significant injector-producer well pairs are represented with weighting coefficients equal to zero.

|          | P1     | P2     | P3    | P4       | P5      | P6      | P7     | P8     | P9     | P10    | P11    |
|----------|--------|--------|-------|----------|---------|---------|--------|--------|--------|--------|--------|
| $R^2$    | 0.3678 | 0.6329 | 0.347 | 0.845566 | 0.58136 | 0.93328 | 0.5547 | 0.8743 | 0.8235 | 0.1757 | 0.069  |
| $B_{oj}$ | -12503 | 60507  | 50131 | -181762  | -42522  | -33929  | 27141  | -15348 | 12990  | 41808  | 5647.3 |
| I1       | 0.03   | 0.51   | 0.00  | 0.17     | 0.67    | -0.36   | 0.00   | -0.46  | -0.25  | 0.14   | 0.00   |
| I2       | 0.09   | -0.65  | 0.18  | 2.93     | 0.00    | 0.24    | -0.06  | 0.35   | -0.18  | 0.09   | 0.02   |
| I3       | 0.00   | 0.42   | 0.00  | -1.37    | 0.00    | 0.46    | 0.36   | 0.29   | 0.49   | 0.00   | 0.00   |
| I4       | 0.00   | 0.00   | 0.00  | -0.39    | 0.00    | 0.00    | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   |

**Table 4.4.** Weighting coefficients  $\beta_{ij}$  for Little Creek Field Phase 2 data. MLR with significant well pairs after hypothesis test, with  $O_d=7.2$

Figure 4.5 is a representation of the data shown in Table 4.4. Similar to the



**Figure 4.5.** Weighting coefficients  $\beta_{ij}$  for Little Creek Field MLR with significant well pairs after hypothesis test, with  $O_d=7.2$  Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients.

previous MLR models, Figure 4.5 shows both positive and negative relationships between injector-producer well pairs. The overall flow of fluid in this model looks similar

to both of the MLR case without diffusivity filters. Injector I4 showed very little connectivity with producers in the model for the MLR case without filters and the hypothesis test suggests that injector I4 does not have a significant positive relationship with any producer in the model. I4 shows a negative relationship with P4 and results suggest that this relationship is significant. This model also suggests that the large negative connectivity between I2 and P4 is significant. In general, this model did not provide more insight that will lead to a better understanding of where fluid is flowing in the reservoir. There are still injector-producer well pairs with significant negative coefficients that cannot be explained. With that being said, this model does serve as a more statistically valid approach for eliminating non-significant well pairs than the SEN and SEP procedures suggested by Albertoni (2002) because significance tests drive the elimination of well pairs.

#### 4.5 RESULT SUMMARY OF THE MLR CASES

|        | MLR MODELS               | $O_d$ | $R^2$ |
|--------|--------------------------|-------|-------|
| CASE 1 | MLR with 0 filters       | 7.4   | 0.83  |
| CASE 2 | MLR with 1 month filter  | 7.2   | 0.86  |
| CASE 3 | MLR with 6 month filter  | 6.2   | 0.78  |
| CASE 4 | MLR with 12 month filter | 5     | 0.72  |
| CASE 5 | MLR with SLM results     | 7.4   | 0.79  |
| CASE 6 | MLR with hypothesis test | 7.4   | 0.82  |

**Table 4.5.** Result summary of the MLR models applied to the Little Creek Field Phase 2 data.

Table 4.5 shows that based on the  $R^2$  values much better results are obtained from the implementation of the MLR approach with 1 month diffusivity filters to the Phase 2 portion of Denbury's Little Creek Field in Mississippi.

Although the  $R^2$  value can be used to determine how closely correlated the

modeled production rate and actual production rates are for the system, the objective of this analysis is to determine injector-producer well connectivity. The difficulty in this analysis becomes how do we evaluate which model is a “better” model? Albertoni (2002) and Dinh (2003) used the  $R^2$  value obtained for each model to determine which model is “better”. The use of only  $R^2$  values would suggest that a better model is chosen based solely on how well fluid rates can be predicted rather than the movement of fluid in the reservoir.

An F-test was applied to the various MLR model cases. The F-test is used to detect the significance of a shift in the standard deviations, i.e., the likelihood in making a mistake in saying the standard deviations are different or the hypothesis that the proposed MLR models fit the actual data. The F-test analysis indicates if there is a (1-pvalue)x100% confidence that the variance between the models are not equal. Results from this test are shown in Table 4.6 and the results shown in Table 4.6 suggests that the variance between the standard deviations of the model and actual rates are not equal.

|        | MLR MODELS                | CONFIDENCE (%) |
|--------|---------------------------|----------------|
| CASE 1 | MLR with 0 filters        | 54             |
| CASE 2 | MLR with 1 month filters  | 55             |
| CASE 3 | MLR with 6 month filters  | 48             |
| CASE 4 | MLR with 12 month filters | 69             |
| CASE 5 | MLR with SLM results      | 50             |
| CASE 6 | MLR with hypothesis test  | 67             |

**Table 4.6.** F-test Result summary of the MLR models applied to the Little Creek Field Phase data.

This suggests that the MLR models are not very different from each other. Also, the use of the F-test would suggest that a “better” model is being chosen based solely on how well fluid rates can be predicted rather than the movement of fluid in the reservoir.

Again, even though the time calculation suggests that there is a maximum of a 1 month

lag between injector-producer response, the MLR models still shows significant negative coefficients and coefficients greater than one for those cases. It is difficult to ascertain which model is correct because all the models have significant negative coefficients that we do not understand. The use of hypothesis testing helped reduce the number of negative weighting coefficients but not all of them. Results from the simple linear model showed positive relationships between all the injector-producer well pairs so this model will be used in Chapter 5 to provide recommendations to improve well performance in Phase 2.

## CHAPTER 5: WELL PERFORMANCE EVALUATION

Once the flow directions in a reservoir have been obtained, the next question to be answered is “can these flow patterns be modified to improve sweep and recovery?.” As shown in Chapter 4, it is not statistically clear which of the models represents flow in Little Creek. For most of this chapter, results from the Simple Linear Model will be used to evaluate performance improvements in the Phase 2 portion of Denbury Resources Little Creek field in Mississippi.

At any given time in the life of a reservoir, the static pressure or the average reservoir pressure is approximately fixed and the flow into the well depends on the pressure drawdown. Well deliverability can be represented with well inflow and wellbore outflow performance relationships. The inflow performance relationship (IPR) of a well is represented as the production rate of a well as a function of its flowing bottomhole pressure. It describes the deliverability of the reservoir. The vertical lift performance or tubing performance relationship (TPR) of a well represents the ability of fluid to flow from the bottomhole through the pipelines and surface facilities to the surface storage tank (Suk Kyoon Choi et. al. , 2008).

### 5.1 Inflow Performance Relationships

“In a vertical geometry with single-phase oil under steady state, Darcy proposed a constitutive equation that describes the flow of a fluid through a porous medium. Darcy can be expressed in various forms according to reservoir geometries. The following form represents the steady state radial flow in a circular drainage area with potential skin effect in the near wellbore” (Suk Kyoon Choi et. al. , 2008). Inflow performance of an oil well

can be expressed with the productivity index ( $J_o$ ).

$$J_o = \frac{q_o}{P_e - P_{wf}} = \frac{kh}{141.2 B_o \mu_o [\ln(r_e/r_w) + s]} \quad (5.1)$$

where,

$J_o$  = productivity index

$q_o$  = inflow rate, STB/day

$k$  = effective oil permeability, md

$h$  = reservoir thickness, ft

$p_e$  = pressure at  $r = r_e$ , psia

$p_{wf}$  = wellbore flowing pressure at  $r = r_w$ , psia

$r_e$  = wells drainage radius, ft

$r_w$  = wellbore radius, ft

$\mu_o$  = oil viscosity,  $cp$

$B_o$  = oil formation volume factor bbl/STB

In a depletion drive system, the standard method to improve the productivity of a well is to increase the pressure drawdown by reducing the flowing bottomhole pressure of the well. The flowing bottomhole pressure of a well can be decreased by reducing the pressure losses between the bottomhole and the separation facility, either by optimizing tubing sizes or by implementing or improving artificial lift procedures (Economides et al, 1994). Another method of improving the productivity of a well is by reducing the skin effect(s) which accounts for near-wellbore damage. This can be done through matrix stimulation and removal of near-wellbore damage (Economides et al, 1994).

For wells that flow mostly water, Equation 5.1 also holds with the knowledge that the oil properties in the equation are now water properties (i.e  $B_w, K_w, \mu_w$  and  $q_w$ ). For wells flowing mostly gas, Equation 5.1 does not generally hold. Because of the small gas viscosity value and the high compressibility, inertial effects are significant unless

flowrates are small. The productivity index then is highly non-linear and not nearly as approximately constant over the reservoir life as is typically assumed for oil or water systems.

Standard nodal analysis techniques (Beggs, 2003) can be used to provide an understanding of the effects that changing reservoir and flowing bottomhole pressure values would have on production or injection rates. Because there was no pressure data provided for the study area, approximate methods should provide insight into the effect that changing flowrates in one well might have on flow rates in another well. Therefore, Equation 5.1 will be used with the reservoir flow rates shown in Figures 3.2 and 3.3. For pressures, it is known that the miscibility pressure in Little Creek is approximately 4500 psi (Senocack et. al, 2008). In addition, data from the operator for recent operations indicate that the surface injection pressures are around 1300 psi. This yields an injection well pressure of approximately 5025psi using standard tubing flow correlations (API RP14B, 1976). For rates similar to those in the data set, wellhead pressures are slightly higher at around 1800 psi yielding a flowing bottomhole pressure of around 4830psi. This suggests an average reservoir pressure of around 4925 psi.

Results from the SLM model indicates that producers P1 and P11 show the least connectivity with injectors in the system. Well P3 and P10 show low connectivity despite having two of the largest production rates in the system. Producer P4 shows a very strong connectivity with injector I2 and Injector I4 shows little connectivity with its surrounding producers. Fluid from injector I3 seems to only be moving towards wells to the south leaving areas between I3 and wells to the north of it unswept.

Producers P1 and P11 both have the lowest production rates in the system which

could be because of their location relative to the sand boundary. Low productivity in these wells could also be due to high flowing bottomhole pressures or low reservoir pressure in the area. To increase well productivity for these wells, typically an increase in the pressure drawdown would be required and would be implemented by reducing the flowing bottomhole pressures since the wells in Phase 2 are currently flowing and not on artificial lift. Because this is a CO<sub>2</sub> flood, bottomhole pressures need to be held above the miscibility pressure (4500 psi) and flowing surface pressures should be higher than 1200 psi in order to keep the CO<sub>2</sub> in a supercritical state.

Based on the flowing bottomhole pressure indicated at 4830 psi, it appears there might be an additional 300 psi of drawdown that could be utilized. This additional 300 psi cannot be attained by changing tubing size in the well, but may be possible if a transfer pump could be utilized at the surface. The assumption here is that the 1800 psi of back pressure is not due to wellhead chokes. If it is, opening the choke would be the easiest way to increase drawdown.

Once drawdown has been increased in a well, that well's area of influence will change and average bottomhole pressure will likely fall unless another well is held back (kept from producing as much as "allowed"). From the SLM results, wells P4, P6, P7 and P9 should be held back (flow rate should be decreased) and wells P2, P3, P5 and P10 should be increased. In addition, wells P1 and P11 should have drawdown increases if possible. Because these wells produce set rates that are so much lower than the other wells, it is not clear at all what to do with them. A pressure transient test may be required to evaluate their productivities.

Table 5.1 shows the productivity index for each producer and the injectivity index

for each injector calculated from the average flow rate over the time period divided by

| Wells | II or PI (bpd/psi) | Oil Rate |
|-------|--------------------|----------|
| I1    | 78.5               |          |
| I2    | 91.35              |          |
| I3    | 89.92              |          |
| I4    | 102.73             |          |
| P1    | 4.89               | 174.24   |
| P2    | 49.41              | 1139.16  |
| P3    | 42.69              | 1824.24  |
| P4    | 33.17              | 702.24   |
| P5    | 33.19              | 1511.4   |
| P6    | 21                 | 2566.08  |
| P7    | 42.39              | 1813.68  |
| P8    | 15.64              | 1209.12  |
| P9    | 15.51              | 3288.12  |
| P10   | 40.78              | 2060.52  |
| P11   | 4.68               | 63.36    |

**Table 5.1** Productivity index for wells in the Phase 2 portion of Denbury Resources Little Creek field in Mississippi.

the pressure difference suggested previously as being somewhat representative of field operations (5025 psi  $P_{wf}$  for injectors, 4830 psi for producers and 4925 psi for average reservoir pressure).

From Eqn. 5.1 an estimate of the new flow rate in each well can be obtained if there is some idea of how the bottomhole flowing pressure and average reservoir pressures change. The estimate can be obtained from

$$q_2 = \frac{q_1}{PI_1 / PI_2} * \frac{\left(1 - P_{wf\ 2} / \bar{P}\right)}{\left(1 - P_{wf\ 1} / \bar{P}\right)} \quad (5.2)$$

This equation can be used either to estimate the new flowrate and/or  $P_{wf}$  combination in a single well or in different wells.

The SLM model also suggests that injector I4 has little connectivity with surrounding producers in the model. Interestingly, for the time period being analyzed, injector I4 has the highest injection rate. This reduced effect between I4 and producers in the system could be a result of its location relative to the reservoir boundary. To check if the SLM model is truly capturing fluid flow from I4 to producers in the model, the injector I4 should be shut-in and nearby producers should be monitored to see the effect in production rate on nearby producers. If there is no rate change observed in nearby producers, then injector I4 should be shut-in permanently because there is no use for this injector in the system. However, if the effect of shutting-in I4 is seen in nearby producers, then the injection rate for I4 should be increased. The average monthly injection rate for I4 is 216,872 RB. This amount could be doubled and the production rates for nearby producers should be monitored to see the effect this rate change has on the system.

Alternatively, if the fluid flow in the reservoir did not show any areas where oil was being left behind, the best option to improve recovery would be to try to increase the amount of the displacing fluid going to those parts of the reservoir with the highest oil cut or simply the highest oil rate. A low oil cut may mean that CO<sub>2</sub> is cycling between the injector and the producer reducing efficiency of the flood.

All of the producers in the Phase II portion of the Little Creek Field have very low oil cut. Most are between 0 and 2%. Wells P6, P9 and P10 are the highest oil producers in the area and also have oil cuts over 2.5%. Wells P3, P7 and P8 also have reasonably high oil rates and oil cuts in excess of 2%. In general, Table 5.1 shows lower oil rates for wells in the northern region of Phase 2 in comparison to wells in the southern part of the area. This would indicate that efforts to improve recovery from this area should be

concentrated on directing more CO<sub>2</sub> to injectors I2, I3 and I4 and production from wells P1, P4, P5 and P11 should be reduced. This would be especially true if the test previously described to evaluate injector I4 was found to be effective at improving recovery from the southern-most wells in the area.

## CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

The determination of the connectivity between injector-producer well pairs to evaluate the effectiveness of the displacement process is a difficult task if reservoir information is limited to injection and production rate data. Various statistical methods have been used to quantify connectivities to guide operational changes to improve the displacement process.

Compared to the Multivariate Linear Regression Model with SEN and SEP, the Simple Linear Model is less complicated and requires fewer *ad hoc* assumptions. SLM can be used to evaluate the relationship between each injector-producer well pair for a selected time period.

The significance of the  $b$  values which represents the relationship between each injector-producer well pair obtained from the Simple Linear Model was determined by  $t$ -test. Plots showing the relationships between the significant  $b$  values for each injector-producer well pair and their cumulative recovery, average production, and other reservoir characteristics were inconclusive based both on statistical hypothesis testing ( $t$ -test) and on the low correlation coefficients obtained.

The Simple Linear Model and hypothesis test (Figure 4.1) suggests that producers P1, P11 and P3 show the least connectivity with injectors in the system. Producer P4 shows a very strong connectivity with injector I2 and Injector I4 shows little connectivity with its surrounding producers. To improve sweep in the northern part of the Phase 2 area, the surface pressure for well P1 should be reduced if possible. The increase in pressure drawdown should increase the productivity for this well. Cutting back on the

production from well P4 which shows a strong connectivity with nearby injectors I2 and I3 may help increase reservoir pressure in the area and allow for fluid to flow from Injectors I2 and I3 to producers to the north of their well location. By increasing the surface pressure for well P4, the operating flow rate for this well should decrease and more fluid will be directed away from this area. SLM results suggest that producer P3 has very little connectivity with injectors in the system while, the actual production data shows that well P3 has the second highest production rate in the model with one of the higher gas-oil-ratios. This information suggests that the area has high heterogeneity, gravity override problems or viscous fingering problems . Cutting back on the production from well P3 should help increase reservoir pressure in the area and allow for more CO<sub>2</sub> may be forced through the apparently bypassed region and the oil would be pushed to the nearby producers. This area may be a good candidate for mobility control test.

In the southern part of the Phase 2 area, injectors I2, I3 seem to have a greater effect on nearby producers in comparison to injector I4. Injector I4 has the highest total injection rate for the time period being analyzed and this raises a concern as to where that injection fluid is going. Injector I4 should be shut-in and nearby producers should be monitored to see the effect in production rate on nearby producers. If there is no rate change observed then injector I4 should be shut-in permanently. If the effect of shutting-in I4 is seen in nearby producers, then to improve sweep the Injection rate for I4 should be increased. The average monthly injection rate for I4 is 216,872 RB. This amount could be doubled and the production rates for nearby producers should be monitored to see the effect this rate change has on the system. As this injection rate is increased, nearby producers P12 and P13 included in both patterns 34-10 and 34-9 which were producing

for this time period but not included in the analysis should be monitored. If more response is seen in producers P12 and P13, then a production cap can be set on one or both wells to increase reservoir pressure in the area; thereby allowing flow of fluid from injector I4 to its surrounding producers in Phase 2.

For monitoring this flood, the best results (in terms of matching predictions to observed rates) were obtained from the Multiple Linear Regression techniques with a one month diffusivity filter or the MLR technique without diffusivity filters. This implies low formation dissipation in Denbury's Little Creek Field: a change in injection rate should quickly cause a response in the production rate of nearby producers. This effect could not be captured in the current work because the production and injection rate information were at one month intervals. Analysis with daily or weekly rate data should be considered in the future.

In the future, daily production and injection rate data can be used in the SLM model to determine the connectivity between injector-producer well pairs and a hypothesis test can be used to test the significance of the relationships seen. Various plots showing the relationships between the significant  $b$  values for each injector-producer well pair and their cumulative recovery, average production, and other reservoir characteristics can be generated. If results attained suggest a significant relationship, this could provide further information about the effectiveness of the displacement process. For the case presented here, these plots did not provide any insight.

## REFERENCES

Albertoni, A. and Lake W.L., “Inferring interwell connectivity from well-rate fluctuations in waterfloods”, Society of Petroleum Engineers, paper SPE 75225, presented at the 2002 SPE improved Oil Recovery Symposium, Tulsa OK, April 2002.

Albertoni, A., Inferring interwell connectivity from well-rate fluctuations in waterfloods, M.S. Thesis, The University of Texas at Austin, May 2002.

Albertoni, A. and Lake W.L., “Inferring interwell connectivity from well-rate fluctuations in waterfloods”, SPE Journal, February, 2003.

Beggs H. Dale, Production Optimization Using Nodal Analysis, 2<sup>nd</sup> Edition. Oklahoma: OGCI, Inc., PetroSkills, LLC and H. Dale Beggs, 2003

Choi Suk Kyoan, Ouyang Liang-Biao, and Huang Wann-Sheng (Bill), “A Comprehensive Comparative Study on Analytical PI/IPR Correlations”, SPE paper 116580 presented at the 2008 SPE Annual Technical Conference and Exhibition, Denver CO, September 2008.

De Sant’Anna Pizarro, J.O., Estimating injectivity and lateral autocorrelation in heterogeneous media, Ph.D. Thesis, The University of Texas at Austin, 1998

Dinh, A., Reservoir characterization for the Delaware-Childers field based on flow rate analysis, M.S. Thesis, The University of Oklahoma at Norman, 2003.

Dinh, A., Tiab.D, “Inferring interwell connectivity from well bottomhole pressure fluctuations in waterfloods”, Society of Petroleum Engineers, paper SPE 106881, 2007.

Dinh, A., Tiab.D, “Inferring interwell connectivity from well bottomhole pressure fluctuations in waterfloods”, SPE Journal, October, 2008.

Dinh, A., Reservoir characterization for the Delaware-Childers field based on flow rate analysis, M.S. Thesis, The University of Oklahoma at Norman, 2003.

Dykstra, H. and Parson, R.L. “The Prediction of oil Recovery by Waterflood” In: A.P. Institute (Editor), Secondary Recovery of Oil in the United States, New York, pp.160-174, 1950.

Economides Michael J., Hill A. Daniel, Ehlig-Economides Christine, Petroleum Production Systems, Prentice Hall, PTR, 1994.

Edwards A.L., An introduction to Linear Regression and Correlation, 2<sup>nd</sup> Edition.  
New York: W.H. Freeman, 1984

Geaghan, James P. Experimental Statistics Course Notes, The Louisiana State University, 2005; [http://www.stat.lsu.edu/faculty/geaghan/EXST3201/Fall2005/PDF/Chapter7\\_SLR01.pdf](http://www.stat.lsu.edu/faculty/geaghan/EXST3201/Fall2005/PDF/Chapter7_SLR01.pdf)

Geaghan, James P. Experimental Statistics Course Notes, The Louisiana State University, 2009; <http://www.stat.lsu.edu/faculty/geaghan/EXST7005/Fall2009/PDF/Lecture08a%20Notes%20Fall2009.pdf>

Guo Boyun, Lyons William C., Ghalambor Ali, Petroleum Production Engineering, 1<sup>st</sup> Edition. Elsevier Inc, 2007

Hansen P.W., A CO<sub>2</sub> Tertiary Recovery Pilot Study and Field Expansion CO<sub>2</sub> process analysis, Shell Oil Company Internal Report, Onshore Division, June 1977

Hansen P.W., “A CO<sub>2</sub> Tertiary Recovery Pilot Little Creek Field, Mississippi”, SPE paper 6747 presented at the 52<sup>ND</sup> Annual Technical Conference and Exhibition, Denver CO, October, 1977.

Heffer, K.J., Fox, R.J., and McGill, C.A., “Novel techniques show links between reservoir flow directionality, earth stress, fault structure and geomechanical changes in mature waterfloods”, Society of Petroleum Engineers, paper SPE 30711, 1995.

Jensen, J.L., Lake, L.W, Corbett, P.W.M. and Goggin, D.J., Statistics for Petroleum Engineers and Geoscientists, Elsevier, 2000.

Jensen, J.L., Lake, L.W, “A Review of Heterogeneity Measures Used in Reservoir Characterization”, SPE paper 20156, 1991.

Kumar N., “Improvements for flow correlations for gas wells experiencing liquid loading”, SPE paper 92049 presented at the 2005 SPE Western Regional Meeting, Irvine CA, March 2005.

Panda, M.N., Chopra, A.K., “An integrated approach to estimate well interactions”, Society of Petroleum Engineers, paper SPE 39563, 1998.

Petroleum Experts Prosper Single Well Systems Analysis Version 9.1

Refunjol, B.T., Reservoir characterization of Northhuck Draw field based on tracer response and production/injection analysis, M.S. Thesis, The University of Texas at Austin, 1996.

Sachs, L., Applied Statistics, 2<sup>nd</sup> Edition, Springer-Verlag New York Inc, 1984

Sayarpour, M., Kabir, C.S, Lake, L. W., “Field Applications of capacitance-resistive models in waterfloods”, Society of Petroleum Engineers, paper SPE 114983, 2008.

Senocak, D., “Effective use of heterogeneity measures in the evaluation of a mature CO<sub>2</sub> Flood”, Society of Petroleum Engineers, paper SPE 113977, presented at the 2008 SPE improved Oil Recovery Symposium, Tulsa OK, April 2008.

Soeriawinata, T., Kelkar, M., “Reservoir management using production data”, Society of Petroleum Engineers, paper SPE 52224, 1999.

# APPENDIX A: MLR WITH 12 MONTH DIFFUSIVITY FILTERS

## A.1 MLR with 12 Month Diffusivity Filters

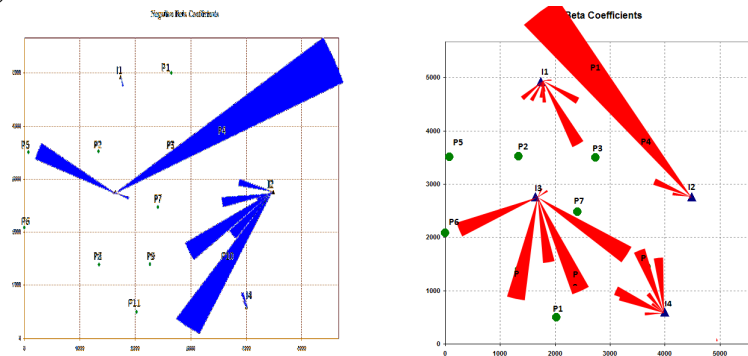
The  $O_d$  factor for this case is 5 which is outside the stated range suggested by Albertoni. We should expect that predictions for this case will be less correlated than in the case with no filters.

The application of MLR with 12-month diffusivity filters to the Phase 2 portion of Denbury Little Creek's Field gives the weighting coefficients shown in Table A.1.

In the results shown in Table A.1, there are both positive and negative linear relationships between the injector-producer pairs. The weighting coefficients shown in Table A.1 are shown in Figure A.1

|              | P1    | P2     | P3   | P4      | P5     | P6      | P7    | P8      | P9      | P10     | P11    |
|--------------|-------|--------|------|---------|--------|---------|-------|---------|---------|---------|--------|
| $R^2$        | 0.23  | 0.66   | 0.38 | 0.96    | 0.76   | 0.75    | 0.29  | 0.72    | 0.76    | 0.17    | 0.23   |
| $\beta_{oi}$ | -9444 | 191118 | 208  | -245658 | 171247 | -192675 | 99576 | -214986 | -107367 | -103565 | -98815 |
| I1           | 0.1   | 0.07   | 0.01 | 0.5     | 0.34   | 0.28    | -0.12 | 0.2     | 0.13    | 0.9     | 0.27   |
| I2           | -0.01 | -0.49  | 0.49 | 3.86    | 0.23   | -0.71   | -0.09 | -0.23   | -1.38   | -2.02   | -0.77  |
| I3           | 0.11  | -0.04  | 0.14 | -3.51   | -1.23  | 1.06    | -0.18 | 1.3     | 1.28    | 1.32    | 0.82   |
| I4           | -0.08 | 0.01   | -0.2 | 0.69    | 0.19   | 0.59    | 0.31  | 0.03    | 0.66    | 0.83    | 0.24   |

**Table A.1** Weighting coefficients  $\beta_{ij}$  for Little Creek Field. MLR with 12 month Diffusivity filters, with  $O_d = 5$



**Figure A.1** Weighting coefficients  $\beta_{ij}$  for Little Creek Field. MLR with 12-month diffusivity filters, with  $O_d = 5$ . Left graph shows positive weighting coefficients; right graph shows negative weighting coefficients.

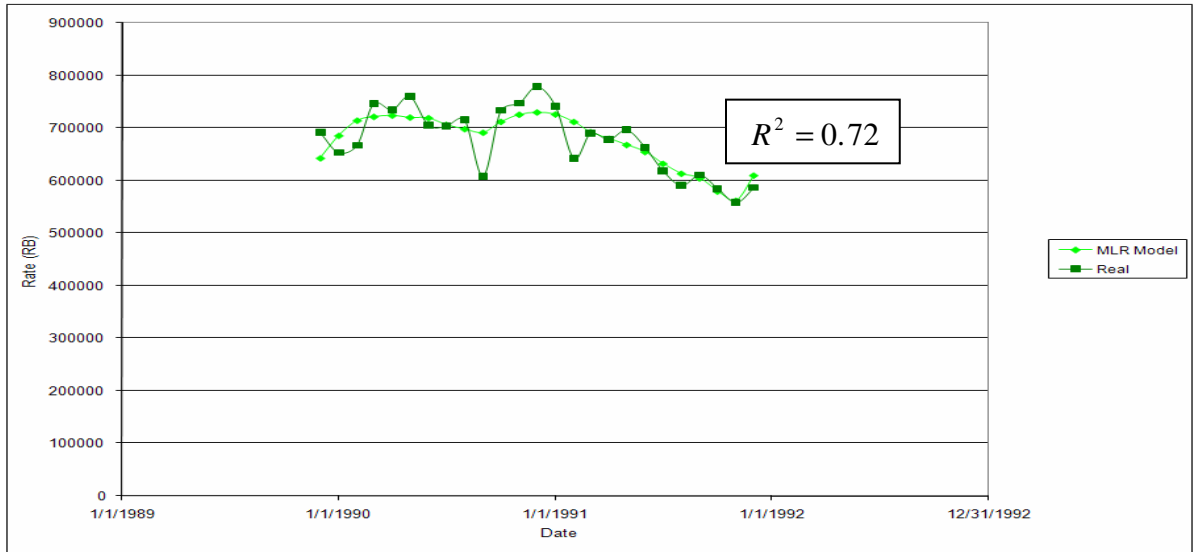
Figure A.1 looks similar to Figure 3.15. The major difference is that Figure A.1 generally shows a larger magnitude of connectivity between injectors I1, I3, I4 and the producers in the system. Similar to the application of 6 month diffusivity filters, injector well I3 seems to be strongly influencing the producers located to the south except for producer P3, with which it has very little connectivity. Again, similar to the case with the application of a 6-month diffusivity filter, injector I3 shows a negative relationship with the producers P2, P4 and P5 above it. These negative coefficients could be indicating the presence of a barrier or fault in between Injector I3 and producers P2, P3 and P5 which is preventing flow from the injector I3 to the north or producers P6, P8, P9 and P10 are at much lower pressures than the P2, P5 and P3 wells drawing fluid south. Interestingly, unlike the previous cases, injector I1 shows the strongest connectivity with producer P10 which is a large distance away.

Figure A.1 shows positive connectivity between injector I4 and all the producers in the system except producers P1 and P3. Also, the magnitude of connectivity increased between injector I4 and the surrounding producers in the system when the 12 month diffusivity filters were applied.

In comparison to the MLR with 6 month diffusivity filter model, the  $R^2$  values for all producers in this model have decreased. This would suggest that the application of the 12-month diffusivity filter did not improve the results of the MLR model as suggested by Albertoni (2002).

Figure A.2 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate. In comparison to the MLR case with 1 month filters, 6 month filters and without filters, the coefficient of determination  $R^2$

value decreased to 0.72. Although this  $R^2$  value is also lower than the MLR case with 6-month diffusivity filters, Figure A.2 looks more like Figure 3.17 in the sense that for both plots, the MLR model rates do not seem to be closely correlating with the actual monthly rates. The modeled rates are smoother than the actual values.



**Figure A.2** Comparison between total modeled liquid production rate and the total observed liquid production rate for the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .

## A.2. Application of the SE-N and SE-P procedure to MLR with 12-month Diffusivity Filters

Table A.2 shows the results obtained after the application of the SEN and SEP procedure to the MLR with 12-month diffusivity filters. Comparing the results in Table A.2 to those in Table A.1 shows that after the SEN and SEP procedures there has been a reduction in the number of injector-producer well pairs to be accounted for in the system.

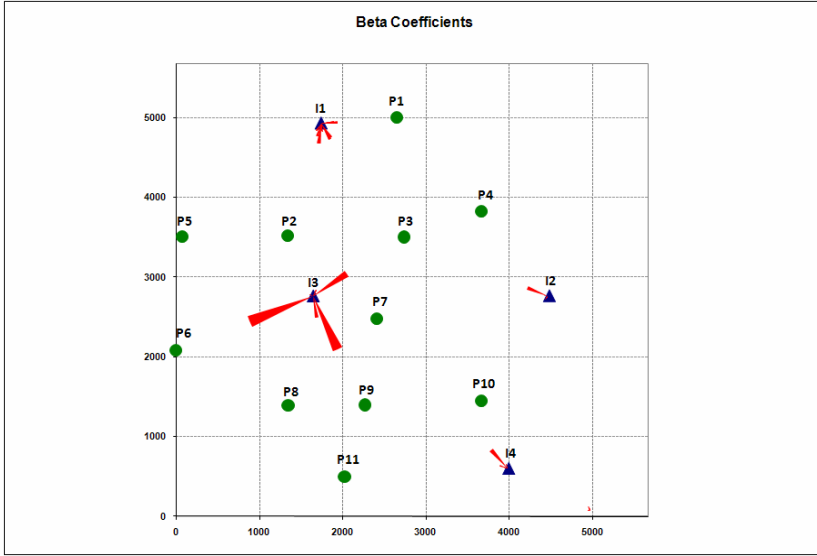
|              | P1     | P2    | P3     | P4    | P5    | P6     | P7    | P8    | P9     | P10   | p11    |
|--------------|--------|-------|--------|-------|-------|--------|-------|-------|--------|-------|--------|
| $R^2$        | 0.22   | 0.07  | 0.37   | 0.00  | 0.00  | 0.72   | 0.13  | 0.00  | 0.43   | 0.03  | 0.12   |
| $\beta_{oi}$ | -23930 | 77692 | -41219 | 88182 | 64302 | -82088 | 46219 | 43714 | -60072 | 51862 | -38089 |
| I1           | 0.14   | 0.11  | 0.14   | 0.00  | 0.00  | 0.07   | 0.00  | 0.00  | 0.00   | 0.15  | 0.06   |
| I2           | 0.00   | 0.00  | 0.18   | 0.00  | 0.00  | 0.00   | 0.00  | 0.00  | 0.00   | 0.00  | 0.00   |
| I3           | 0.06   | 0.00  | 0.33   | 0.00  | 0.00  | 0.55   | 0.00  | 0.00  | 0.48   | 0.00  | 0.18   |
| I4           | 0.00   | 0.00  | 0.00   | 0.00  | 0.00  | 0.07   | 0.2   | 0.00  | 0.00   | 0.04  | 0.00   |

**Table A.2** Weighting coefficients  $\beta_{ij}$  after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12 month Diffusivity filters, with  $O_d = 5$

After the application of the SEN and SEP procedure, 15 negative weighting coefficients and 13 positive weighting coefficients were eliminated leaving 16 injector-producer relationships.

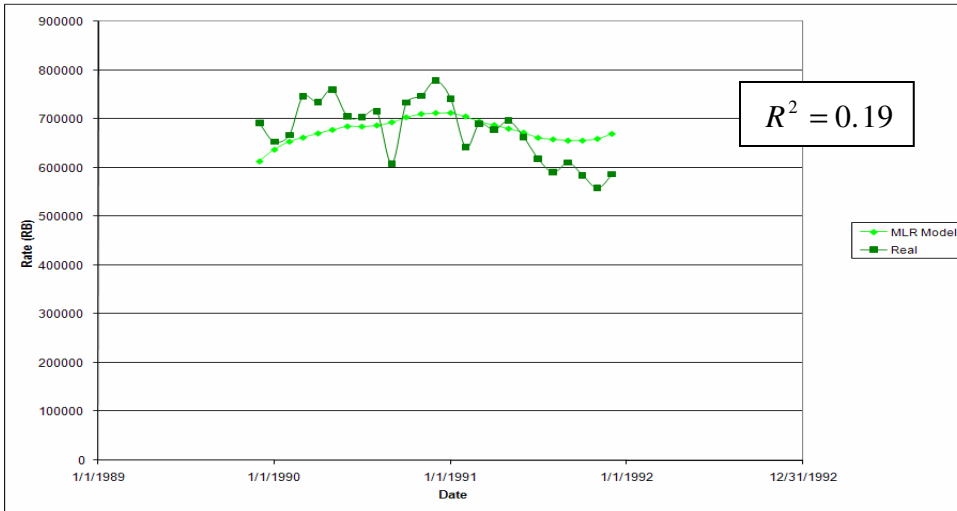
As shown in Table A.2, the  $R^2$  values for all the 11 producers after the implementation of the SEN and SEP procedure have decreased in comparison to the MLR case with 12 month diffusivity filters. This implies that the SEN and SEP procedure did not help improve results.

The weighting coefficients determined after the application of the SEN and SEP procedure are shown in Figure A.3. Both the magnitude and the directionality of the weighting coefficients between injector-producer well pairs are significantly altered relative to Figure A.1. The representation of the weighting coefficients in Figure A.3 shows that, there is barely any computed connectivity between injector-producer well pairs after the application of the SEN and SEP procedure.



**Figure A.3** Representation of the positive weighting coefficients  $\beta_{ij}$  after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .

Figure A.4 shows a comparison between total modeled liquid production rate and the total observed liquid production rate after the application of the SEN and SEP procedure to the MLR case with 12-moth diffusivity filters. The  $R^2$  value decreased from 0.72 to 0.19 after the SEN and SEP procedure was applied.



**Figure A.4** Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to the Little Creek Field Phase 2 data. MLR with 12-month diffusivity filters, with  $O_d=5$ .

## APPENDIX B: APPLICATION OF THE HYPOTHESIS TEST ON THE SLOPE OF VARIOUS CORRELATIONS

### B.1 T statistic- Hypothesis test discussion

The  $R^2$  values shown in Figures B.1-B.3 represent how accurately the two data populations are correlated. The  $R^2$  values are very low indicating weakness or nearly nonexistence in the relationships. A statistical hypothesis test (t-test) on the slope can be used to determine the significance of the slopes shown in Figures B.1-B.3. The null hypotheses  $H_0$  states that the slope of the figures are zero and so therefore the relationships are insignificant. The alternative hypothesis  $H_1$ , states that the slope is not equal to zero implying that the relationship is significant. According to Geaghan (2005), the t-value can be determined using

$$T^* = \frac{b_1 - 0}{\sqrt{\frac{MSE}{S_{xx}}}} \quad (\text{B.1})$$

where,

$b_1$  is the least squares estimate of the slope

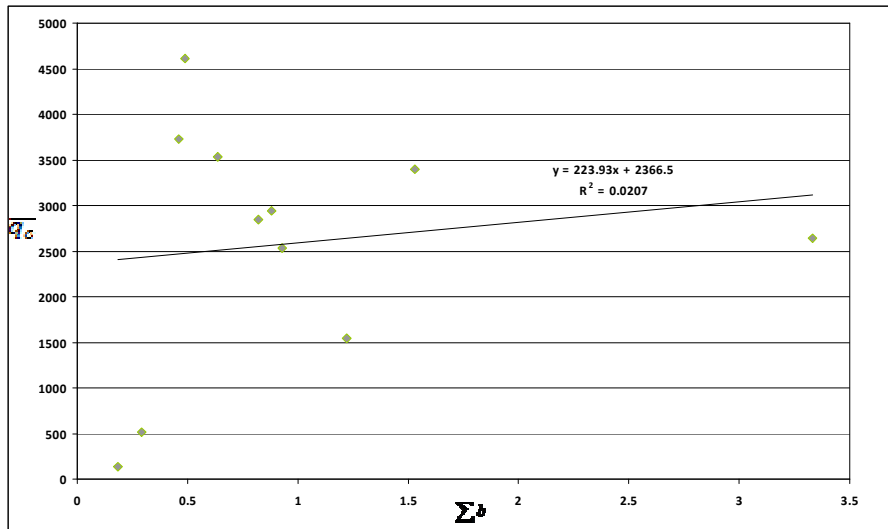
$MSE$  is the estimate of the common variance around the slope

$S_{xx}$  is the sum of squares

### Choices of decisions

According to Geaghan (2009), the choices of decisions to be made for the t statistic hypothesis test for the slope of the figures, will be to reject the null hypothesis  $H_0$  (and therefore conclude  $H_1$ ) if the t value is in the interval of -1.65 and 1.65 or to not reject the null hypothesis  $H_0$ .

The relationship between the summation of the  $b$  values for each producer and the average oil production (BOPM) for each producer is shown in Figure B.1 and is positive as expected. A greater  $\sum b$  value for a producer  $j$  would imply that there is a larger effect of the injectors acting on that producer which would result in an increase in the average production (oil) from that well. It is important to note that although this is a positive trend, the hypothesis test for the slope of Figure B.1 was found to be 0.44 which lies between the interval of -1.65 and 1.65; This implies that the slope is insignificant. The  $R^2$  value is also very low which shows very low correlation.



**Figure B.1** showing a weak relationship between the average oil production for each producer and summed  $b$  values for each producer.

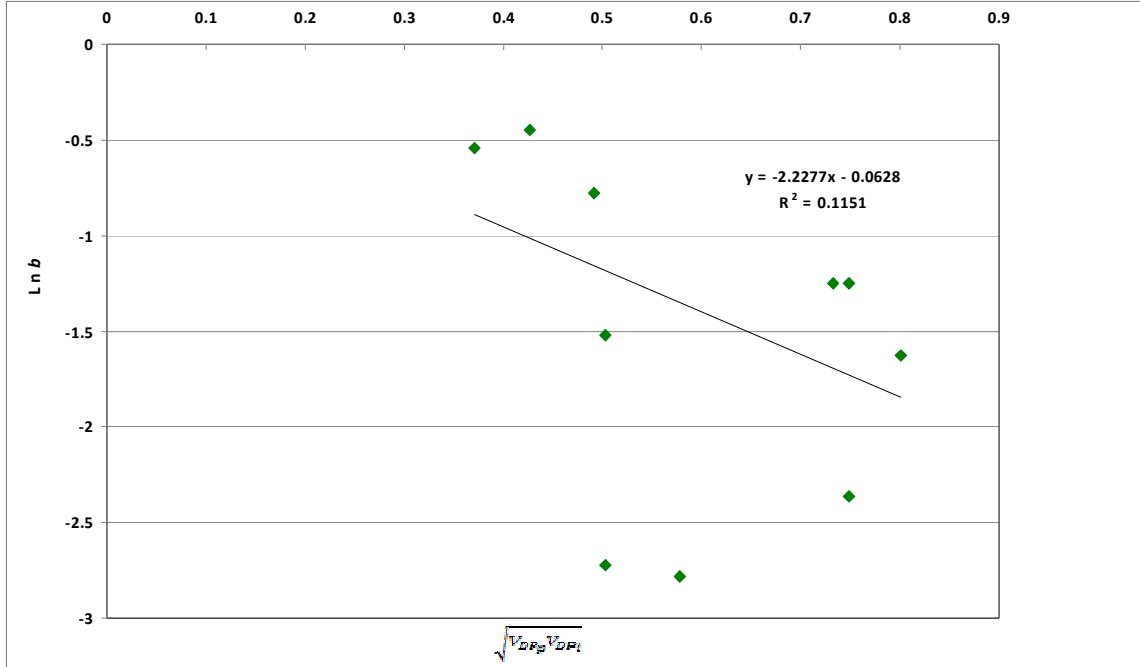
A similar relationship between the cumulative oil recovery for each producer and the  $\sum b$  values for each producer was found. The  $R^2$  value is 0.0207 which is very low indicating very low correlation and the hypothesis test resulted in a t-value of  $9.73 \times 10^{-7}$  which indicates that the slope is insignificant.

The most common measures of heterogeneity in the industry are the Dykstra-Parsons coefficient,  $V_{DP}$  and the Lorenz coefficient,  $L_C$ . Both measures range from zero

to one where higher values scaled between about one-half and one correspond to higher heterogeneity (Lake and Jensen, 1991). Therefore, a value of zero is for a completely homogeneous reservoir while a value of one is for an infinitely heterogeneous reservoir. The Dykstra-Parsons coefficient is based on permeability distribution and is calculated as follows

$$V_{DP} = \frac{k_{50} - k_{84.1}}{k_{50}} \quad (\text{B.2})$$

where the  $k_{50}$  term in equation B.2 “is the median permeability and the  $k_{84.1}$  term is the permeability one standard deviation above  $k_{50}$  on a log-normal plot” (Dykstra and Parsons, 1950; Jensen and Lake, 1991). A high Dykstra-Parsons coefficient would imply heterogeneity in the reservoir which may cause an increase or decrease in the effect an injector  $i$  has on a producer  $j$ . Figure B.2 shows a negative relationship between the natural log of the  $b$  values for each producer and the Dykstra Parsons coefficient for each injector-producer well pair. This suggests that there is a decrease in the magnitude of connectivity between injector-producer well pairs in the more heterogeneous parts of the reservoir. Since the Dykstra-Parsons coefficient is based on permeability distribution, the decrease in magnitude of connectivity would suggest that due to the variability in permeability in the heterogeneous parts of the field, movement of fluid would be directed towards areas with higher permeabilities; thereby causing a decrease in connectivity between producer-injector well pairs in areas with lower permeability. Figure B.2 shows the coefficient of determination value for this correlation is very low and the significance test found the slope to be insignificant with a T-value of -1.08.



**Figure B.2** showing a weak relationship between the log normal  $b$  values for each producer and the Dykstra Parsons coefficient for each injector-producer well pair.

The Lorenz coefficient is computed from a plot of cumulative flow capacity,  $F_j$  versus storage capacity  $C_j$ . According to Jensen, et al. (2000), the flow capacity and storage capacity can be obtained using equations B.3 and B.4 below

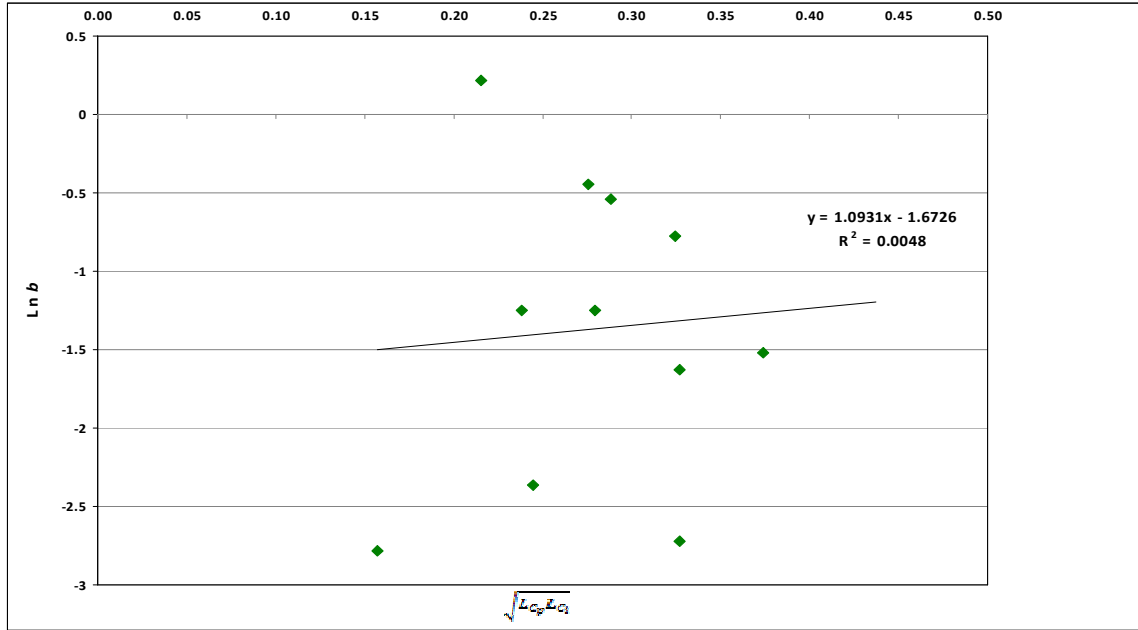
$$F_j = \frac{\sum_{j=1}^J k_j h_j}{\sum_i^N k_i h_i} \quad (\text{B.3})$$

$$C_j = \frac{\sum_{j=1}^J \phi_j h_j}{\sum_{i=1}^N \phi_i h_i} \quad (\text{B.4})$$

A high Lorenz coefficient,  $L_c$  would imply heterogeneity in the reservoir which

could cause an increase or decrease in the effect an injector  $i$  has on a producer  $j$  .

Figure B.3 shows a positive relationship between the natural log of the  $b$  values for each producer and the Lorenz coefficient for each injector-producer well pair. This suggests that more connectivity is seen in areas of the reservoir with higher heterogeneity. This does not agree with the results shown in Figure B.2. As stated earlier, Figure B.2 suggests that in the more heterogeneous parts of the reservoir, there is a decrease in the magnitude of connectivity between injector-producer well pairs which makes sense because injection fluid would flow to the more permeable areas in the reservoir as opposed to the areas with lesser permeability thereby resulting in a lesser magnitude of connectivity between injector-producer well pairs in the lesser permeable areas. The difference in the results shown between the Dykstra-Parsons and the Lorenz Coefficient measures could be due to the fact that the Dykstra-parsons model only accounts for variability in permeability while the Lorenz model accounts for the variability in permeability and porosity in a heterogeneous reservoir. In the Lorenz coefficient case, the high heterogeneity is as a function of the flow capacity and storage capacity in the reservoir. With that being said, Figure B.3 shows a very low coefficient of determination value and the significance test found the slope to be insignificant with a t-value of 1.34.



**Figure B.3** showing the relationship between the log normal beta values for each producer and the Lorenz Coefficient for each injector-producer well pair.

## VITA

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