THREE-ECHELON SUPPLY CHAIN DELIVERY POLICY WITH TRADE CREDIT CONSIDERATION

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Farhana Rahman
B.S., Bangladesh University of Engineering and Technology, Bangladesh, 2003
M.S., Bangladesh University of Engineering and Technology, Bangladesh, 2006
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ABSTRACT

In recent years, collaboration in supply chain approach widely discussed in the literature; but most have dealt with the two-echelon systems. This study focuses on the just-in-time delivery policy of three-echelon supply chain by collaborative approach, where any of the information from the supply chain is available to all the subsystems involved; manufacturer, distribution center and retailer. In the first part of the study a simple model has been developed for a three-echelon supply system that consists of a single manufacturer, a single distribution center and a single retailer. The other part of the study extends this model by considering a upstream integrated delivery supply chain system consisting of a single manufacturer, multiple distribution centers and multiple retailers. In both cases the retailer enjoys a permissible delay in payment. The joint annual cost of the supply chain is obtained by summing the annual relevant costs at all the subsystems. Using the convex property of the cost function, the optimum values of the decision values are initially obtained that minimizes the total cost. Then, these values are adjusted according to feasibility criteria of the credit conditions and other constraints using an algorithm. A numerical example illustrating the solution reveals that total supply chain cost is less by the presented collaborative approach compared to typical delivery policy. A sensitivity analysis also showed the robustness of the new model. This model considers lot-splitting and deferred payment simultaneously. That has not been studied for three-echelon system before. Future extension of this study involves assumption of random demand with cross-transfer delivery, unequal cycle time, shortage consideration, etc.

CHAPTER 1

INTRODUCTION

In this era of extreme competition, each subsystem in different echelons of supply chain thrives to improve their operations, reduce costs and increase profitability. Currently, the competition is not confined to the subsystems of the same echelon levels, the necessity of long term and reliable business relation has created competition among the supply chains. Hence, the consideration of joint optimization of supply chain cost is of interest.

In recent years, the just-in-time policy is widely practiced to gain and maintain a competitive advantage. The literature indicates that if all entities of a supply chain agree to collaborate and follow the joint shipment size, the total relevant cost within the supply chain can be significantly reduced compared to the typical delivery policy [Kreng and Chen 2007]. Hence, both purchaser and vendor may benefit from negotiation.

In this study, a three-echelon delivery supply chain of a single product has been considered. In the first part of the study a relatively simple supply chain has been considered that consists of a single entity in each echelon level—a single manufacturer, a single distribution center and a single retailer. In the second part, this model has been upstream integrated to study a more generic and complex structure of a three-echelon supply chain system consisting of a single manufacturer, multiple distribution centers and multiple retailers. In both of the models, the lot produced is delivered to any distribution center in equal size multiple shipments and the lot received by any distribution center is again delivered to any retailer in equal size multiple shipments based on the principle of just-in-time delivery policy. It is important to note that the shipment size might differ among the distribution centers and among the retailers in accordance with their demand rates and related cost parameters. The demand information is known and consistent, and is available to all the echelon levels. Moreover, there is no need to wait for the whole lot to be produced (at manufacturer) or received (at distribution center) to feed the downstream echelon levels. In both of the models, the retailer enjoys a permissible delay period to settle the credited account without interest to the corresponding distribution center. During the period before the account has to be settled, the retailer can sell the product and continue to accumulate revenue and earn interest. The retailer can pay the distribution center either at the end of the credit period or later incurring interest charges on the unpaid balance for the overdue period. Permissible delay period is a common attribute, in practice and is an incentive to the retailer, the most important subsystem of the supply chain being the closest one to the consumer.

1.1. Literature Review

Current literature has dealt with collaborative approach of supply chain. However, the joint optimization for supplier and buyer was introduced by Goyal (1976). He suggested a joint economic lot size model where the objective is to minimize the total relevant costs for both the vendor and the buyer. Goyal (1976) stated that if all parties, instead of determining their policies independently, decide to cooperate and determine the economic joint inventory policy, then considerable savings can be achieved. Later on his approach was reinforced by, among others, Monahan (1984), Banerjee (1986), Joglekar and Tharthare (1990), Zahir and Sarker (1991), Hall (1995), Miller and Kelle (1998). All of the research outcomes conclude that joint determination of the economic lot size for both parties can reduce their total cost substantially. Goyal (1988) studied a joint economic-lot-size model for a single purchaser and a single vendor considering vendor's lot size as an integer multiple of the purchaser's order size. Lu (1995) also proposed a lot-splitting model, that is, single set-up, multiple deliveries, but with the assumption that the vendor delivers a number of equal small lot size even before producing the entire lot. Both of the studies show that implementing such equal size multiple shipment increases the transportation cost substantially, but the overall cost reduction is achieved for reducing the holding cost at the downstream echelon levels that is usually sufficiently high to compensate the increase in transportation cost. In recent past, Khouja (2003) studied a three-echelon system assuming that the whole lot has to be produced before delivering the lot. Kim and Ha (2003) have developed a lot-splitting model and discussed how and when the optimal policy for buyer and supplier can be achieved. Kreng and Chen (2007) have further extended their model from two-echelon to three-echelon system that accommodates a distribution center as intermediary. They have proposed a two stage integration of the model that finds out optimal delivery policy in two steps for all the entity involved and thus minimized the supply chain cost.

The traditional economic ordering quantity model considers that the retailer pays the purchasing cost for the product as soon as the products are received; but in reality, the supplier usually offers different delay period, known as **trade credit period** or **deferred payment period**, sometimes with different price discounts to encourage the retailer to order more quantity. Goyal (1985) developed an economic ordering quantity model under the conditions of permissible delay in payments for an inventory system. Shah *et al.*(1988) extends the model by incorporating inventory shortage. Aggarwal and Jaggi (1995) developed a model to determine the optimum ordering quantity for deteriorating items with similar trade credit consideration. The other notable works considering similar issues relating to payment period are those of Jamal *et al.*(1997, 2000), Sarker *et al.*(2000, 2001), Salameh *et al.* (2003), Goyal and Teng (2007).

The following shortcomings have been found in the literatures mentioned:

- **Three-echelon system:** The literatures mentioned above have studied integrated model mostly limited to two-echelon system.

- **Lot-splitting:** In most cases, the research involving three-echelon system assumes that the production lot needs to be fully produced before it can start delivering to distribution center, which is relaxed in this study.
- **Different shipment sizes and number of shipments:** Some other research assumptions include equal shipment size and number of shipments per order for all the subsystems of any echelon level; this assumption is relaxed in this study.
- **Trade credit consideration in a three-echelon supply chain:** None of the researchers have considered permissible delay period for settling the account in a three-echelon supply chain delivery policy, which is rather a very common occurrence among practitioners.

Based on the shortcomings identified above, this research aims to overcome the stated deficiencies. Here, two three-echelon supply chain models have been considered, one containing a single manufacturer, a single distribution center and a single retailer, and the other one containing a single manufacturer, multiple distribution centers and multiple retailers. In both of the models the retailer is allowed credit from the distribution center. It is assumed that every subsystem is ready to collaborate for the maximum benefit of supply chain. In the solution process of both of the models, firstly, the annual total relevant cost is developed for each subsystem and secondly, they are summed up to find out the joint annual cost of the supply chain. The cost function is then optimized to find the decision variables within the constraints. Under the condition, two different cost functions are possible for two different scenarios of permissible delay of the retailer. Both of the functions are solved and tested to obtain a feasible solution.

CHAPTER 2

RESEARCH OBJECTIVE AND SCOPE

The motivation and objectives of the research are addressed in this chapter. The solutions that can overcome some shortcomings addressed above are stated here specifically. It is followed by addressing the scope of problems, applications and solution strategies.

2.1 Motivation

This research aims to reduce the total cost of a three-echelon delivery supply chain system by coordinating among the echelon levels. The value of any inventory increases as a product moves down the distribution chain, and therefore the associated holding cost also increase. The system under consideration follows just-in-time delivery policy of lot-splitting for shipping to subsequent downstream levels from manufacturer to retailer, through distribution center. This system lowers the overall supply chain cost by reducing the holding cost in the downstream levels when the transportation cost is not high compared to the holding cost. In addition, in the model there is no need to wait for the entire lot to be produced to feed to the downstream levels that suppose to further lower the overall cost. So all these conditions, to enhance the reduction of the total cost of three-echelon delivery supply chain and to incorporate the trade credit benefit as an incentive for the retailer, provide the motivation fors this research.

2.2 Objective

The objective of the research is to ascertain an optimum just-in-time delivery policy of a product for three-echelon supply chain that minimizes the overall cost of the entire system. The specific type of problems dealt in this research may be stated as follows:

2.2.1 Single-channel, Multi-echelon Distribution System

This problem addresses the just-in-time (JIT) delivery policy of a single product that follows a three-echelon supply chain from a manufacturer to a distribution center, and finally to a retailer. The demands at all echelon levels are equal and known, and every echelon level collaborates among themselves—the retailer, the distribution center and the manufacturer, to reduce the overall system cost. In the model, the retailer is allowed some credit period before settling the account with the distribution center. In order to comply with the principle of JIT delivery policy, the manufacturer delivers, in multiple shipments, the outstanding order to the distribution center, likewise the distribution center also splits the order quantity into multiple shipments to deliver to the retailer. This phenomenon of delivering order quantity in multiple shipments from the immediately proceeding echelon level to a downstream echelon is known as **lot-splitting**. The objective of the problem is to determine the shipment quantity and number of shipments per order of the product through each echelon level that minimizes the total cost of the integrated supply chain.

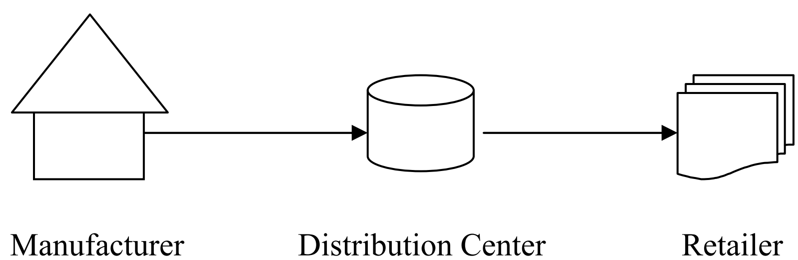

Figure 2.1 Single-Channel, Multi-echelon Distribution System

2.2.2 Multi-channel, Multi-echelon Distribution System

This part of the study considers the inventory model of a product that follows a upstream integrated three-echelon supply chain from a manufacturer to a number of distribution centers, finally to number of retailers. The term, **upstream integrated**, is used in the sense that a number of subsystems in a downstream level gets shipments from any dedicated subsystem of the immediate upstream level and it proceeds downstream. It should be noted here that the upstream integrated model is considered instead of **cross-transfer** (any downstream subsystem can get delivery from any subsystem of the immediate uptream level) model because the total demand at each echelon is equal to that at any other echelon level. These echelon demands are deterministic and known. Cross-transfer model should be assumed where there is uncertainty of demand.

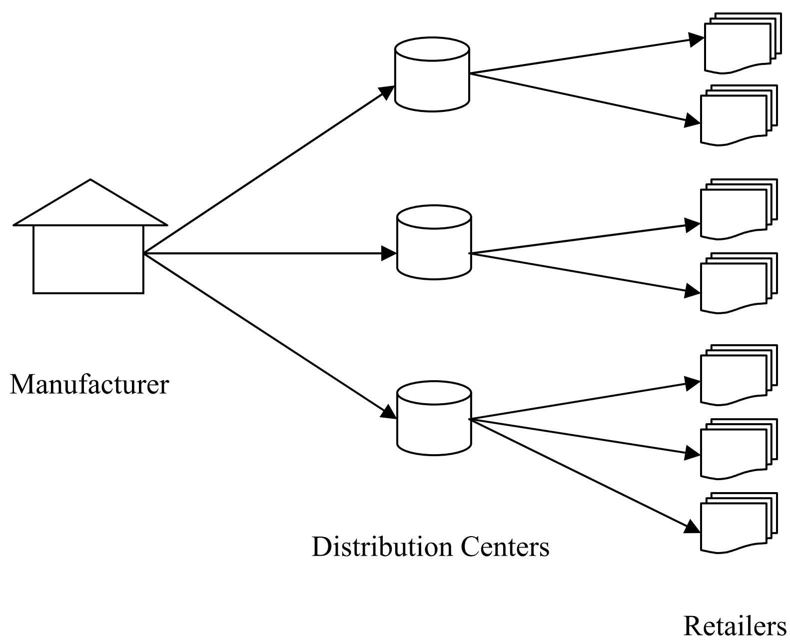

Figure 2.2 Multi-channel, Multi-echelon Distribution System

This model also follows **lot-splitting** (equal-size and multiple shipments per order) at each echelon level. However, the shipment size and the number of shipments might differ among distribution centers and among retailers in accordance with their demand rates and related cost parameters. All retailers enjoy permissible delay period of payment to the corresponding distribution centers. Thus, the objective of the problem is to determine delivery policy of the product through each echelon level that minimizes the total associated cost of the integrated supply chain. The delivery policy includes shipment size and number of shipments per order of each subsystem. This multi-channel, multi-echelon model is the more generic form of the three-echelon delivery supply chain model, but it is more computationally intensive compared to the single-channel, multi-echelon model.

2.3 Scope

When the demand is random, the multi-channel, multi-echelon system can be extended to cross-transfer delivery, where any retailer can get delivery from any of the distribution centers, instead of upstream integrated delivery model to meet the uncertainty of the demand. In such a case, possible shortages can also be considered. Moreover, to provide more flexibility of the model, different cycle time can be assumed for different subsystems.

2.4 Applications

The three-echelon supply chain system of manufacturer, distribution centers and retailers is a commonly used sturcture of a supply chain system. It is applicable for many product supply chain system from the manufacturer to the retailer through the distribution center. For instance, in a study of auto industry, Helper (1991) reveals that 52% of suppliers follow JIT delivery policy to meet customer’s JIT requirement. So the model under consideration can be applied to many delivery chains, including auto industries, having three-echelon levels where retailers are allowed credit benefit.

2.5 Solution Strategy

After identifying the problems and determining the scope and objectives, the strategy of this research will be described in this section. In order to minimize the total cost of the supply chains, the total relevant annual cost of each echelon level are quantified. In order to quantify the cost of each subsystem, average inventory at each retailer, distribution center and manufacturer are identified. Once the model is formulated and the total integrated cost is quantified, the cost function is minimized with respect to the decision variables. The solutions must maintain the feasibility criteria of problem assumptions and constriants; for example, the relative length of the cycle time and permissible delay period. This methodology led to determine the production quantity at the manufacturer and order quantity and the number of shipments at each echelon level, with the aim of minimizing the overall cost.

2.6 Overview

The just-in-time delivery policy of a single product is considered in this research where delivery supply chain consists of three-echelon levels. The goal of the study is to determine the optimal or near-optimal shipment size and number of shipment of each subsystem, in order to minimize the overall supply chain cost. The literature review of this study was presented earlier. The research is organized as follows. Chapter 3 discusses the single-channel, multi-echelon supply chain model where the retailer enjoys permissible delay in payment. The notation and the model formulation are presented. A numerical example is explained in detail after depicting solution methodology. The result of the model is compared to the result of typical delivery model. Next, multi-channel, multi-echelon supply chain model is explained in Chapter 4 with additional notation and the model formulation of the model. The second developed algorithm is presented. Computational results are given based on three different numerical examples. A sensitivity analysis is performed for some parameters of the multi-channel model based on 18 different problems. In Chapter 5, operational schedules are done for the more generic multi-channel model. Finally, the conclusion of the research is discussed in Chapter 6. The significance of the research and possible future extensions are summarized in the last Chapter.

CHAPTER 3

SINGLE-CHANNEL, MULTI-ECHELON DISTRIBUTION SYSTEM

In this chapter the JIT delivery policy of a single product is developed, that flows through a single-channel, multi-echelon supply chain system consisting of a single manufacturer, a single distribution center (DC) and a single retailer as shown in Figure 3.1. In addition, the distribution center offers the retailer a delay period, known as **trade credit period**, to pay back purchasing cost. During trade credit period, the retailer can accumulate revenues by selling items and by earning interests. However, beyond the credit period the retailer must pay for interest charges on the unpaid balance for the overdue period.

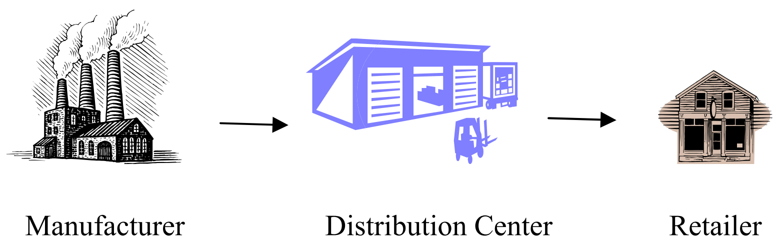

Figure 3.1 Single-channel, multi-echelon supply chain system

3.1 The Problem

This section considers the inventory model of a product that follows a three-echelon supply chain from a manufacturer to a distribution center to a retailer. The demand at the retailer is deterministically known. This demand information is shared by all the upstream echelon levels. Hence, the demands at all echelon levels are equal and known. According to the principle of JIT delivery policy **lot-splitting** strategy has been applied in the model. The objective of the problem is to determine the decision variables of the delivery policy of the product through each echelon level that minimizes the total relevant cost of the integrated supply chain.

3.2 Model Formulation

In order not to allow any shortage, the production rate of the manufacturer, P, is assumed to be higher than the demand rate of the product, D. Given that in each ordering cycle, the manufacturer delivers m_D shipments to the distribution center, each shipment having q_D units of products, the manufacturer uses a policy of producing $m_D q_D$ units every time it produces to satisfy the demand that exactly equals DT, the demand of product in cycle time, T. So, unlike the increasing inventory build-up in a traditional economic manufacturing model with a continuous demand, a saw-tooth inventory model is built up by the manufacturer during the production period as shown in Figure 3.2.

The distribution center again splits the quantity, q_D into n shipments and delivers q units of products to the retailer in each shipment. So, the inventory of the distribution center resembles a step function, each step having the height of quantity q $(= \frac{q_D}{n})$, whereas the inventory of the retailer resembles the saw-tooth having $m_R (= n m_D)$ number of iterations in cycle time T, where each step follows traditional economic ordering model with instantaneous replenishment.

Under this policy, the retailer and the distribution center are willing to place an order for an extended period of time to get the advantage of saving both ordering cost and decreasing inventory carrying cost, since each shipment is made only when the buyer is about to deplete his stock. However, concerns about product changes and long-term commitments may result in risk of losing flexibility to change to another supplier or to a different product. So, in such a policy, disadvantages of increasing shipment cost and decreasing flexibility is to be expected. Kelle *et al.* (2003) introduced a new cost parameter to quantify such risk, that represents the managerial concern of having a very large order quantity. This cost parameter, known as **flexibility loss**

cost, can be defined by the cost of committed, but undelivered, supply that makes the buyer to lose flexibility for changing to a different supplier or a product. Kreng and Chen (2007) also used this cost parameter in their model. Since such risk is a consequence of committed, but undelivered, supply and may take place at any time, average order quantity is used to estimate this risk. Such risk also involves the distribution center.

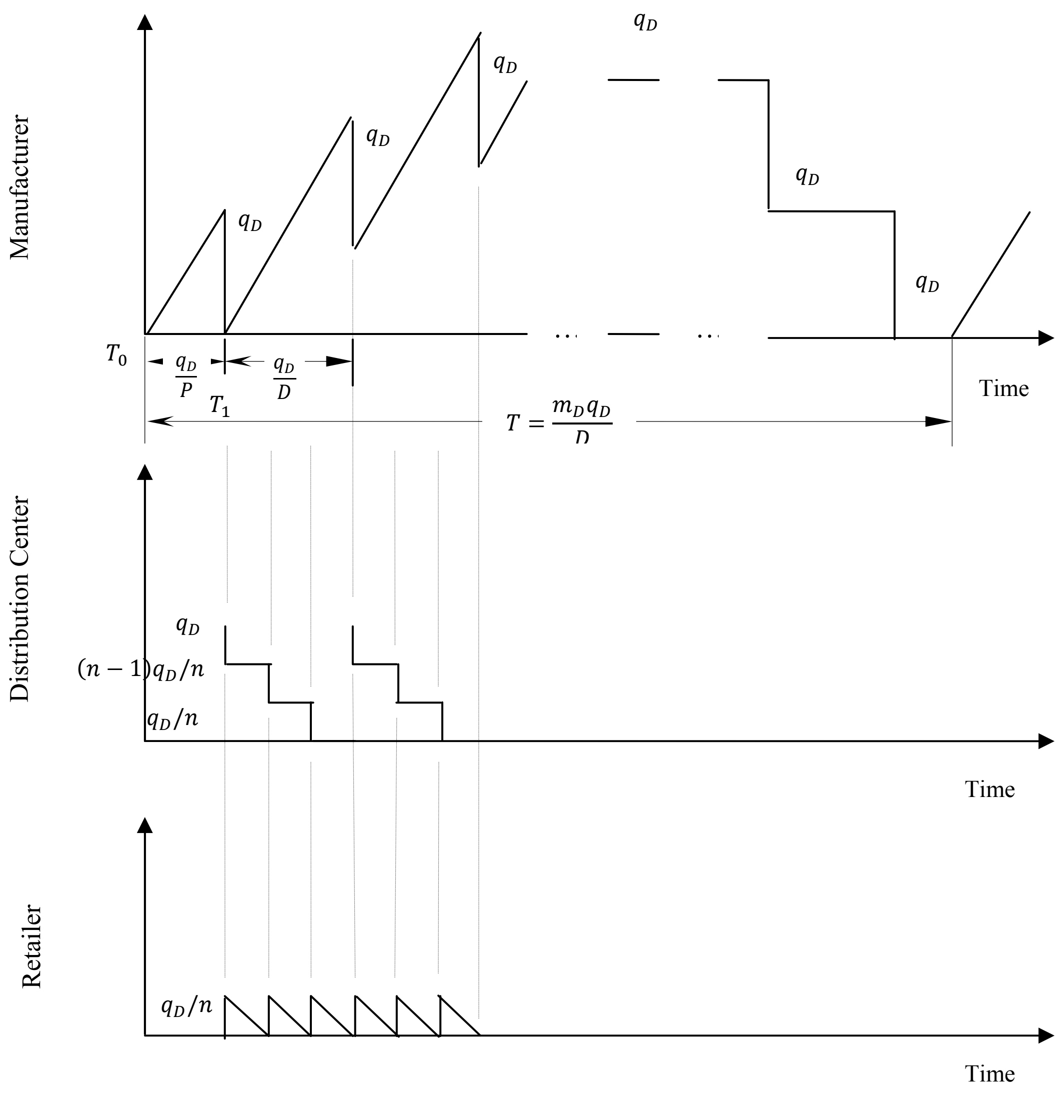

Figure 3.2 Inventory of product through a manufacturer, a distribution center and a retailer.

Again, the distribution center allows the retailer a trade credit without interest during a permissible delay period. During the period before the account has to be settled, the retailer can sell the items and continue to accumulate revenue and earn interest, instead of paying off the overdraft that might, at other instances, be necessary, if the supplier requires settlement of the account immediately after replenishment. The retailer can pay the distribution center either at the end of the credit period or later incurring interest charges on the unpaid balance for the overdue period. Hence, interest cost of the retailer from credit consists of the interest earned on the revenue and interest paid to the distribution center that is illustrated in detail in the latter part of this chapter. So, retailer's total annual relevant cost consists of flexibility loss cost, interest cost, ordering, transportation and holding costs. Distribution center's relevant cost consists of flexibility loss cost, ordering, transportation and holding costs and manufacturer's cost consists of set-up cost, tranportation and holding costs.

3.2.1 Notation and Assumptions

The following notations are used to model the single-channel, multi-echelon supply chain system with trade credit consideration:

3.2.1.1 Retailer's Parameters

D Annual demand rate of the retailer (units/year)

A_R The retailer's ordering cost per contract (dollars/order)

h_R Stock holding cost per unit per year for the retailer (dollars/unit/year)

τ_R The fixed transportation cost of receiving a shipment from DC (dollars/shipment)

$\emptyset_R$ Flexibility loss cost (dollars/unit/year)

c Unit purchase price from DC (dollars/unit)

k Profit rate on purchase price, *i.e.,* retailer's selling price =$(1 + k)c$

I_e Interest earned per dollar per year (/dollar/year)

I_p Interest payable to DC per dollar per year (/dollar/year)

t Permissible delay in settling the account (year)

3.2.1.2 <u>Distribution Center's (DC) Parameters</u>

A_D The DC's ordering cost per contract (dollars/order)

h_D Stock holding cost per unit per year for the DC (dollars/unit/year)

τ_{1D} The transportation cost of receiving a shipment from the manufacturer (dollars/shipment)

τ_{2D} The transportation cost of the DC of delivering a shipment to retailer (dollars/shipment)

$\emptyset_D$ Flexibility loss cost per unit per year (dollars/unit/year)

3.2.1.3 <u>Manufacturer's Parameters</u>

P Annual production rate of the manufacturer (units/year)

A_M Fixed production setup cost per lot size (dollars/batch)

h_M Stock holding cost per unit per year for the manufacturer (dollars/unit/year)

τ_M The transportation cost of a shipment from the manufacturer to the DC (dollars/shipment)

3.2.1.4 <u>Relevant Variables</u>

m_R Number of shipments per order from DC to the retailer, $m_R \geq 1$

m_D Number of shipments per order from manufacturer to the DC, $m_D \geq 1$

q Shipment quantity from DC to the retailer in each shipment (units)

q_D Shipment quantity from manufacturer to the DC in each shipment (units)

n Number of shipments to the retailer per quantity q_D, *i.e.,* $n = {}^{q_D}/_{q}$

T Common cycle time of production/ordering cycle (year)

It is observed that the same transportation cost is divided between two subsystems. This is done to give more flexibility to the model regarding the share of the cost. For example, suppose the shipment cost for the delivery from the manufacturer to the distribution center is \$1,000/shipment. According to the collaboration, the manufacturer can agree to carry 80% of the shipment cost and the distribution center can agree to carry the rest. So in such a case, τ_M = \$800/shipment and τ_{1D}= \$200/shipment. On the other hand if the manufacturer agrees to carry 100% of the shipment cost, τ_M = \$1,000/shipment and τ_{1D}= 0.

The following assumptions are made to model the system:

(1) The retailer's ordering quantity from the DC has to be on a JIT basis, that may require small and frequent replenishment.
(2) All shipments are of equal size.
(3) Demand rate is constant and deterministic.
(4) Production rate and lead time are constant and deterministic.
(5) Shortage is not allowed, *i.e.,* $P \geq D$.
(6) All the cost parameters are known and constant.
(7) The retailer is allowed permissible credit (delay) period for payment after the purchase of goods from the distribution center. The retailer is subjected to pay the interest on the purchase amount if the account is not settled before the delay period expires.

(8) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account.

(9) The ordering cycle times (time interval in successive orders) are equal for both the distribution center and retailer, that is the same as the production cycle time of the manufacturer.

3.3 The Joint Annual Cost Function

The cost involved in the entire supply chain system containing a single manufacturer, a single distribution center and a retailer is derived here. The system follows JIT delivery policy of frequent deliveries in small lots. The total quantity of the product manufactured during the production cycle time (the time between subsequent production start-ups) of the manufacturer must be equal to the demand of the common cycle time.

The joint annual total relevant cost of the entire three-echelon system, TC^S consists of the annual total relevant cost of the manufacturer, ${TC_M}^S$, the annual total relevant cost of the distribution center, ${TC_D}^S$, and the total annual cost of the retailer, ${TC_R}^S$, and it is given by:

$$TC^S = {TC_M}^S + {TC_D}^S + {TC_R}^S. \tag{3.1}$$

3.3.1 Cost at the Retailer

The retailer enjoys a trade credit from the distribution center that allows it to delay payment until the end of an allowed period. The retailer does not pay any interest during the fixed period to settle the account, but if the payment is delayed beyond the specified period, interest is charged. This is an incentive for the retailer in that it can sell the product, continue to accumulate revenue, and earn interest at a rate of I_e on the accumulated money instead of paying off the amount even after the permissible credit period expires. So interest cost of the retailer consists of the interest earned on the revenue and interest paid to the distribution center. However, two different scenarios need to be considered for such a case; (a) when the cycle time, T at least equals permissible delay period, t , *i.e.,* $T \geq t$ and (b) when the cycle time, T is less than permissible delay period, t , *i.e.*, $T < t$ (Goyal 1985).

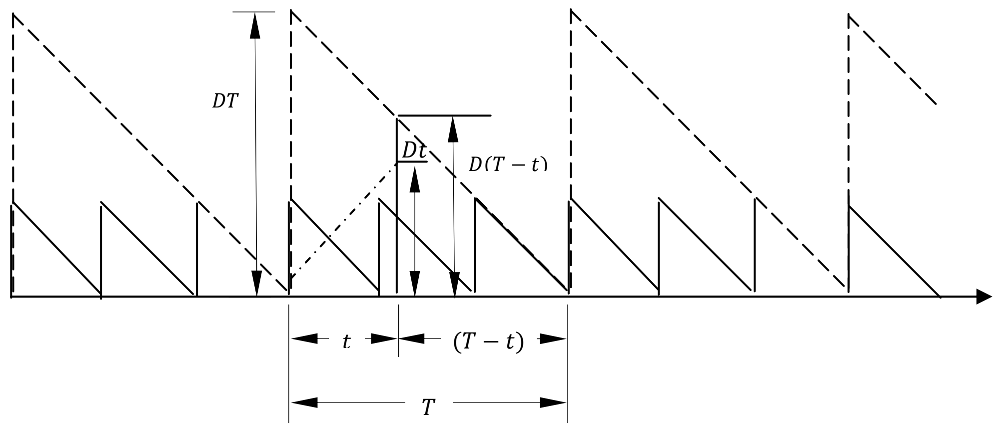

Figure 3.3 Time weighted demand at retailer when $T \geq t$

3.3.1.1 Case I: $T \geq t$ (Shorter permissible delay)

The interest is payable during time $(T - t)$. The demand at the time of settling the replenishment account equals $D(T - t)$ which is shown in Figure 3.3. The average shipment quantity received from the distribution center during time $(T - t)$ is $D(T - t)/2$, whose total purchase price is $cD(T - t)/2$, where c is the unit purchase price. For an interest rate, I_p, the interest payable in one ordering cycle in time T is given by $cD(T - t)^2 I_p/2$. So interest payable per year can be expressed by $cD(T - t)^2 I_p/(2T)$. Conversely, the retailer earns interest during the permissible settlement period, t on the average revenue incurred during that period, which is given by $(1 + k)cDt/2$, where $(1 + k)c$ is the retailer's selling price. Hence, the interest earned in one ordering cycle in time T is given by $(1 + k)cDt^2 I_e/2$. So interest earned per year can be expressed by $(1+k)cDt^2I_e/(2T)$. Therefore, for $T \geq t$, the annual cost of interest of the retailer, I_I is given by

$$I_I = \frac{cD(T-t)^2 I_p}{2T} - \frac{(1+k)cDt^2 I_e}{2T}. \tag{3.2}$$

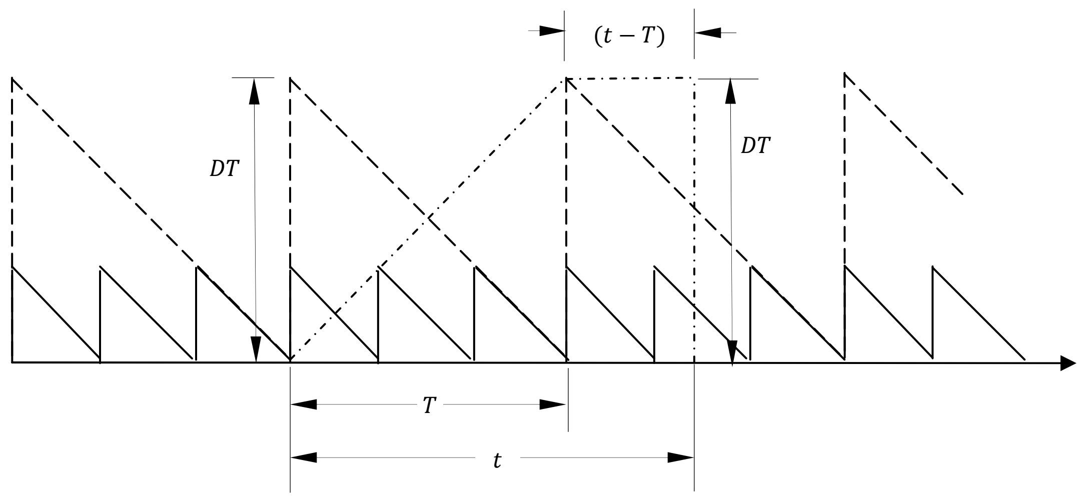

Figure 3.4 Time-weighted demand at the retailer when $T < t$

3.3.1.2 <u>Case II: $T > t$ (Larger permissible delay)</u>

In such a case the retailer does not incur any interest to be paid to distribution center; on the contrary it earns the interest on the revenue (Goyal 1985). Both credit period and payment period in this system is shown along with the inventory level in Figure 3.4. It should be noted that during period T the interest is earned on the average revenue during that time. Based on the average demand $DT/2$ during period T, the corresponding average revenue is given by $(1+k)cDTI_e/2$ which accumulates an interest of $(1+k)cDT^2I_e/2$, where I_e is the interest rate. Whereas, during time $(t-T)$, the interest is earned on the accumulated revenue $(1+k)cDT$, hence, the interest earning during $(t-T)$ is given by $(1+k)cDT(t-T)I_e$. Therefore, for $T < t$, the annual cost of interest of the retailer, I_{II} is given by

$$I_{II} = -\frac{(1+k)cDT^2I_e}{2T} - \frac{(1+k)cDT(t-T)I_e}{T} \tag{3.3}$$

$$= -(1+k)cD(t-\tfrac{T}{2})I_e.$$

Now, the annual total relevant cost of retailer is obtained by summing over ordering cost, transportation cost of receiving shipment, holding cost, flexibility loss cost and interest cost due to trade credit. For a fixed ordering cost A_R, the annual ordering cost of the retailer is given by A_R/T. Denoting number of shipments in cycle time T as m_R, the transportation cost is given by $\tau_R m_R/T$, with shipment cost τ_R in each shipment. For the retailer, $m_R q/2$ is the average delivered quantity per ordering cycle. So, having the flexibility loss cost per unit per year, $\emptyset_R$, the annual flexibility loss cost of the retailer is quantified as $\emptyset_R(m_R q/2)$. The average inventory level of the retailer, I_R is denoted by $q/2$ over the cycle time T, hence the annual holding cost of the retailer can be quantified by $h_R q/2$. Since the annual interest cost varies for two situations, the annual total relevant cost of the retailer will also be different in those two situations.

For *Case I* ($T \geq t$), the annual total relevant cost of the retailer, ${TC_{R,I}}^S$, is given by

$${TC_{R,I}}^S(m_R, q, T) = \frac{A_R}{T} + \frac{\tau_R m_R}{T} + \frac{h_R q}{2} + \emptyset_R\left(\frac{m_R q}{2}\right) + \frac{cD(T-t)^2 I_p}{2T} - \frac{(1+k)cDt^2 I_e}{2T}. \tag{3.4}$$

The total demand, DT during cycle time T should be exactly equal to the total quantity delivered, $m_R q$ in m_R shipments with quantity q units/shipment.

$${TC_{R,I}}^S(m_R, q) = \left[A_R + m_R\tau_R + \frac{c\{I_p - (1+k)I_e\}Dt^2}{2}\right]\frac{D}{m_R q} + h_R\frac{q}{2} + \left(\emptyset_R + cI_p\right)\frac{m_R q}{2}$$

$$-cDtI_p. \tag{3.5}$$

For *Case II* ($T < t$), the annual total relevant cost of the retailer, ${TC_{R,II}}^S$, is given by

$${TC_{R,II}}^S(m_R, q, T) = \frac{A_R}{T} + \frac{\tau_R m_R}{T} + \frac{h_R q}{2} + \emptyset_R\left(\frac{m_R q}{2}\right) - (1+k)cD\left(t - \frac{T}{2}\right)I_e, \tag{3.6}$$

which can be expressed in terms of two variables m_R and q,

$${TC_{R,II}}^S(m_R, q) = [A_R + m_R\tau_R]\frac{D}{m_R q} + h_R\frac{q}{2} + [\emptyset_R + (1+k)cI_e]\frac{m_R q}{2} - (1+k)cDtI_e. \tag{3.7}$$

3.3.2 Cost at the Distribution Center

In order to determine the annual holding cost, the average inventory of the distribution center, I_D must be known. In one shipment the distribution center gets quantity q_D from the manufacturer, which is delivered to retailer in n shipments. Hence, at that distribution center, stock of the product consists of $(n-1)$ rectangles as shown in Figure 3.2 and the inventory in time ${q_D}/{D}$ is given by Kreng and Chen (2007) by

$$\left[(n-1)\,{q_D}/{n} + (n-2)\,{q_D}/{n} \dots \dots + {q_D}/{n}\right]\left[{q_D}/{nD}\right].$$

Hence, yearly average inventory of the distribution center is

$$I_D = \frac{q_D(n-1)}{2n}. \tag{3.8}$$

So, the annual holding cost of the distribution center is $h_D q_D(n-1)/2n$. The annual ordering cost of the distribution center is given by A_D/T and the flexibility loss cost of the distribution center is $\emptyset_D(m_D q_D/2)$. Unlike other echelon levels, the distribution center incurs two annual shipment cost elements, one for receiving shipments from the manufacturer and the other one for delivering shipments to the retailer. The shipment costs of the distribution center for receiving and delivering are expressed by $\tau_{1D}m_D/T$ and $\tau_{2D}m_R/T$, respectively, where τ_{1D} and τ_{2D} denote unit transportation cost to receive a shipment from the manufacturer and to deliver a shipment to the retailer, respectively; m_D denotes number of shipments to the distribution center and m_R denotes number of shipments to the retailer.

The annual total relevant cost of the distribtuion center, ${TC_D}^S$, is obtained by summing over the ordering cost, transportation cost of receiving as well as delivering shipment, holding cost and flexibility loss cost and is given by

$$
{TC_D}^S(m_R, m_D, n, q_D, T) = \frac{A_D}{T} + \frac{\tau_{1D}m_D}{T} + \frac{\tau_{2D}m_R}{T} + h_D\frac{q_D(n-1)}{2n} + \emptyset_D\frac{m_D q_D}{2}. \quad (3.9)
$$

By assumption, $m_D q_D = m_R q = DT$. Again, the distribution center splits the quantity q_D into n shipments to deliver to the retailer with quantity q in each shipment, *i.e.*, $q_D = nq$ which establishes the relationship $nm_D = m_R$. Based on these relationships, ${TC_D}^S$ can be expressed in terms of three decision variables m_R, m_D and q as,

$$
{TC_D}^S(m_R, m_D, q) = [A_D + m_R\tau_{2D} + m_D\tau_{1D}]\frac{D}{m_R q} + \left[(\frac{m_R}{m_D} - 1)h_D + \emptyset_D\right]\frac{m_R q}{2}. \quad (3.10)
$$

3.3.3 Cost at the Manufacturer

The manufacturer schedules production start-up in such a way that the first q_D units are produced by the time they are to be shipped. The remaining $(m_D - 1)q_D$ units are produced continuously during the remaining uptime (production time), $(m_D - 1)\,{q_D}/{P}$, where P is the production rate of the manufacturer. The inventory under such condition has been discussed and derived by Joglekar (1988), as shown in Appendix A. Thus, the unit time average inventory is obtained from Eq. (A.4) as

$$I_M{}^S = \frac{m_D q_D}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right). \qquad (3.11)$$

For annual setup cost, A_M/T, and the annual transportation cost, $\tau_M m_D/T$, the annual total associated cost of the manufacturer, $TC_M{}^S$, with unit inventory cost, h_M, is given by

$$TC_M{}^S(m_D, q_D, T) = \frac{A_M}{T} + \frac{\tau_M m_D}{T} + h_M \frac{m_D q_D}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right). \qquad (3.12)$$

Since $m_D q_D = m_R q = DT$, Eq. (3.12) can be expressed as

$$TC_M{}^S(m_R, m_D, q) = (A_M + m_D \tau_M)\frac{D}{m_R q} + h_M \frac{m_R q}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right). \quad (3.13)$$

Finally, the annual total cost of the entire supply chain, TC^S is composed of the manufacturer's annual cost, $TC_M{}^S$, distribution center's annual cost, $TC_D{}^S$ and the retailer's annual cost, $TC_R{}^S$. It is important to note that, having cycle time, T, and permissible delay period, t, the two distinct cases of retailer's trade credit condition, namely $T \geq t$ and $T < t$, incur different annual cost to the retailer. Hence, the annual total relevant cost of the entire system will also be different for following two cases:

Case I: $T \geq t$.

When $T \geq t$, the annual total cost of the system can be written as

$$TC_I{}^S(m_R, m_D, q) = TC_M{}^S + TC_D{}^S + TC_{R,I}{}^S$$

$$= \left\{(A_M + m_D \tau_M)\frac{D}{m_R q} + h_M \frac{m_R q}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right)\right\}$$

$$+\left\{[A_D + m_R\tau_{2D} + m_D\tau_{1D}]\frac{D}{m_R q} + \left[(\frac{m_R}{m_D} - 1)h_D + \emptyset_D\right]\frac{m_R q}{2}\right\}$$

$$+\left\{\left[A_R + m_R\tau_R + \frac{c\{I_p - (1+k)I_e\}Dt^2}{2}\right]\frac{D}{m_R q} + h_R\frac{q}{2} + \left(\emptyset_R + cI_p\right)\frac{m_R q}{2}\right.$$

$$\left. - cDtI_p\right\}$$

$$= \left[A_R + A_D + A_M + m_R(\tau_R + \tau_{2D}) + m_D(\tau_{1D} + \tau_M) + \frac{c\{I_p - (1+k)I_e\}Dt^2}{2}\right]\frac{D}{m_R q}$$

$$+ \left[h_R + \left(\frac{m_R}{m_D} - 1\right)h_D + h_M m_R\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right)\right.$$

$$\left. + \left(\emptyset_R + \emptyset_D + cI_p\right)m_R\right]\frac{q}{2} - cDtI_p. \qquad (3.14)$$

Case II: $T < t$

When $T < t$, the annual total cost of the system can be written as,

$$TC_{II}{}^S(m_R, m_D, q) = TC_M{}^S + TC_D{}^S + TC_{R,II}{}^S$$

$$= \left\{(A_M + m_D\tau_M)\frac{D}{m_R q} + h_M\frac{m_R q}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right)\right\}$$

$$+\left\{[A_D + m_R\tau_{2D} + m_D\tau_{1D}]\frac{D}{m_R q} + \left[\left(\frac{m_R}{m_D} - 1\right)h_D + \emptyset_D\right]\frac{m_R q}{2}\right\}$$

$$+\left\{[A_R + m_R\tau_R]\frac{D}{m_R q} + h_R\frac{q}{2} + [\emptyset_R + (1+k)cI_e]\frac{m_R q}{2} - (1+k)cDtI_e\right\}$$

$$= [A_R + A_D + A_M + m_R(\tau_R + \tau_{2D}) + m_D(\tau_{1D} + \tau_M)]\frac{D}{m_R q} + \{h_R + \left(\frac{m_R}{m_D} - 1\right)h_D$$

$$+h_M m_R\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right) + [\emptyset_R + \emptyset_D + (1+k)cI_e]m_R\}\frac{q}{2} - (1$$

$$+ k)cDtI_e. \qquad (3.15)$$

Using the relation $\frac{m_R q}{D}=\frac{m_D q_D}{D}=T$ and after some algebraic manipulations Eqs (3.14) and (3.15) can be expressed by

$$TC_I^S(q,q_D,m_D)=\frac{A}{m_D q_D}+\frac{E}{q}+\frac{F}{q_D}+qL+q_D\mathrm{V}+\mathrm{B}m_D q_D-cDIpt. \tag{3.16}$$

and $$TC_{II}^S(q,q_D,m_D)=\frac{A'}{m_D q_D}+\frac{E}{q}+\frac{F}{q_D}+qL+q_D\mathrm{V}+\mathrm{B'}m_D q_D-(1+\mathrm{k})cDI_e t, \tag{3.17}$$

where $$A=D\left[A_R+A_D+A_M+\frac{1}{2}c\{I_p-(1+k)I_e\}Dt^2\right], \tag{3.16a}$$

$$B=\frac{1}{2}\left[h_M\left(1-\frac{D}{P}\right)+\left(\emptyset_R+\emptyset_D+cI_p\right)\right], \tag{3.16b}$$

$$E=D(\tau_R+\tau_{2D}), \tag{3.16c}$$

$$F=D(\tau_M+\tau_{1D}), \tag{3.16d}$$

$$L=\frac{1}{2}(h_R-h_D), \tag{3.16e}$$

$$V=\frac{1}{2}\left(h_D-h_M+\frac{2D}{P}h_M\right), \tag{3.16f}$$

$$A'=D(A_R+A_D+A_M),\ \text{and} \tag{3.17a}$$

$$B'=\frac{1}{2}\left[h_M\left(1-\frac{D}{P}\right)+\{\emptyset_R+\emptyset_D+(1+k)cI_e\}\right]. \tag{3.17b}$$

Now, in order to determine the optimum value of the decision variables, the cost equations are analyzed with respect to each of variables q, q_D and m_D with certain parameters. Figure 3.4, 3.5 and 3.6 show how the total cost functions for *Case I* (TC1) and *Case II* (TC2) behave with varying shipment sizes q and q_D and number of shipment, m_D, respectively. The graphs reveal that both of the cost equations are convex in nature with respect to all of the variables q, q_D and m_D.

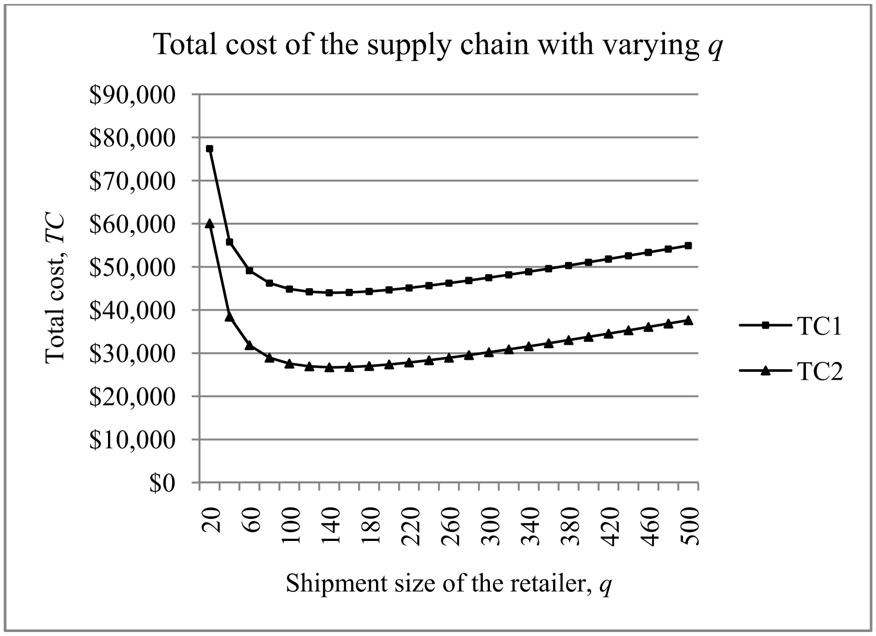

Figure 3.5 Total cost of the supply chain with respect to q

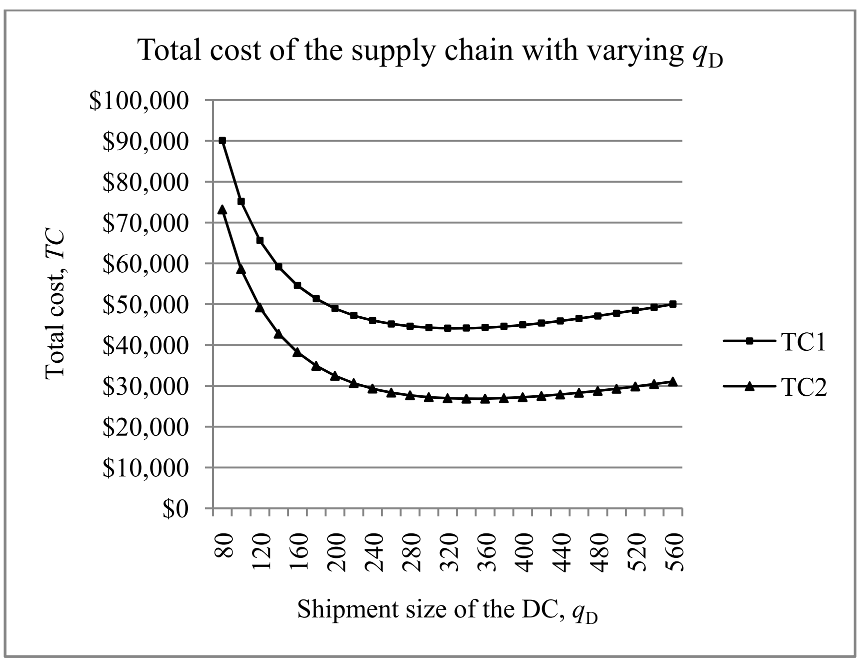

Figure 3.6 Total cost of the supply chain with respect to q_D

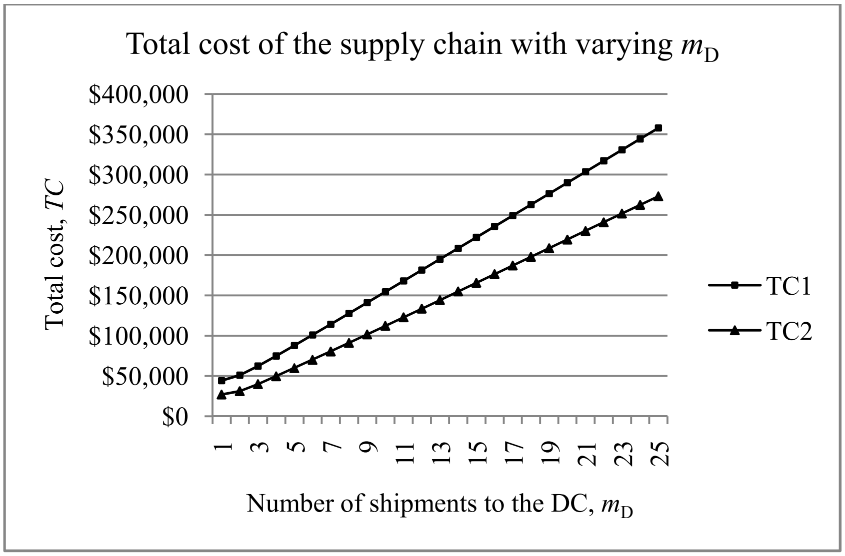

Figure 3.7 Total cost of the supply chain with respect to m_D

3.4 Solution Methodology

Now, for both cases I and II , the total cost function is a convex function in q, q_D and m_D . So relaxing the integer requirement of the variables, q, q_D and m_D, the simultaneous solutions of $\frac{\partial TC}{\partial q} = 0, \frac{\partial TC}{\partial q_D} = 0$ and $\frac{\partial TC}{\partial m_D} = 0$ will lead to the near-optimal solution for the non-integer variables q and q_D and integer variables m_D.

For Case I: $T \geq t$, the optimum values of the variables q^*, q_D^* and m_D^* are obtained by solving the simultaneous equation below using the software package MATLAB 7.0:

$$\frac{\partial TC_I^S}{\partial q} = L - \frac{E}{q^2} = 0, \tag{3.18}$$

$$\frac{\partial TC_I^S}{\partial q_D} = V + Bm_D - \frac{F}{q_D^2} - \frac{A}{m_D q_D^2} = 0, \text{ and} \tag{3.19}$$

$$\frac{\partial TC_I^S}{\partial m_D} = Bq_D - \frac{A}{q_D m_D^2} = 0, \tag{3.20}$$

which leads to $$q^* = \sqrt{\frac{2D(\tau_R + \tau_{2D})}{(h_R - h_D)}}\,, \tag{3.21}$$

$$q_D^* = \sqrt{\frac{2D(\tau_M + \tau_{1D})}{\left(h_D - h_M + \frac{2D}{P} h_M\right)}}. \tag{3.22}$$

and $$m_D^* = \sqrt{\frac{\left[A_R + A_D + A_M + \frac{1}{2} c\{I_p - (1+k)I_e\} D t^2\right]\left(h_D - h_M + \frac{2D}{P} h_M\right)}{(\tau_M + \tau_{1D})\left[h_M\left(1 - \frac{D}{P}\right) + \left(\emptyset_R + \emptyset_D + cI_p\right)\right]}}. \tag{3.23}$$

Since the decimal value for number of shipment, m_D^* is unacceptable, the value of m_D^* must be rounded using the following inequalities,

$$TC(m_D^* - 1) - TC(m_D^*) \geq 0 \tag{3.24}$$

and $$TC(m_D^* + 1) - TC(m_D^*) \geq 0 \tag{3.25}$$

Using the method of induction, the values of $TC(m_D^* - 1)$, $TC(m_D^*)$ and $TC(m_D^* + 1)$ are

$$TC_I^S(m_D{}^* - 1) = \frac{A}{(m_D{}^* - 1)q_D} + \frac{E}{q} + \frac{F}{q_D} + qL + q_D\mathrm{V} + \mathrm{B}(m_D{}^* - 1)q_D - cDIpt. \tag{3.26}$$

$$TC_I^S(m_D{}^*) = \frac{A}{m_D{}^* q_D} + \frac{E}{q} + \frac{F}{q_D} + qL + q_D\mathrm{V} + \mathrm{B}m_D{}^* q_D - cDIpt. \tag{3.27}$$

$$TC_I^S(m_D{}^* + 1) = \frac{A}{(m_D{}^* + 1)q_D} + \frac{E}{q} + \frac{F}{q_D} + qL + q_D\mathrm{V} + \mathrm{B}(m_D{}^* + 1)q_D - cDIpt. \tag{3.28}$$

Substituting Eqs (3.26), (3.27) and (3.28) into Eqs (3.24) and (3.25) and simplifying the expressions, the following inequality results,

$$m_D^*(m_D^* + 1) \geq \frac{1}{q_D^2}\frac{A}{B} \geq m_D^*(m_D^* - 1). \tag{3.29}$$

For Case II: $T < t$, solving the following equations simultaneously leads to the optimal solutions to the Case II.

$$\frac{\partial TC_{II}^{S}}{\partial q} = L - \frac{E}{q^2} = 0, \tag{3.18}$$

$$\frac{\partial TC_{II}^{S}}{\partial q_D} = V + B'^{m_D} - \frac{F}{q_D^2} - \frac{A'}{m_D q_D^2} = 0, \text{ and} \tag{3.30}$$

$$\frac{\partial TC_{II}^{S}}{\partial m_D} = B' q_D - \frac{A'}{q_D m_D^2} = 0 \tag{3.31}$$

which leads to $$q^* = \sqrt{\frac{2D(\tau_R + \tau_{2D})}{(h_R - h_D)}}\ , \tag{3.21}$$

$$q_D^* = \sqrt{\frac{2D(\tau_M + \tau_{1D})}{\left(h_D - h_M + \frac{2D}{P} h_M\right)}}, \tag{3.22}$$

and $$m_D^* = \sqrt{\frac{(A_R + A_D + A_M)\left(h_D - h_M + \frac{2D}{P} h_M\right)}{(\tau_M + \tau_{1D})\left[h_M\left(1 - \frac{D}{P}\right) + \{\emptyset_R + \emptyset_D + (1+k)cI_e\}\right]}}. \tag{3.32}$$

Again, as before, the value of m_D^* must be rounded using the following inequality,

$$m_D^*(m_D^* + 1) \geq \frac{1}{q_D^2}\frac{A'}{B'} \geq m_D^*(m_D^* - 1). \tag{3.33}$$

Based on the optimal solution, the cycle time, T, needs to be identified to verify the feasibility conditions for two different cases of the retailer's trade credit situation. That is why, an algorithm is necessary to solve the model, which is given as

Algorithm 1: Adjusting for integer solution and credit period conditions for single-channel

Step 1: Initialize $A_R,\ A_D,\ A_M,\ h_R,\ h_D,\ h_M,\ \tau_R, \tau_{1D},\ \tau_{2D},\ \tau_M,\ \emptyset_R,\ \emptyset_D,\ c,\ k,\ I_e,\ I_p,\ t,\ P$ and D.

Step 2: Compute, $q^*,\ q_D^*$ and m_D^* using Eqs (3.21), (3.22), and (3.23), respectively.

Set $q_D \leftarrow q_D^*$ and $m_D \leftarrow m_D^*$. Compute $T = \frac{m_D q_D}{D}$.

For $T < t$, go to step 6.

Step 3: Compute $q_D^* = \frac{TD_1}{\lceil m_D^* \rceil}$. Set $q_D \leftarrow q_D^*$.

If, $\lfloor m_D^* \rfloor = 0$, set $m_D \leftarrow 1$.

Else, if $\lceil m_D^* \rceil(\lceil m_D^* \rceil + 1) \geq \frac{1}{q_D^2}\frac{A}{B} \geq \lceil m_D^* \rceil(\lceil m_D^* \rceil - 1)$, set $m_D \leftarrow \lceil m_D^* \rceil$

Else, set $m_D \leftarrow \lfloor m_D^* \rfloor$. Compute $q_D^* = \frac{TD_1}{\lfloor m_D^* \rfloor}$. Set $q_D \leftarrow q_D^*$.

Step 4: Compute $m_R^* = \frac{TD}{q^*}$

If $\lfloor m_R^* \rfloor = 0$, $m_R \leftarrow 1$.

Else, set $m_R \leftarrow \text{round}(m_R^*)$. Set $q \leftarrow \frac{TD}{m_R}$.

Step 5: Find ${TC_I}^S$ using Eq. (3.16).

Step 6: Compute, $q^*,\ q_D^*$ and m_D^* using Eqs (3.21), (3.22), and (3.32), respectively.

Set $q_D \leftarrow q_D^*$ and $m_D \leftarrow m_D^*$. Compute $T = \frac{m_D q_D}{D}$.

For $T \geq t$, go to step 8.

Compute $q_D^* = \frac{TD_1}{\lceil m_D^* \rceil}$. Set $q_D \leftarrow q_D^*$.

If, $\lfloor m_D^* \rfloor = 0$, set $m_D \leftarrow 1$.

Else, if $\lceil m_D^* \rceil (\lceil m_D^* \rceil + 1) \geq \frac{1}{q_D^2} \frac{A'}{B'} \geq \lceil m_D^* \rceil (\lceil m_D^* \rceil - 1)$, set $m_D \leftarrow \lceil m_D^* \rceil$

Else, set $m_D \leftarrow \lfloor m_D^* \rfloor$. Compute $q_D^* = \frac{TD_1}{\lfloor m_D^* \rfloor}$. Set $q_D \leftarrow q_D^*$.

Repeat step 4.

Step 7: Find ${TC_{II}}^S$ using Eq. (3.17). Go to step 9.

Step 8: Set $T \leftarrow t$, Repeat steps 3 and 4.

Compute ${TC_t}^S$ by using Eq. (3.16).

Step 9: Set $TC^m \leftarrow min({TC_I}^S,\ {TC_{II}}^S, {TC_t}^S)$. Take $q^*, q_D^*,$ and m_D^* with least annual cost value.

Step 10: Stop ■

An example is devised to illustrate the model of the joint optimal delivery policy of the three echelon supply chain system.

- **Example 3.1:** Single-channel, multi-echelon model

As an illustration of the single-channel, multi-echelon supply chain model, a numerical example is presented for a single product by putting together the numerical problems from Kreng and Chen (2007) and Sarker *et al.* (2000). The numerical example from Kreng and Chen (2007) is exactly taken and other required information for permissible delay period, interest payable rate and interest earning rate are taken from Sarker *et al.*(2000) . In addition to those given variables, since profit rate is not considered in any of those examples, it is estimated reasonably as k = 0.05.

In the example, $P = 2000$ unit/year, $D = 1800$ unit/year, and t = 30 days. The other cost parameters are given in the Table below:

Table 3.1: Cost parameters of Example 3.1

Manufacturer	**Distribution Center**	**Retailer**
$\boldsymbol{A_M}$ = \$1600/order	A_D = \$400/order	A_R = \$400/order
$\boldsymbol{\tau_M}$ = \$864/shipment	τ_{1D}= \$216/shipment	τ_R = \$100/shipment
	τ_{2D}= \$400/shipment	h_R = \$108/unit/year
$\boldsymbol{h_M}$ = \$21.6/unit/year	h_D = \$21.6/unit/year	$\emptyset_R$ = \$18/unit/year
	$\emptyset_D$ = \$8.1/unit/year	c = \$270/unit $k = 0.05$
		I_p = 0.20* I_e = 0.13*

* Sarker *et al.* (2000)

The optimal model of individual subsystem is unable to operate independently in a supply chain system. In such a case the uncoordinated inventory underestimates the minimum cost and hence, the total cost is not practical (Kelle *et al.* 2003). The typical delivery policy was defined by Kreng and Chen (2007). Such a policy sets the production lot size equal to the integer multiple of the delivery quantity of dominating subsystem of the supply chain. In this policy, if the retailer is the focal company in the supply chain, which is true for most cases, it forces others to adopt its own optimal policy. On the other hand, if the manufacturer is the sole supplier, the retailer has to accept the optimal policy from the manufacturer. Following the typical delivery policy and taking the retailer as the focal company of the system, the optimal policy gives $q_D \cong$ 174 units, m_D = 1, $q \cong$ 58 units and m_R = 3, with the total supply chain cost \$ 57,830. Under these optimum values, $T \cong$ 36 days, which fulfills the condition of $T \geq t$, having t = 30 days.

Following the proposed algorithm, for *Case I*, the optimum $q_D \cong$ 331, m_D = 1, $q \cong$ 166 and m_R = 2. Under these optimum values, $T \cong$ 68 days, which fulfills the condition of $T \geq t$, since t = 30 days. Considering *Case II*, the optimum values are, $q_D \cong$ 364, m_D = 1, $q \cong$ 121 and m_R = 3; but under these optimum values $T \cong$ 74 days, that violates the condition $T < t$ and makes the solution infeasible. Hence, *Case I* gives the optimum solution for which $q \cong$ 166 units and the joint optimal cost of the three-echelon supply chain is calculated as $44,139/year. It is observed that the total annual cost derived by the algorithm is about 23.67% less than the total cost derived from the typical delivery plan. Hence, the presented approach gives a better solution compared to the typical delivery policy to reduce the total cost of the supply chain.

3.5 Discussion

From the solution of the numerical example, it is evident that there can be considerable savings on the overall supply chain cost if all the subsystems agree to collaborate and follow the optimum operational plan based on the integrated approach of the system presented here. The solution methodology presented here is simple and can be easily adapted in practice. The cost savings on the system improves the overall supply chain performance and all of the subsystems can get the benefit in the long run. However, there might be instances where the cost of any subsystem derived by individual optimum model is less compared to the cost derived by the optimum collaborative model. In such cases, the losing party should be given some incentive. For example, the share of the shipment cost should be reduced on the losing party and should be increased on the winning party. Such share does not have any impact on the optimum solution of the collaborative supply chain model.

In the succeeding chapter, this research addresses the problem that is more general and relatively complex, and thus, it enhances the scope of the application.

CHAPTER 4

MULTI-CHANNEL MULTI-ECHELON DISTRIBUTION SYSTEM

In this chapter a multi-channel, multi-echelon JIT delivery policy of a single product is developed. The product flows through three-echelon upstream integrated supply chain consists of single manufacturer, multiple distribution centers (DC) and multiple retailers. The distribution centers allow trade credit benefit to their retailers. This model is an extension of the model of previous chapter and a more generic form of supply chain system.

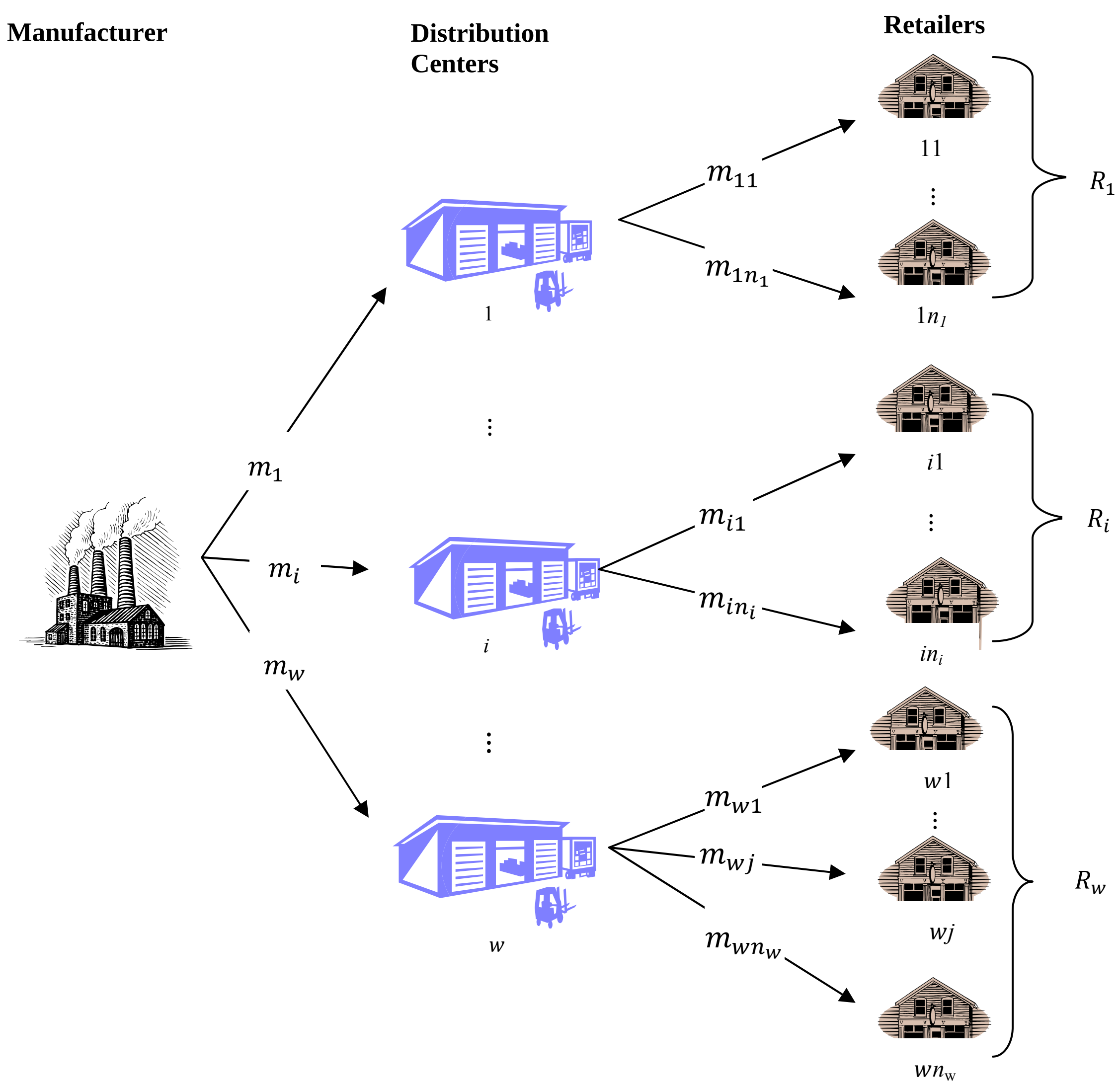

Figure 4.1 Three-echelon upstream integrated supply chain

4.1 The Problem

This chapter considers the inventory model of a product that follows a upstream integrated, three-echelon supply chain from a manufacturer to retailers as shown in Figure 4.1. As shown in Figure 4.1, the manufacturer delivers the product to distribution center i $(i = 1, 2, \cdots, w)$, whereas distribution center i supplies the product to particular retailers ij $(j = 1, 2, \cdots, n_i)$; that is, each retailer is dedicated to a particular distribution center as its supplier. The total demand at each echelon level, that is, the total demand of all the retailers is equal to the total demand of all the distribution centers, which is exactly equal to the demand at the manufactuer's level. The manufacturer and all the distribution centers adapt **lot-splitting** delivery policy. However, due to different demand rates and related cost parameters, the distribution centers and retailers might have different optimum shipment sizes and number of shipments.

4.2 Model Formulation

Production is organized in such a way that the first shipment to each distribution center is done in a sequence. Following this sequence, the first delivery starts from the first distribution center followed by the second, the third, and so on. In this model, the order cycle time for each retailer, distribution center and the production cycle time for the manufacturer are equal. The objective of the problem is to determine the delivery policy of the product through each echelon level, such that it minimizes the total cost of the integrated supply chain. Hence, the total annual cost of each entity of each echelon level is determined and they are summed together to determination of the total cost of the supply chain. This leads to determine the production/ordering quantity and the number of shipments of each channel in each echelon level, with an aim to minimize the overall cost.

Denoting the number of shipments from the manufacturer to ith distribution center by m_i and the quantity per shipment in each ordering cycle by q_i, the manufacturer produces the quantity $\sum_{i=1}^{w} m_i q_i$ in cycle time T that exactly equals DT, the total demand of product in each cycle. So, as in the previous model of single channel multi-echelon system, a saw-tooth fashion inventory model is built up for the manufacturer during the production period as shown in Figure 4.2.

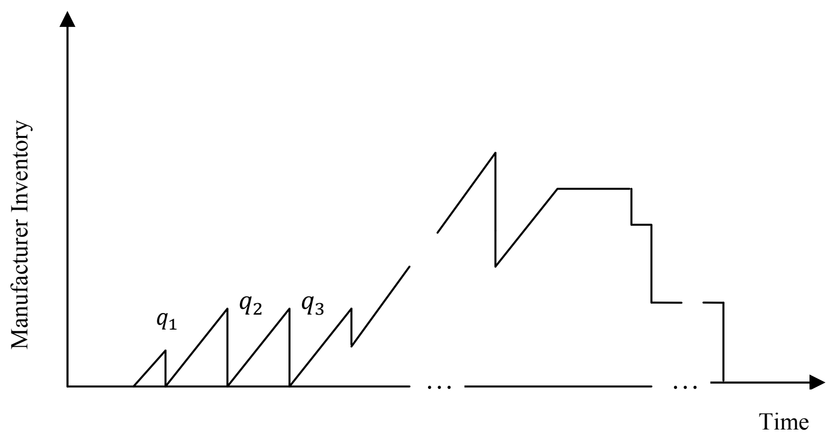

Figure 4.2 The manufacturer's inventory level under a upstream integrated supply chain

The ith distribution center again splits the quantity, q_i into number of shipments so that it can deliver n_{ij} shipments having q_{ij} units of products in each to the corresponding retailer ij, *i.e.,* $q_i = \sum_{j=1}^{n_i} n_{ij} q_{ij}$. So, the inventory of the distribution center resembles the sum of step-down step functions, where each step corresponds to each retailer of the respective distribution center. Moreover, the ith distribution center ships an order to corresponding retailer ij by m_{ij} number of shipments, each shipment having quantity q_{ij}. The inventory of the corresponding retailer resembles the saw-tooth having m_{ij} number of iterations in cycle time, T, where each iteration having height q_{ij}, follows traditional economic ordering model with instantaneous replenishment.

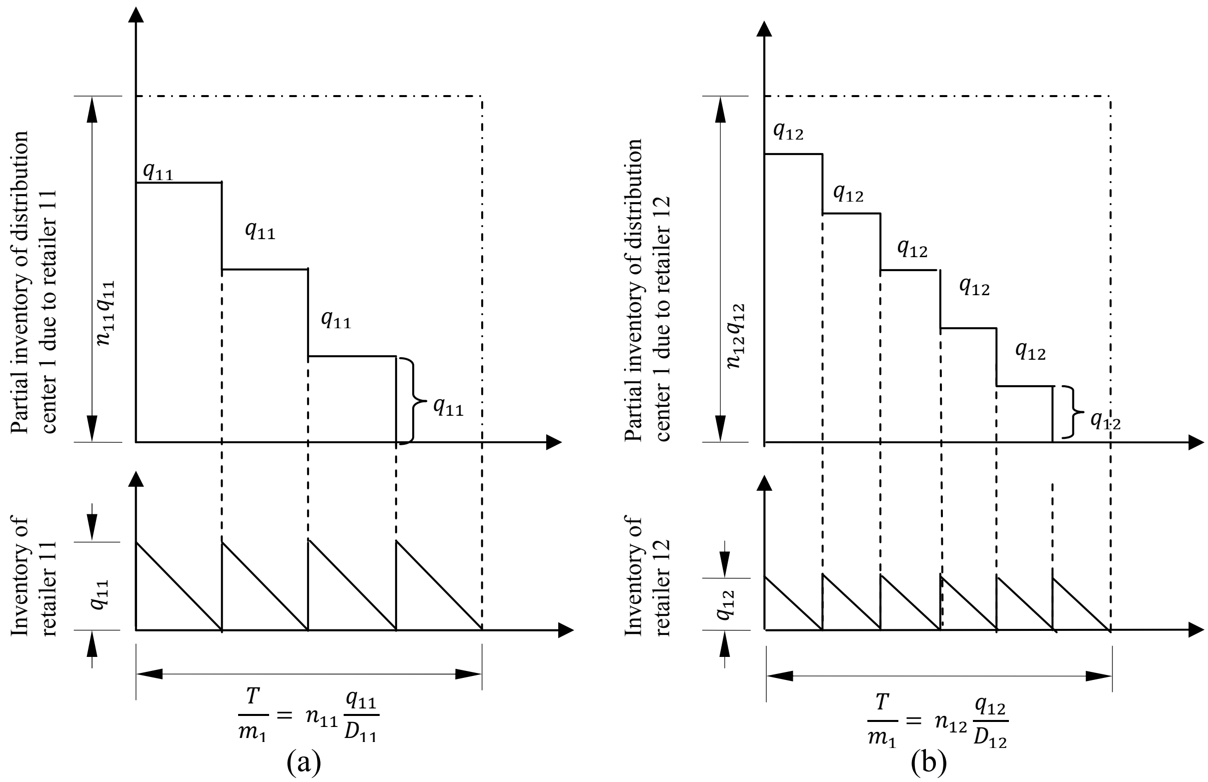

Figure 4.3 Partial inventory of distribution center 1 and inventory of corresponding retailers

For example, suppose in the supply chain system the manufacturer has three distribution centers, $N = 1, 2$ and 3. Now distribution center 1 ships to two dedicated retailers, $n_1 = 2$, having index $ij = 11$ and 12, distribution center 2 ships to two retailers, $n_2 = 2$, having index $ij = 21$ and 22; and distribution center 3 ships to three retailers, $n_3 = 3$, having index $ij = 31$ and 33, as shown in Figure 4.1. Now considering distribution center 1, it receives m_1 number of shipments from the manufacturer during the cycle time T having q_1 amount of products in each shipment. Hence, the time between two shipments from the manufacturer to the distribution center 1 is T/m_1. The distribution center 1 needs to deliver this q_1 quantity of products among each of its retailers 11 and 12. Now the distribution center 1 splits the quantity q_1 in such a way that retailer 11 gets n_{11} number of shipments with each having quantity q_{11} and retailer 12 gets n_{12} number of shipments with each having quantity q_{12}, *i.e.,* $q_1 = n_{11}q_{11} + n_{12}q_{12}$. So the inventory of the distribution center 1 pertaining to two step functions, one corresponds to retailer 11 and the other one corresponds to retailer 12 as shown in (a) and (b) of Figure 4.3, respectively. Hence, the total inventory of distribution center 1 can be calculated by sum of the areas under both of the step-ladders of (a) and (b) and thus the average inventory of distribution center 1 can be found out by dividing the total area by the time T/m_1.

Resembling the single channel multi-echelon system, every retailer and distribution center incurs flexibilibty loss cost. Moreover, due to the trade credit benefit for every retailer, in addition to ordering cost (alternatively set-up cost), transportation cost and holding cost, every retailer's total annual relevant cost includes, flexibility loss cost and interest cost due to trade credit. Every distribution center also incurs flexibility loss cost as a consequence of committed but undelivered supply.

4.2.1 Notation and Assumptions

The following notations are used to model this upstream integrated three-echelon supply chain delivery system with trade credit consideration:

4.2.1.1 Retailer's Demand and Cost Parameters

ij Subscripts used to identify retailer j ($j = 1, 2, \cdots, n_i$) by DC i ($i = 1, 2, \cdots, w$), for example when $ij = 32$, it represents 2nd retailer of 3rd DC.

D_{ij} Annual demand rate of the retailer ij (units/year)

A_{ij} Ordering cost per contract of the retailer ij (dollars/order)

h_{ij} Stock holding cost per unit per year for the retailer ij (dollars/unit/year)

τ_{ij} The fixed transportation cost of receiving a shipment from DC i (dollars/shipment)

$\emptyset_{ij}$ Flexibility loss cost of the retailer ij (dollars/unit/year)

c Unit purchase price from DC (dollars/unit)

k Profit rate on purchase price, that is, retailer's selling price =$(1 + k)c$

I_e Interest earned per dollar per year (/dollar/year)

I_p Interest payable to DC per dollar per year (/dollar/year)

t Permissible delay in settling the account (year)

4.2.1.2 Distribution Center's (DC) Cost Parameters

i A subscript used to represent different DC, $i = 1, 2, \cdots, w$.

D_i Annual demand rate of the DC i (units/year), $i.e.,$ $D_i = \sum_{j=1}^{n_i} D_{ij}$

A_i Ordering cost per contract of DC i (dollars/order)

h_i Stock holding cost per unit per year for the DC i (dollars/unit/year)

τ_{Di} The transportation cost of receiving a shipment by DC i from the manufacturer (dollars/shipment)

τ_{Dij} The transportation cost of delivering a shipment from DC i to retailer ij(dollars/shipment)

$\emptyset_i$ Flexibility loss cost per unit per year of DC i (dollars/unit/year)

4.2.1.3 Manufacturer's Production and Cost Parameters

P Annual production rate of the manufacturer (units/year)

D Annual demand rate (units/year)

A_M Fixed production setup cost per lot size (dollars/batch)

h_M Stock holding cost per unit per year at the manufacturer (dollars/unit/year)

τ_{Mi} The transportation cost of a shipment from the manufacturer to the DC i (dollars/shipment)

4.2.1.3 Relevant Variables

m_{ij} Number of shipments per order from DC i ($i = 1, 2, \cdots, w$) to the retailer ij ($j = 1, 2, \cdots, n_i$), $m_{ij} \geq 1$ for $\forall i, j$.

m_i Number of shipments per order from manufacturer to the DC i, $m_i \geq 1$ for $\forall i, j$.

q_{ij} Shipment quantity from DC i to the retailer ij in each shipment (units/shipment)

q_i Shipment quantity from manufacturer to the DC i in each shipment (units)

n_{ij} Number of shipments to the retailer ij per quantity q_i, *i.e.*, $q_i = \sum_i n_{ij} q_{ij}$

T Common cycle time, that is, the time between successive production runs as well as time difference between successive orders of all echelon level

In addition to the assumptions of the previous model, the following assumptions are made to model the multi-channel three-echelon supply chain system:

(1) Equal-sized shipments are assumed for any particular DC or retailer; but the shipment size might differ among retailers or DCs in accordance with their demand rate and other cost parameters.

(2) All echelons share the equal demand information and demand rate for each entity is constant and deterministic. Hence, $D = \sum_{i=1}^{w} D_i = \sum_{i=1}^{w} \sum_{j=1}^{n_i} D_{ij}$.

(3) The production is organized in such a way that the first delivery initiates from the first DC followed by the second DC, the third DC and it proceeds likewise sequentially. Hence, $\frac{T}{m_i} \geq \frac{T}{P}\sum_{i=1}^{w}\frac{D_i}{m_i}$.

4.3 The Joint Annual Cost Function

This section derives the cost involved in the entire supply chain delivery system containing a single manufacturer, multiple distribution center and multiple retailers. The system follows the JIT delivery policy of frequent deliveries in small lots. Since the demand is known and no shortage is allowed, the manufacturing quantity must be equal to the total demand of the common cycle time. The joint annual cost of the upstream integrated three-echelon system, TC^m consists of the annual cost of the manufacturer, ${TC_M}^m$, the annual cost of the all the distribution centers, ${TC_D}^m$ and the annual cost of all the retailers, ${TC_R}^m$, which is:

$$TC^m = {TC_M}^m + {TC_D}^m + {TC_R}^m. \tag{4.1}$$

4.3.1 Cost at the Retailers

The inventory model of each retailer of the system exactly resembles the inventory model of the retailer of the model addressed in section 3. Hence, for $T \geq t$, the annual total cost of the retailer ij is given by

$$\begin{aligned} &{TC_{ij}}^m(m_{ij}, q_{ij}, T) \\ &\quad = \frac{A_{ij}}{T} + \frac{Z_{ij}m_{ij}}{T} + \frac{h_{ij}q_{ij}}{2} + \emptyset_{ij}\left(\frac{m_{ij}q_{ij}}{2}\right) + \frac{cD_{ij}(T-t)^2 I_p}{2T} \\ &\quad - \frac{(1+k)cD_{ij}t^2 I_e}{2T} \,. \end{aligned} \tag{4.2}$$

Hence, for $T \geq t$ the total annual cost of all the retailers is given by

$TC_{R,I}{}^{m}(m_{ij}, q_{ij}, T)$

$$= \sum_{i=1}^{w} \sum_{j=1}^{n_i} \left[\frac{A_{ij}}{T} + \frac{\tau_{ij} m_{ij}}{T} + \frac{h_{ij} q_{ij}}{2} + \emptyset_{ij}\left(\frac{m_{ij} q_{ij}}{2}\right) + \frac{c D_{ij} (T-t)^2 I_p}{2T} - \frac{(1+k) c D_{ij} t^2 I_e}{2T}\right]. \quad (4.3)$$

Again for $T < t$, the annual total relevant cost of the retailer ij is given by

$$TC_{ij}{}^{m}(m_{ij}, q_{ij}, T) = \frac{A_{ij}}{T} + \frac{\tau_{ij} m_{ij}}{T} + \frac{h_{ij} q_{ij}}{2} + \emptyset_R\left(\frac{m_{ij} q_{ij}}{2}\right) - (1+k) c D_{ij}\left(t - \frac{T}{2}\right) I_e . \quad (4.4)$$

Hence, for $T < t$, the total annual cost of all the retailers is given by

$TC_{R,II}{}^{m}(m_{ij}, q_{ij}, T)$

$$= \sum_{i=1}^{w} \sum_{j=1}^{n_i} \left[\frac{A_{ij}}{T} + \frac{\tau_{ij} m_{ij}}{T} + \frac{h_{ij} q_{ij}}{2} + \emptyset_R\left(\frac{m_{ij} q_{ij}}{2}\right) - (1+k) c D_{ij}\left(t - \frac{T}{2}\right) I_e\right]. \quad (4.5)$$

4.3.2 Cost at the Distribution Centers

In order to obtain the annual holding cost, the average inventory of the distribution center i, I_i should be known. The method of finding average inventory at distribution center i has already been discussed in Section 4.1. The inventory at distribution center i during time T/m_i can be obtained by summing over the areas under the step-down step functions, each step function corresponds to retailer of the corresponding distribution center. Suppose distribution center 1 has two retailers, having indices 11 and 12, as shown in Figure 4.1. The corresponding inventory at distribution center 1 is shown by the step functions in (a) and (b) of Figure 4.3. Here, in time T/m_1 the distribution center 1 delivers n_{11} number of shipments to retailer 11, each containing quantity q_{11} and n_{12} number of shipments to retailer 12, each containing quantity q_{12}; hence,

$q_1 = n_{11}q_{11} + n_{12}q_{12}$. Given that D_{11} and D_{12} are annual demand rate at retailer 11 and retailer 12, respectively, the partial inventory of distribution center 1 due to retailer 11 can be expressed by

$$I_{11} = q_{11}\left[\frac{q_{11}}{D_{11}}(n_{11}-1) + \frac{q_{11}}{D_{11}}(n_{11}-2) + \cdots + \frac{q_{11}}{D_{11}}\right]$$

$$= \frac{q_{11}^2}{D_{11}}\frac{n_{11}(n_{11}-1)}{2} \quad . \tag{4.6}$$

Likewise the other part of inventory of distribution center 1 due to retailer 12 can be expressed by

$$I_{12} = \frac{q_{12}^2}{D_{12}}\frac{n_{12}(n_{12}-1)}{2} \, . \tag{4.7}$$

Now the average inventory of the distribution center 1 is given by

$$I_1 = \frac{1}{T_1}[I_{11} + I_{12}], \tag{4.8}$$

where $T_1 = \frac{T}{m_1} = n_{11}\frac{q_{11}}{D_{11}} = n_{12}\frac{q_{12}}{D_{12}}$. Now, putting the value of I_{11} and I_{12} and after some simplification, the above equation can be expressed by

$$I_1 = \frac{D_{11}}{n_{11}q_{11}} \times \frac{q_{11}^2}{D_{11}}\frac{n_{11}(n_{11}-1)}{2} + \frac{D_{12}}{n_{12}q_{12}} \times \frac{q_{12}^2}{D_{12}}\frac{n_{12}(n_{12}-1)}{2}$$

$$= \frac{1}{2}[q_1 - (q_{11} + q_{12})] \, . \tag{4.9}$$

Hence, in more generic form, the average inventory at distribution center i can be given by

$$I_i = \frac{1}{2}\left[q_i - \sum_{j=1}^{n_i} q_{ij}\right] . \tag{4.10}$$

So the annual holding cost of the distribution center i is $h_i \times \frac{1}{2}\left[q_i - \sum_{j=1}^{n_i} q_{ij}\right]$, where h_i is the annual holding cost per unit product. The annual ordering cost of the distribution center i is given by A_i/T. Annual shipment cost for receiving shipments from the manufacturer and for delivering shipments to the corresponding retailers can be expressed by $\tau_{Di}m_i/T$ and $\sum_{j=1}^{n_i} \tau_{Dij}m_{ij}/T$, respectively. The flexibility loss cost of the distribution center i is quantified by $\emptyset_i(m_i q_i/2)$. The annual associated cost of the distribtuion center i, TC_i^m, is the total of ordering cost, transportation cost of receiving as well as delivering shipment, holding cost and flexibility loss cost, which is given by:

$$TC_i{}^m(m_{ij}, m_i, q_{ij}, q_i, T) = \frac{A_i}{T} + \frac{\tau_{Di}m_i}{T} + \sum_{j=1}^{n_i} \frac{\tau_{Dij}m_{ij}}{T} + \frac{h_i}{2}\left[q_i - \sum_{j=1}^{n_i} q_{ij}\right] + \emptyset_i \frac{m_i q_i}{2}. \quad (4.11)$$

Hence, the total cost of all the distribution centers is given by

$$TC_D{}^m(m_{ij}, m_i, q_{ij}, q_i, T) = \sum_{i=1}^{w}\left[\frac{A_i}{T} + \frac{\tau_{Di}m_i}{T} + \sum_{j=1}^{n_i} \frac{\tau_{Dij}m_{ij}}{T} + \frac{h_i}{2}\left(q_i - \sum_{j=1}^{n_i} q_{ij}\right) + \emptyset_i \frac{m_i q_i}{2}\right]. \quad (4.12)$$

4.3.3 Cost at the Manufacturer

The manufacturer organized the production in such a way that the first shipment for each distribution center is done in a sequence. The first delivery starts from the first distribution center followed by the second, the third, and so on (ref. Figure 4.2). Moreover, the first q_1 units are produced exactly by the time they are to be shipped, then the first q_2 units, then the first q_3 units and so on. The inventory under such condition has been discussed and derived by Siajadi, Ibrahim and Lochert (2006), which is illustrated in Appendix B.

From Appendix B, the manufacturer's average inventory, ${I_M}^m$ is obtained as,

$$
\begin{aligned}
&{I_M}^m \\
&= \frac{T}{2P}\left\{\left(\sum_{i=1}^{w} D_i\right)^2\right. \\
&+ 2\sum_{i=1}^{w} D_i \left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right] \\
&\left.-\sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\}. \qquad (4.13)
\end{aligned}
$$

Hence, the annual holding cost of the manufacturer is $h_M {I_M}^m$. Annual setup cost of the manufacturer is A_M/T and the annual transportation cost is identified as $\sum_{i=1}^{w} \tau_{Mi} m_i/T$. The annual associated cost of the manufacturer is the total of annual setup cost, transportation cost and holding cost, which is given by:

$$
\begin{aligned}
&{TC_M}^m(m_i, T) \\
&= \frac{A_M}{T} + \sum_{i=1}^{w} \frac{\tau_{Mi} m_i}{T} \\
&+ h_M \frac{T}{2P}\left\{\left(\sum_{i=1}^{w} D_i\right)^2 + 2\sum_{i=1}^{w} D_i \left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right]\right. \\
&\left.-\sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} \frac{D_l}{m_l}\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\}. \qquad (4.14)
\end{aligned}
$$

Finally, the annual total relevant cost of the entire supply chain, TC^m, can be written as

$$TC^m = {TC_M}^m + {TC_D}^m + {TC_R}^m .$$

Since there are two distinct cases of trade credit, the annual cost to the retailers will be different. Hence, having cycle time, T and permissible delay period, t, the annual total associated cost of the entire system will be different for two distinct cases of retailers' trade credit condition, namely for $T \geq t$ and $T < t$.

Case I: $T \geq t$. When $T \geq t$, the annual cost of the system can be written by

$$TC_I{}^m\left(m_i, m_{ij}, q_i, q_{ij}, T\right)$$

$$= \frac{A_M}{T} + \sum_{i=1}^{w} \frac{\tau_{Mi} m_i}{T}$$

$$+ h_M \frac{T}{2P}\left\{\left(\sum_{i=1}^{w} D_i\right)^2 + 2\sum_{i=1}^{w} D_i \left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right]\right.$$

$$\left. - \sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\}$$

$$+ \sum_{i=1}^{w}\left[\frac{A_i}{T} + \frac{\tau_{Di} m_i}{T} + \sum_{j=1}^{n_i} \frac{\tau_{Dij} m_{ij}}{T} + \frac{h_i}{2}\left(q_i - \sum_{i=1}^{w} q_{ij}\right) + \emptyset_i \frac{m_i q_i}{2}\right]$$

$$+ \sum_{i=1}^{w}\sum_{j=1}^{n_i}\left[\frac{A_{ij}}{T} + \frac{\tau_{ij} m_{ij}}{T} + \frac{h_{ij} q_{ij}}{2} + \emptyset_{ij}\left(\frac{m_{ij} q_{ij}}{2}\right) - \frac{cD_{ij}(T-t)^2 I_p}{2T}\right.$$

$$\left. + \frac{(1+k)cD_{ij} t^2 I_e}{2T}\right]. \tag{4.15}$$

Case II: $T < t$. When $T < t$, the annual cost is given by

$$TC_{II}{}^m\left(m_i, m_{ij}, q_i, q_{ij}, T\right) = \frac{A_M}{T} + \sum_{i=1}^{w} \frac{\tau_{Mi} m_i}{T}$$

$$+ h_M \frac{T}{2P}\left\{\left(\sum_{i=1}^{w} D_i\right)^2 + 2\sum_{i=1}^{w} D_i \left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right]\right.$$

$$\left. - \sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\}$$

$$+\sum_{i=1}^{w}\left[\frac{A_i}{T}+\frac{\tau_{Di}m_i}{T}+\sum_{j=1}^{n_i}\frac{\tau_{Dij}m_{ij}}{T}+\frac{h_i}{2}\left(q_i-\sum_{j=1}^{n_i}q_{ij}\right)+\emptyset_i\frac{m_iq_i}{2}\right]$$

$$+\sum_{i=1}^{w}\sum_{j=1}^{n_i}\left[\frac{A_{ij}}{T}+\frac{\tau_{ij}m_{ij}}{T}+\frac{h_{ij}q_{ij}}{2}+\emptyset_{ij}\left(\frac{m_{ij}q_{ij}}{2}\right)-(1+k)cD_{ij}(t-\frac{T}{2})I_e\right]. \tag{4.16}$$

Eq (4.15) and (4.16) are subjected to

$$m_i, m_{ij} \geq 1. \text{ for } \forall i, j \tag{4.16a}$$

$$\frac{T}{m_i} \geq \frac{T}{P}\sum_{i=1}^{w}\frac{D_i}{m_i}. \tag{4.16b}$$

$$q_i, q_{ij}, T > 0 \tag{4.16c}$$

In order to simplify the cost equations, some relationships have been established among the variables. By relaxing the integer requirement of the variables, it is found that the optimum shipment size for any retailer, without considering the other subsystems, is given by

$$m_{ij} = \sqrt{\frac{A_{ij}h_{ij}}{\emptyset_{ij}\tau_{ij}}},$$

which establishes a logical relationship

$$\frac{m_{i1}}{m_{ij}} = \sqrt{\frac{A_{i1}h_{i1}}{\emptyset_{i1}\tau_{i1}}} \times \sqrt{\frac{\emptyset_{ij}\tau_{ij}}{A_{ij}h_{ij}}} \quad \text{for } \forall i, j. \tag{4.17}$$

Again, since all the subsystems have equal cycle time, T, the following relationship exists,

$$\frac{m_{i1}q_{i1}}{D_{i1}} = \frac{m_{i2}q_{i2}}{D_{i2}} = \cdots = \frac{m_{in_i}q_{in_i}}{D_{in_i}} = T. \tag{4.18}$$

So, from Eq. (4.17) and (4.18),

$$q_{ij} = \alpha_{ij} q_{i1}, \tag{4.19}$$

where
$$\alpha_{ij} = \frac{D_{ij}}{D_{i1}} \times \sqrt{\frac{A_{i1} h_{i1}}{\emptyset_{i1} \tau_{i1}}} \times \sqrt{\frac{\emptyset_{ij} \tau_{ij}}{A_{ij} h_{ij}}}, \quad \text{for} \quad \forall i, j \tag{4.20}$$

Relaxing the integer requirement of the variables it is found that the optimum shipment size for any distribution center, without considering the other subsystems, is given by

$$m_i = T \sqrt{\frac{D_i h_i}{2\tau_{Di}}},$$

which also establishes a logical relationship:

$$\frac{m_1}{m_i} = \sqrt{\frac{D_1 h_1}{\tau_{D1}}} \times \sqrt{\frac{\tau_{Di}}{D_i h_i}} \quad \text{for} \quad \forall i. \tag{4.21}$$

So,
$$m_i = \lambda_i m_1, \tag{4.22}$$

where
$$\lambda_i = \sqrt{\frac{D_i h_i \tau_{D1}}{\tau_{Di} D_1 h_1}}. \tag{4.23}$$

Again, as before, since all the subsystems have equal cycle time, T, the following relationship exists:

$$\frac{m_i q_i}{D_i} = \frac{m_1 q_1}{D_1} = T. \tag{4.24}$$

So, from Eq. (4.20) and (4.21),

$$q_i = \beta_i q_1, \tag{4.25}$$

where $$\beta_i = \sqrt{\frac{D_i \tau_{Di} h_1}{h_i D_1 \tau_{D1}}} \quad \text{for} \quad \forall i. \tag{4.26}$$

Based on the common cycle time, T, the following relations apply,

$$\frac{m_{11} q_{11}}{D_{11}} = \frac{m_{i1} q_{i1}}{D_{i1}} = T. \tag{4.27}$$

From Eq. (4.17) and (4.22) the following relationship holds:

$$q_{i1} = \delta_i q_{11}, \tag{4.28}$$

where $$\delta_i = \frac{D_{i1}}{D_{11}} \times \sqrt{\frac{\emptyset_{i1} \tau_{i1}}{A_{i1} h_{i1}}} \times \sqrt{\frac{A_{11} h_{11}}{\emptyset_{11} \tau_{11}}} \quad \text{for} \quad \forall i. \tag{4.29}$$

Using Eq. (4.18), (4.19), (4.22), (4.24), (4.25) and (4.28) in Eq. (4.15) and (4.16); and after some algebraic manipulation,

$$TC_I{}^m (m_1, q_1, q_{11}) = \frac{G}{m_1 q_1} + \frac{H}{q_1} + \frac{Q}{q_{11}} + \frac{q_{11}}{2} S + m_1 q_1 J + q_1 R - cDtI_p, \tag{4.30}$$

and $$TC_{II}{}^m (m_1, q_1, q_{11}) = \frac{G'}{m_1 q_1} + \frac{H}{q_1} + \frac{Q}{q_{11}} + \frac{q_{11}}{2} S + m_1 q_1 J' + q_1 R - (1+k) cDtI_e, \tag{4.31}$$

where $$G = D_1 \left[A_M + \sum_{i=1}^{w} A_i + \sum_{i=1}^{w} \sum_{j=1}^{n_i} A_{ij} + \frac{D}{2} \{ ct^2 [I_p - (1+k) I_e] \} \right], \tag{4.30a}$$

$$H = \sum_{i=1}^{w} \frac{D_i}{\beta_i} (\tau_{Mi} + \tau_{Di}), \tag{4.30b}$$

$$J = \frac{1}{D_1} \left(\sum_{i=1}^{w} \sum_{j=1}^{n_i} \frac{\emptyset_{ij} D_{ij}}{2} + \frac{cDI_p}{2} \right) + \frac{D h_M}{2 D_1} \left(1 - \frac{D}{P} \right) + \sum_{i=1}^{w} \frac{\emptyset_i \lambda_i \beta_i}{2}, \tag{4.30c}$$

$$R = h_M\left[D\left(\frac{1}{P}-\frac{1}{D_1}\right)-\frac{1}{2D_1}\sum_{i=1}^{w}\frac{D_i}{\lambda_i}+\frac{1}{PD_1}\sum_{i=1}^{w}D_i\left(P-D_1+\sum_{l=1}^{i}\frac{D_l}{\lambda_l}\right)\right]+\sum_{i=1}^{w}\frac{\beta_i h_i}{2}, \quad (4.3od)$$

$$Q = \sum_{i=1}^{w}\sum_{j=1}^{n_i}\frac{D_{ij}}{\delta_i\alpha_{ij}}\left(\tau_{Dij}+\tau_{ij}\right), \quad (4.30e)$$

$$S = \sum_{i=1}^{w}\sum_{j=1}^{n_i}\delta_i\alpha_{ij}\left(h_{ij}-h_i\right), \quad (4.30f)$$

$$G' = D_1\left[A_M+\sum_{i=1}^{w}A_i+\sum_{i=1}^{w}\sum_{j=1}^{n_i}A_{ij}\right], \text{ and,} \quad (4.31a)$$

$$J' = \frac{1}{D_1}\left(\sum_{i=1}^{w}\sum_{j=1}^{n_i}\frac{\emptyset_{ij}D_{ij}}{2}+\frac{(1+k)cDI_e}{2}\right)+\frac{Dh_M}{2D_1}\left(1-\frac{D}{P}\right)+\sum_{i=1}^{w}\frac{\emptyset_i\lambda_i\beta_i}{2}. \quad (4.31b)$$

Now, in order to determine the optimum values of the decision variables, it is essential to analyze the cost equations with certain parameters. Figure 4.4, 4.5 and 4.6 show how the total cost functions for Case I (TC1) and Case II (TC2) behave with varying shipment sizes q_{11} and q_1 and number of shipment, m_1, respectively. It can be concluded that both of the cost functions follow the characteristics of a convex function with respect to all of decision variables q_{11}, q_1 and m_1. Hence, the minimum total cost of the supply chain can be obtained at optimal shipment sizes, q_{11}, q_1 and number of cycles, m_1. In the graphs the total costs decrease with increasing values of shipment size at the beginning, and increases again because of convexity property.

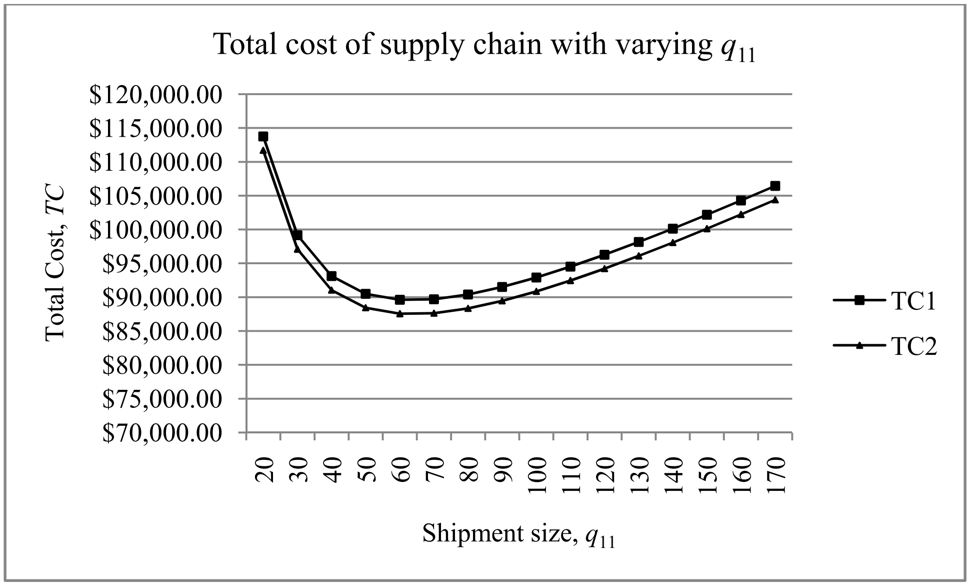

Figure 4.4 Total cost of the supply chain with respect to q_{11}

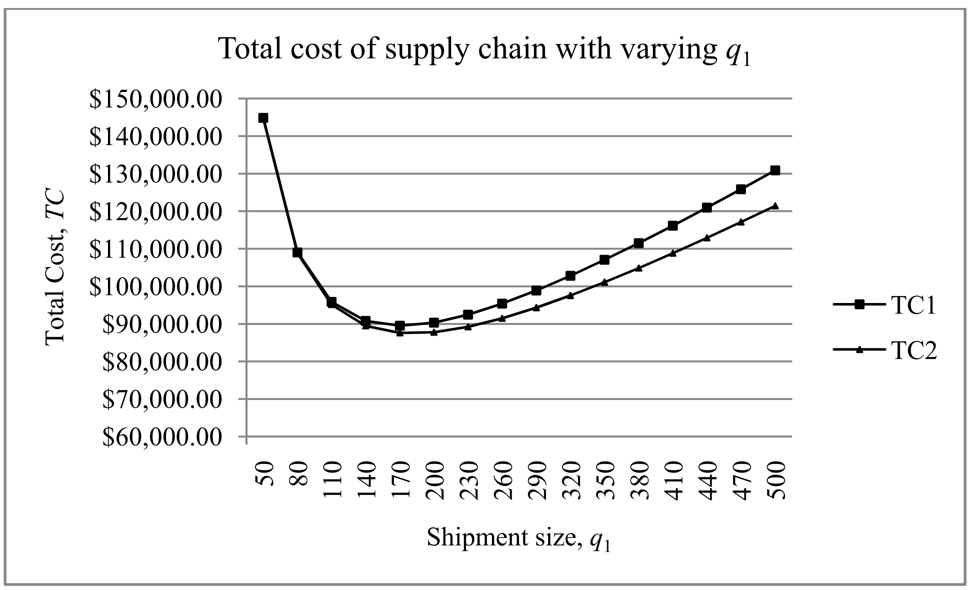

Figure 4.5 Total cost of the supply chain with respect to q_1

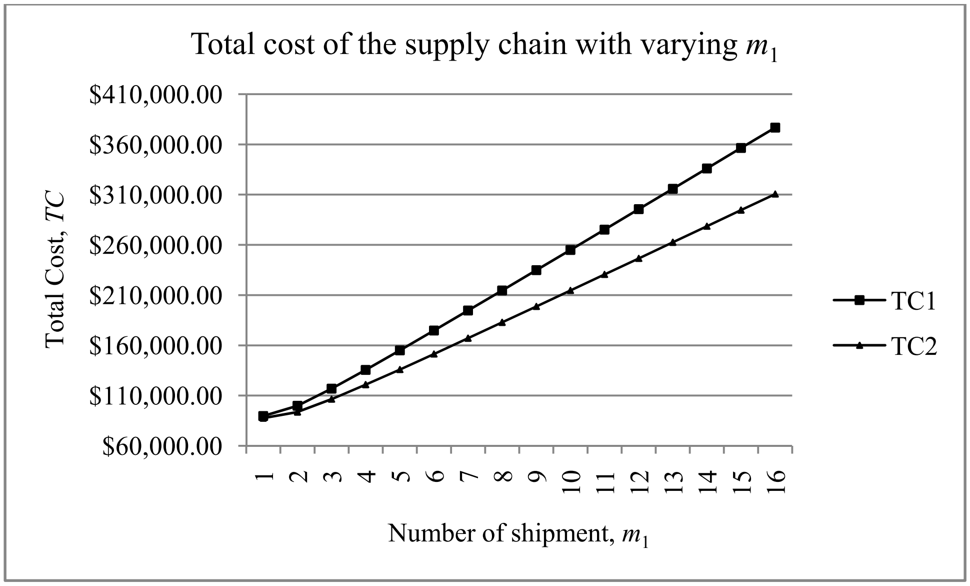

Figure 4.6 Total cost of the supply chain with respect to m_1

4.4 Solution Methodology

The total cost functions are convex in nature with respect to q_{11}, q_1 and m_1. So, the simultaneous solutions of the equations $\frac{\partial TC}{\partial q_{11}} = 0$, $\frac{\partial TC}{\partial q_1} = 0$ and $\frac{\partial TC}{\partial m_1} = 0$ will lead to the near-optimal solution for the variables q_{11}, q_1 and m_1, if the integer requirement of the variables is relaxed. The optimal ordering quantity, shipment size and number of shipments of every other subsystems can be determined once q_{11}, q_1 and m_1 are obtained.

For Case I: $T \geq t$, the optimum values of the variables ${q_{11}}^*$, q_1^* and m_1^* are obtained by simultaneously solving the following equations by using the software package MATLAB 7.0:

$$\frac{\partial TC_I^m}{\partial q_{11}} = \frac{S}{2} - \frac{Q}{{q_{11}}^2} = 0, \tag{4.32}$$

$$\frac{\partial TC_I^m}{\partial q_1} = R + Jm_1 - \frac{H}{q_1^2} - \frac{G}{m_1 q_1^2} = 0, \tag{4.33}$$

and $$\frac{\partial TC_I^m}{\partial m_1} = Jq_1 - \frac{G}{q_1 m_1^2} = 0, \tag{4.34}$$

which leads to $$q_{11}^* = \sqrt{\frac{2\sum_{i=1}^{w}\sum_{j=1}^{n_i}\frac{D_{ij}}{\delta_i\alpha_{ij}}\left(\tau_{Dij}+\tau_{ij}\right)}{\sum_{i=1}^{w}\sum_{j=1}^{n_i}\delta_i\alpha_{ij}\left(h_{ij}-h_i\right)}}. \tag{4.35}$$

$$q_1^* = \left(\frac{\sum_{i=1}^{w}\frac{D_i}{\beta_i}(\tau_{Mi}+\tau_{Di})}{h_M\left[D\left(\frac{1}{P}-\frac{1}{D_1}\right)-\frac{1}{2D_1}\sum_{i=1}^{w}\frac{D_i}{\lambda_i}+\frac{1}{PD_1}\sum_{i=1}^{w}D_i\left(P-D_1+\sum_{l=1}^{i}\frac{D_l}{\lambda_l}\right)\right]+\sum_{i=1}^{w}\frac{\beta_i h_i}{2}}\right)^{1/2}, \tag{4.36}$$

and $$m_1^* = \sqrt{\frac{GR}{JH}}. \tag{4.37}$$

Since it is impossible to have number of shipment in decimal value, the value of m_1^* has to be rounded using the inequalities

$$TC(m_1^*-1) - TC(m_1^*) \geq 0, \tag{4.38}$$

and $$TC(m_1^*+1) - TC(m_1^*) \geq 0. \tag{4.39}$$

Using the method of induction, the values of $TC(m_1^*-1)$, $TC(m_1^*)$ and $TC(m_1^*+1)$ are

$$TC_I^m(m_1{}^*-1) = \frac{G}{(m_1{}^*-1)q_1} + \frac{Q}{q_{11}} + \frac{H}{q_1} + \frac{S}{2}q_{11} + Rq_1 + J(m_1{}^*-1)q_1 - cDIpt, \tag{4.40}$$

$$TC_I^m(m_1{}^*) = \frac{G}{m_1{}^*q_1} + \frac{Q}{q_{11}} + \frac{H}{q_1} + \frac{S}{2}q_{11} + Rq_1 + Jm_1{}^*q_1 - cDIpt, \tag{4.41}$$

and $$TC_I^m(m_1{}^*+1) = \frac{G}{(m_1{}^*+1)q_1} + \frac{Q}{q_{11}} + \frac{H}{q_1} + \frac{S}{2}q_{11} + Rq_1 + J(m_1{}^*+1)q_1 - cDIpt. \tag{4.42}$$

Substituting Eqs (4.40), (4.41) and (4.42) into Eqs (4.38) and (4.39) and simplifying the resulting expressions, the following inequality results,

$$m_1^*(m_1^*+1) \geq \frac{1}{q_1^2}\frac{G}{J} \geq m_1^*(m_1^*-1). \tag{4.43}$$

For Case II: $T < t$, the optimum values of the variables ${q_{11}}^*$, q_1^* and m_1^* are similarly obtained.

$$\frac{\partial TC_{II}^m}{\partial q_{11}} = \frac{S}{2} - \frac{Q}{{q_{11}}^2} = 0, \tag{4.32}$$

$$\frac{\partial TC_{II}^m}{\partial q_1} = R + J'm_1 - \frac{H}{q_1^2} - \frac{G'}{m_1 q_1^2} = 0, \text{ and} \tag{4.44}$$

$$\frac{\partial TC_{II}^m}{\partial m_1} = J'q_1 - \frac{G'}{q_1 m_1^2} = 0, \tag{4.45}$$

which leads to

$$q_{11}^* = \sqrt{\frac{2\sum_{i=1}^{w}\sum_{j=1}^{n_i}\frac{D_{ij}}{\delta_i\alpha_{ij}}\left(\tau_{Dij}+\tau_{ij}\right)}{\sum_{i=1}^{w}\sum_{j=1}^{n_i}\delta_i\alpha_{ij}\left(h_{ij}-h_i\right)}}, \tag{4.35}$$

$$q_1^* = \left(\frac{\sum_{i=1}^{w}\frac{D_i}{\beta_i}(\tau_{Mi}+\tau_{Di})}{h_M\left[D\left(\frac{1}{P}-\frac{1}{D_1}\right)-\frac{1}{2D_1}\sum_{i=1}^{w}\frac{D_i}{\lambda_i}+\frac{1}{PD_1}\sum_{i=1}^{w}D_i\left(P-D_1+\sum_{l=1}^{i}\frac{D_l}{\lambda_l}\right)\right]+\sum_{i=1}^{w}\frac{\beta_i h_i}{2}}\right)^{1/2}, \tag{4.36}$$

and

$$m_1^* = \sqrt{\frac{G'R}{J'H}}. \tag{4.46}$$

Again m_1^* must be rounded using as before by

$$m_1^*(m_1^*+1) \geq \frac{1}{q_1^2}\frac{G'}{J'} \geq m_1^*(m_1^*-1). \tag{4.47}$$

Based on the optimal solutions obtained from above equations, the cycle time, T, needs to be identified to check the feasibility conditions for two different cases of the retailers' trade credit situation. Moreover, the constraints of the cost equations must be maintained in the solution. That is why, an algorithm is necessary to solve the model which is:

Algorithm 2: Adjusting for integer solution and feasibility condition of credit period and constraints for multi-channel model:

Step 1: Initialize A_{ij}, A_i, A_M, h_{ij}, h_i, h_M, τ_{ij}, τ_{Dij}, τ_{Di}, τ_{Mi}, $\emptyset_{ij}$, $\emptyset_i$, D_{ij}, c, k, I_e, I_p, t and P for $\forall i, j$.

Step 2: Compute q_{11}^*, q_1^* and m_1^* using Eqs (4.35), (4.36), and (4.37), respectively.

Set $q_1 \leftarrow q_1^*$ and $m_1 \leftarrow m_1^*$. Compute $T = \frac{m_1 q_1}{D_1}$.

For $T < t$, go to step 7.

Step 3: Compute $q_1^* = \frac{TD_1}{\lceil m_1^* \rceil}$. Set $q_1 \leftarrow q_1^*$.

If, $\lfloor m_1^* \rfloor = 0$, set $m_1 \leftarrow 1$

Else, if $\lceil m_1^* \rceil (\lceil m_1^* \rceil + 1) \geq \frac{1}{q_1^2} \frac{G}{J} \geq \lceil m_1^* \rceil (\lceil m_1^* \rceil - 1)$, set $m_1 \leftarrow \lceil m_1^* \rceil$

Else, set $m_1 \leftarrow \lfloor m_1^* \rfloor$. Compute $q_1^* = \frac{TD_1}{\lfloor m_1^* \rfloor}$. Set $q_1 \leftarrow q_1^*$.

Step 4: Compute q_{ij}^* for all $\forall i, j$ (except q_{11}^*) using Eq.(4.19) and (4.28).

Compute $m_{ij}^* = \frac{TD_{ij}}{q_{ij}^*}$ for $\forall i, j$.

If $\lfloor m_{ij}^* \rfloor = 0$, $m_{ij} \leftarrow 1$.

Else, set $m_{ij} \leftarrow \text{round}(m_{ij}^*)$. Set $q_{ij} \leftarrow \frac{TD_{ij}}{m_{ij}}$ for $\forall i, j$.

Compute m_i^* for all $\forall i$ (except m_1^*) using Eq.(4.22).

If $\lfloor m_i^* \rfloor = 0, m_i \leftarrow \lceil m_i^* \rceil$.

Else, set $m_i \leftarrow \text{round}(m_i^*)$. Set $q_i \leftarrow \frac{TD_i}{m_i}$ for $i = 2, \cdots, w$.

Step 5: If not $\frac{T}{m_i} \geq \frac{T}{P} \sum_{i=1}^{w} \frac{D_i}{m_i}$, re-structure the hierarchy of the DCs in the order by

$$m_1' \geq m_2' \geq \cdots \geq m_y{}' \geq \cdots\cdots \geq m_w{}'$$

Find $m_y{}'$ for which $\frac{P}{m_y{}'} \geq \sum_{i=1}^{w} \frac{D_i{}'}{m_i}$ is invalid.

Set $m_1', m_2', \cdots, m_y' = m$.

Find the largest integer value of m for which $\frac{P}{m} \geq \sum_{i=1}^{y} \frac{D_i{}'}{m} + \sum_{i=y+1}^{w} \frac{D_i{}'}{m_i{}'}$

Compute $q_i' = \frac{TD_i{}'}{m}$ for $i = 1,2, \dots, y$.

Step 6: Find $TC_I{}^m$ using Eq. (4.15).

Step 7: Compute q_{11}^*, q_1^* and m_1^* using Eqs (4.35), (4.36) and (4.46), respectively.

Set $q_1 \leftarrow q_1^*$ and $m_1 \leftarrow m_1^*$. Compute $T = \frac{m_1 q_1}{D_1}$.

For $T \geq t$, go to step 10.

Step 8: Compute $q_1^* = \frac{TD_1}{\lceil m_1^* \rceil}$. Set $q_1 \leftarrow q_1^*$.

If, $\lfloor m_1^* \rfloor = 0$, set $m_1 \leftarrow 1$.

Else, if $\lceil m_1^* \rceil(\lceil m_1^* \rceil + 1) \geq \frac{1}{q_1^2}\frac{G'}{J'} \geq \lceil m_1^* \rceil(\lceil m_1^* \rceil - 1)$, set $m_1 \leftarrow \lceil m_1^* \rceil$

Else, set $m_1 \leftarrow \lfloor m_1^* \rfloor$. Compute $q_1^* = \frac{TD_1}{\lfloor m_1^* \rfloor}$. Set $q_1 \leftarrow q_1^*$.

Repeat steps 4 and 5.

Step 9: Find ${TC_{II}}^m$ using Eq. (4.16). Go to step 11.

Step 10: Set $T \leftarrow t$, Repeat Step 3, 4 and 5.

Compute ${TC_t}^m$ by using Eq. (4.15).

Step 11: Set $TC^m \leftarrow min({TC_I}^m,\ {TC_{II}}^m, {TC_t}^m)$. Take $q_{ij}^*, m_{ij}^*, q_i^*$ and m_i^* with least annual cost value.

Step 12: Stop ■

4.5 Computational Result

As an illustration of the multi-channel three-echelon supply chain model, three numerical examples are presented for a single product by considering together the given values from the numerical problems in Kreng and Chen (2007) and Sarker *et al.* (2000). Since Kreng and Chen (2007) have considered single-channel three-echelon model, for the parameter values of multi-channel model, a reasonable estimation can be done. Other required information for permissible delay period, interest payable rate and interest earning rate are taken from Sarker *et al*.(2000). The reason for estimating some of the values and combining numerical examples of more than one work is that no previous study has been done in this area to incorporate trade credit consideration in three-echelon supply chain model.

The structure of supply chain models considered in these examples is shown in Figure 2.2, where the manufacturer supplies the product to three distribution centers. Two distribution centers distribute the product to a total of four retailers, each distribution center to two retailers, and the other distribution center distributes the product to three others retailers. So, there are altogether seven retailers.

- **Example 4.1** Multi-channel, multi-echelon model without lot-splitting

In the example, c = \$270/unit, k = 0.05, I_p= 0.20, I_e= 0.13 and t = 30 days. Other input parameters are shown in the following Figure (see Subsection 4.2.1 for units of each parameter):

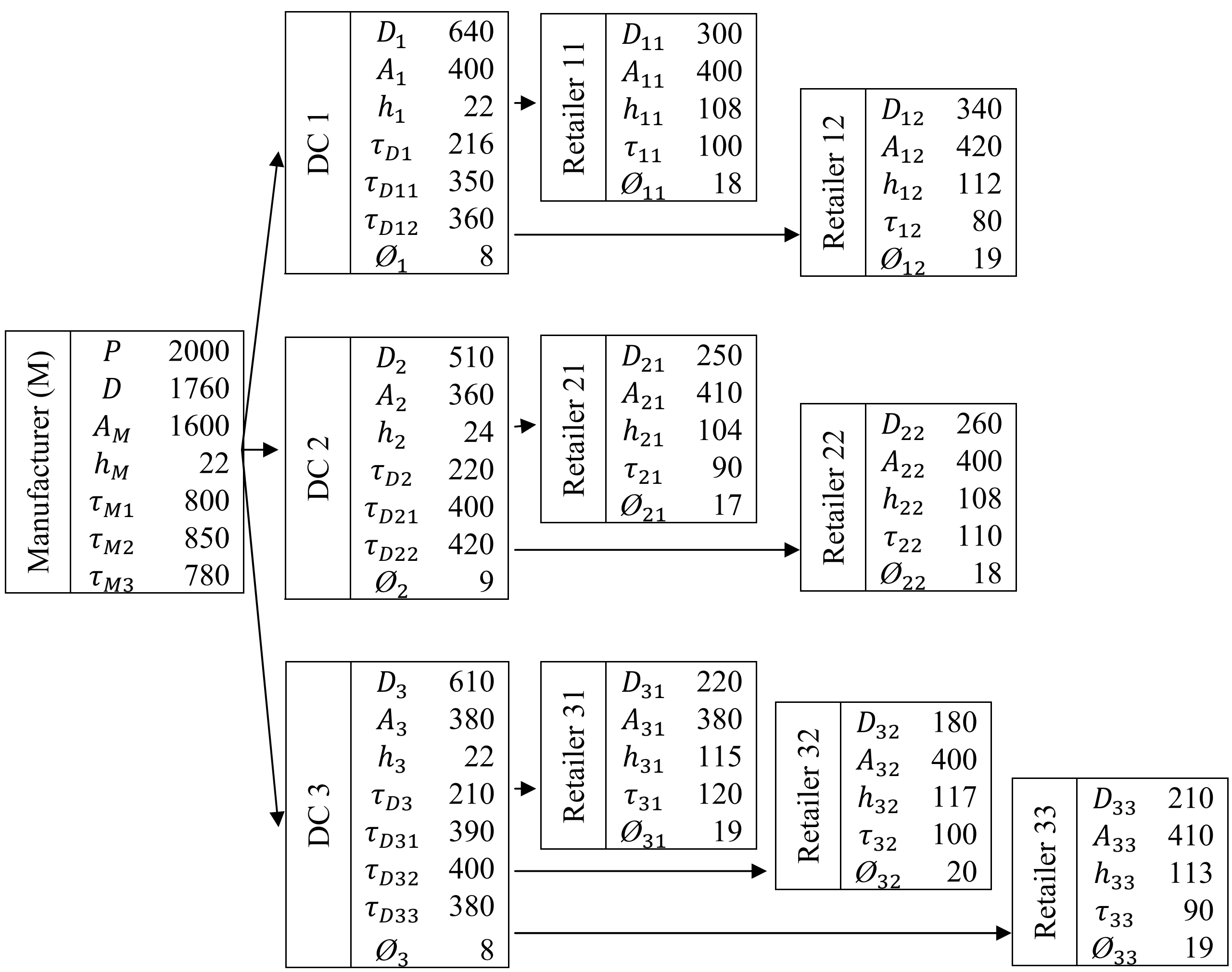

Figure 4.7: Input parameters of Example 4.1

Table 4.1 shows the solution of Example 4.1. It is observed from Table 4.1 that the optimum model does not require lot-splitting for any of the shipments ($m_{ij} = 1, m_i = 1$ for $\forall i, j$), in other words, the outstanding orders are shipped all at once in each cycle from the manufacturer to the distribution centers and from the distribution centers to the corresponding retailers. This might be the case where the shipment cost is relatively high compared to holding cost. However, later on, a sensitivity analysis will be done to clarify this fact. In this example, the solution is feasible when $T \geq t$, having $T \cong 101$ days and t = 30 days.

Table 4.1: The solution for Example 4.1

<table>
<tr><td rowspan="2">R11</td><td>q_{11}</td><td>83</td><td rowspan="2">R12</td><td>q_{12}</td><td>94</td><td rowspan="2">DC1</td><td>q_1</td><td>178</td><td rowspan="2" colspan="2">T</td><td rowspan="2">101 days</td></tr>
<tr><td>m_{11}</td><td>1</td><td>m_{12}</td><td>1</td><td>m_1</td><td>1</td></tr>
<tr><td rowspan="2">R21</td><td>q_{21}</td><td>69</td><td rowspan="2">R22</td><td>q_{22}</td><td>72</td><td rowspan="2">DC2</td><td>q_2</td><td>142</td><td rowspan="2" colspan="2">TC</td><td rowspan="2">$84,182</td></tr>
<tr><td>m_{21}</td><td>1</td><td>m_{22}</td><td>1</td><td>m_2</td><td>1</td></tr>
<tr><td rowspan="2">R31</td><td>q_{31}</td><td>61</td><td rowspan="2">R32</td><td>q_{32}</td><td>50</td><td rowspan="2">R33</td><td>q_{33}</td><td>58</td><td rowspan="2">DC3</td><td>q_3</td><td>169</td></tr>
<tr><td>m_{31}</td><td>1</td><td>m_{32}</td><td>1</td><td>m_{33}</td><td>1</td><td>m_3</td><td>1</td></tr>
</table>

- **Example 4.2** Multi-echelon, multi-channel model with lot-splitting

In this example, only the shipment costs for all the subsystems have been modified keeping all other parameters of Example 4.1 unchanged. Now, the changed shipment costs and corresponding optimal solution are shown in Tables 4.2 and 4.3. It is observed that the optimal solution given by the algorithm suggests lot-splitting ($m_{ij} > 1$) while delivering shipments from the distribution centers to the retailers. Due to decrease of shipment costs by 50% at all the subsystems compared to Example 4.1, the optimal solution has increased number of shipments from the distribution centers to the retailers and as a result the total cost also has been reduced. However, optimal number of shipments from the manufacturer to the distribution center has remained unchanged due to constraint subjected to the total cost function. In this example, the solution is feasible when $T \geq t$, having $T \cong 101$ days and t = 30 days just as Example 4.1.

Table 4.2: Modified cost parameters (in $/shipment) of Examples 4.2 from 4.1

R11	τ_{11}	50	DC1	τ_{D1}	108
R12	τ_{12}	40		τ_{D11}	175
R21	τ_{21}	45		τ_{D12}	180
R22	τ_{22}	55	DC2	τ_{D2}	110
R31	τ_{31}	60		τ_{D21}	200
R32	τ_{32}	50		τ_{D22}	210
R33	τ_{33}	45	DC3	τ_{D3}	105
M	τ_{M1}	400		τ_{D31}	195
	τ_{M2}	425		τ_{D32}	200
	τ_{M3}	390		τ_{D33}	190

Table 4.3: The solution for Example 4.2

R11	q_{11}	42	R12	q_{12}	47	DC1	q_1	178	T		101 days	
	m_{11}	2		m_{12}	2		m_1	1				
R21	q_{21}	35	R22	q_{22}	36	DC2	q_2	142	TC		$67,974	
	m_{21}	2		m_{22}	2		m_2	1				
R31	q_{31}	31	R32	q_{32}	25	R33	q_{33}	29	DC3	q_3	170	
	m_{31}	2		m_{32}	2		m_{33}	2		m_3	1	

- **Example 4.3** Multi-channel, multi-echelon model with longer permissible delay period

Suppose the permissible delay period, t is now 120 days instead of 30 days of the supply chain model of Example 4.1. It is observed that algorithm suggests lot-splitting only for a few deliveries from the distribution centers to the retailers. In this example, the solution is feasible for $T < t$, having $T \cong 114$ days and $t = 120$ days, which reduces the total cost of the supply chain.The optimum solution by the algorithm is shown in Table 4.4.

Table 4.4: The solution for Example 4.3

R11	q_{11}	93	R12	q_{12}	52	DC1	q_1	198	T		114 days
	m_{11}	1		m_{12}	2		m_1	1			
R21	q_{21}	39	R22	q_{22}	80	DC2	q_2	157	TC		\$65,897
	m_{21}	2		m_{22}	1		m_2	1			
R31	q_{31}	68	R32	q_{32}	56	R33	q_{33}	32	DC3	q_3	188
	m_{31}	1		m_{32}	1		m_{33}	2		m_3	1

- **Special Case:** Single-channel multi-echelon level

When each echelon level has got only one subsystem, the multi-channel becomes single-channel multi-echelon supply chain system. So, there is only one distribution center (i = 1 only) and one retailer (j = 1 only) and hence $D_{11} = D_1 = D$. Using these and substituting the subscript for the only retailer to R and for the only distribution center to D, in Eqs (4.32)-(4.39),

$$G = D\left[A_M + A_D + A_R + \frac{D}{2}\{ct^2[I_p - (1+k)I_e]\}\right] = A, \text{ and}$$

$$H = D(\tau_{M1} + \tau_{1D}) = F, \quad \text{where } \tau_{1D} = \tau_{D1},$$

$$J = \frac{1}{D}\left(\frac{\emptyset_R D}{2} + \frac{cDI_p}{2}\right) + \frac{Dh_M}{2D}\left(1 - \frac{D}{P}\right) + \frac{\emptyset_D}{2} = \frac{1}{2}\left[h_M\left(1 - \frac{D}{P}\right) + \left(\emptyset_R + \emptyset_D + cI_p\right)\right] = B,$$

$$R = h_M\left[D\left(\frac{1}{P} - \frac{1}{D}\right) - \frac{D}{2D} + \frac{1}{PD}(P - D + D)\right] + \frac{h_D}{2} = \frac{1}{2}\left(h_D - h_M + \frac{2D}{P}h_M\right) = V, \text{and}$$

$$Q = D(\tau_{2D} + \tau_R) = E, \quad \text{where } \tau_{2D} = \tau_{D11}.$$

Also, $S = h_R - h_D = 2L.$

$$G' = D[A_M + A_D + A_R] = A', \text{ and}$$

$$J' = \frac{1}{D}\left(\frac{\emptyset_R D}{2} + \frac{(1+k)cDI_e}{2}\right) + \frac{Dh_M}{2D}\left(1 - \frac{D}{P}\right) + \frac{\emptyset_D}{2}$$

$$= \frac{1}{2}\left[h_M\left(1 - \frac{D}{P}\right) + \{\emptyset_R + \emptyset_D + (1+k)cI_e\}\right] = B'.$$

Now, replacing $G,\ H,\ J,\ R,\ Q,\ S,\ G'$ and J' by $A,\ F,\ B,\ V,\ E,\ L,\ A'$ and B', respectively, Eqs (4.30) and (4.31) become,

$$TC_I(q, q_D, m_D) = \frac{A}{m_D q_D} + \frac{F}{q_D} + Bm_D q_D + Vm_D + \frac{E}{q} + qL - cDI_p t,$$

and $$TC_{II}(q, q_D, m_D) = \frac{A'}{m_D q_D} + \frac{F}{q_D} + B'm_D q_D + Vm_D + \frac{E}{q} + qL - (1+k)cDI_e t.$$

These are the cost equations for single-channel multi-echelon model derived in Chapter 3. Now, the constraints become, $m_D,\ m_R \geq 1$ and $P \geq D$, which are the basic assumption of the single-channel model of Chapter 3.

4.6 Sensitivity Analysis

Sensitivity analysis has been done to test the robustness of the proposed solution approach in facing changes of given parameter, holding cost, transportation cost and the permissible delay period on the decision variables and total supply chain cost.

To analyze the sensitivity of the decision variables with respect to change of transportation cost, six different problems (problems 1–6) are presented. The transportation costs at subsystems are decreased by 1.5, 2, 3, 4 and 5 times in the subsequent problems starting from problem 1 and the corresponding values are given in Table C.1 of Appendix C. The other input parameters are taken from Example 4.1. The optimum solutions of the problems are summarized in Tables C.2–C.4 of Appendix C. The sensitivity of different decision variables with the decreasing transportation costs are shown in Figures 4.8-4.13. It is observed from Figure 4.9 and 4.11 that the optimum number of delivery per order increases with the decreasing shipment cost, which as a result reduces the total supply chain cost and shown in Figure 4.12.

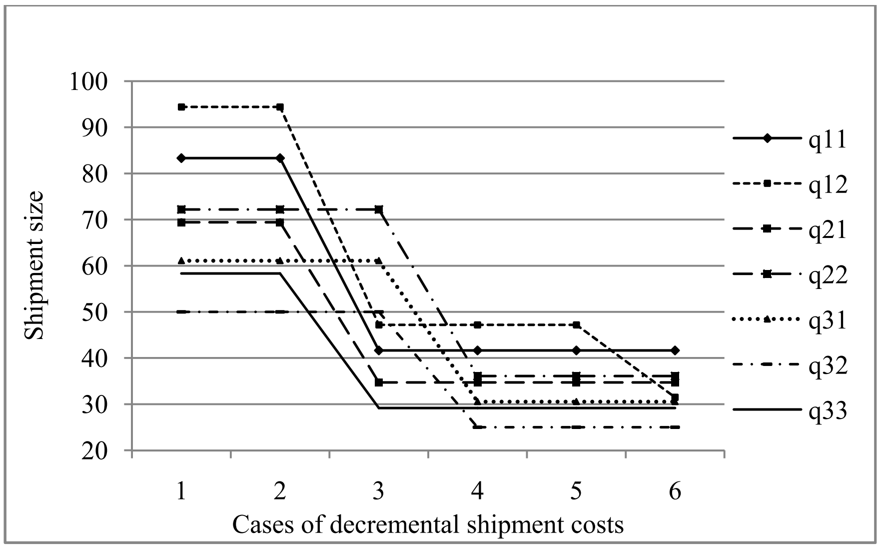

Figure 4.8 Sensitivity of retailers' shipment size to shipment cost

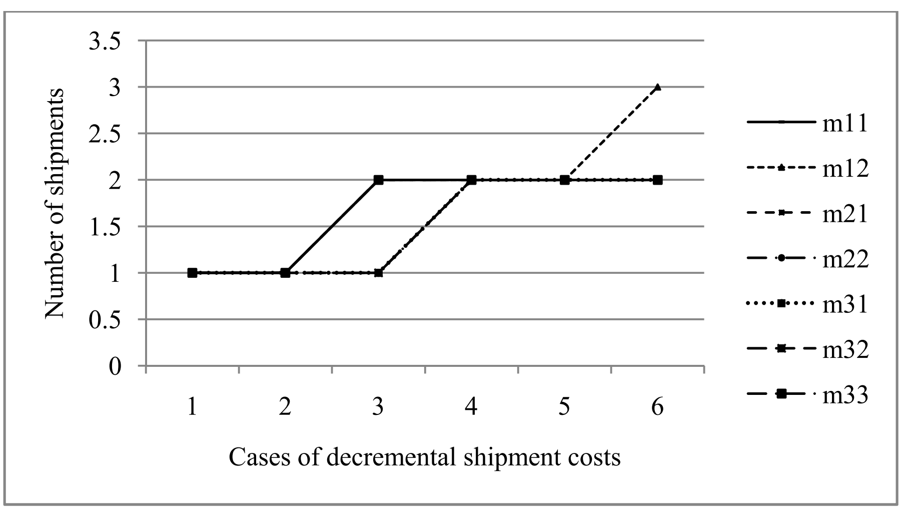

Figure 4.9 Sensitivity of retailers' number of shipments to shipment cost

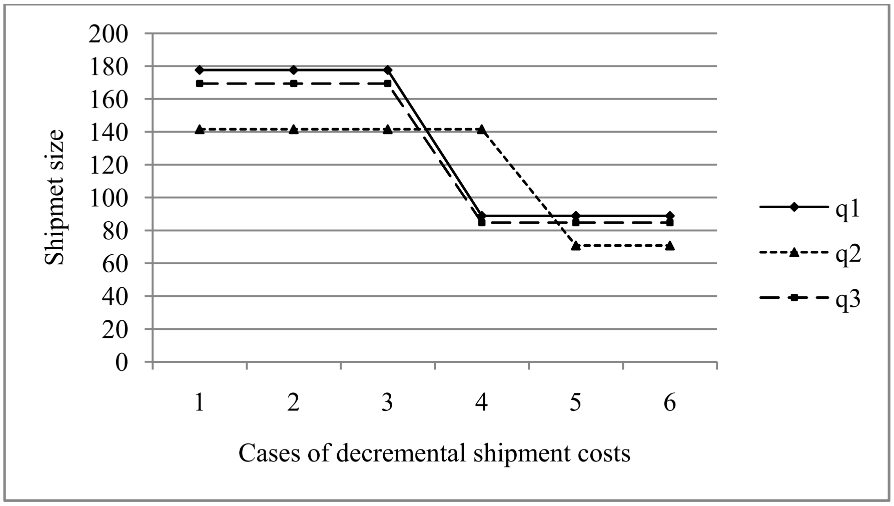

Figure 4.10 Sensitivity of DCs' shipment size to shipment cost

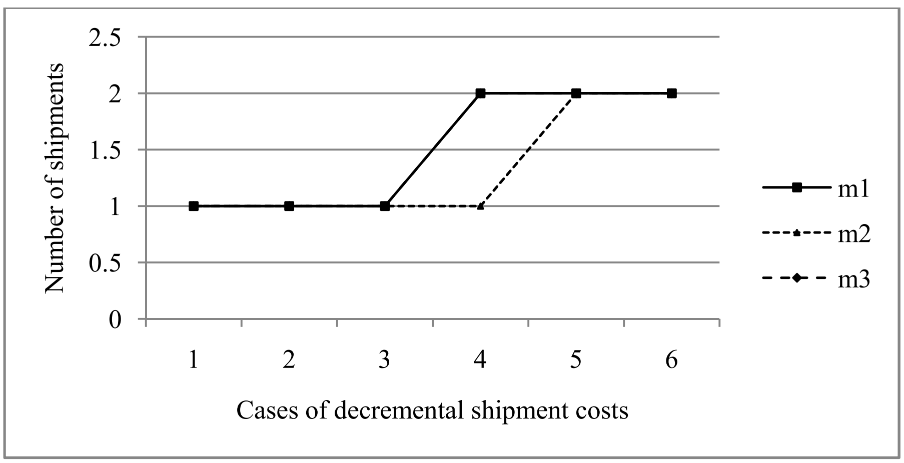

Figure 4.11 Sensitivity of DCs' number of shipments to shipment cost

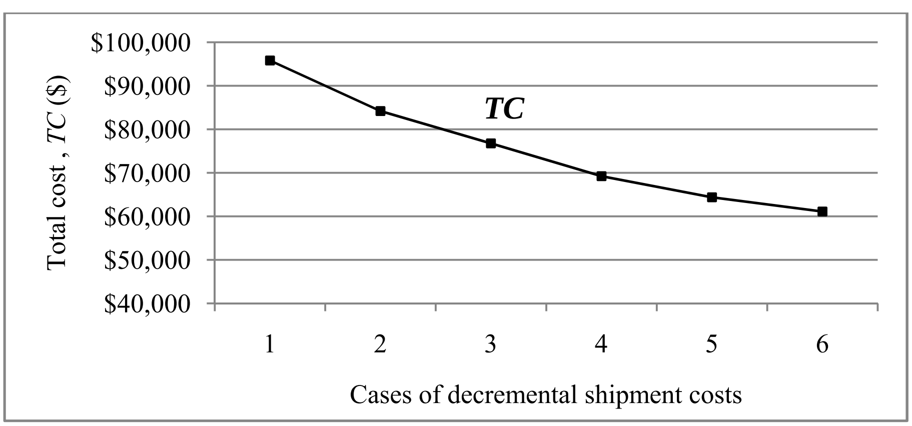

Figure 4.12 Sensitivity of total supply chain cost to shipment cost

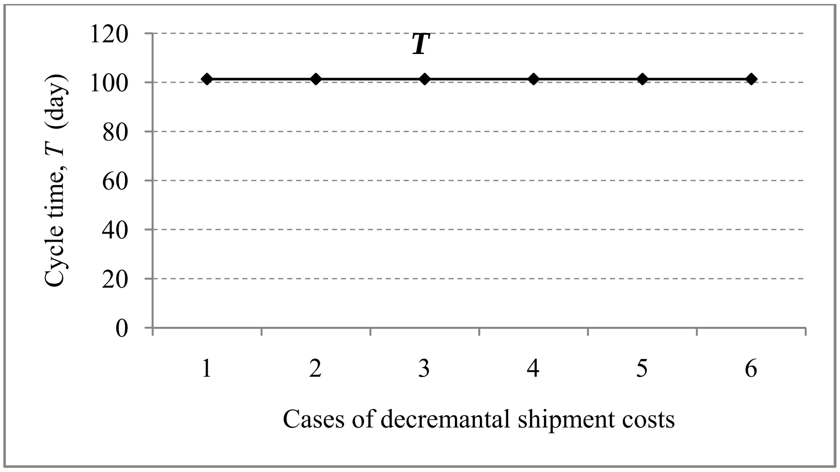

Figure 4.13 Sensitivity of cycle time to shipment cost

Figure 4.13 reveals that the shipment cost has no impact on the optimum cycle time. Hence shipment size varies only when the optimum number of shipment varies.

Six other problems (problems 7–12) have been presented to analyze the sensitivity of the decision variables to the holding cost. The holding costs at the subsystems are increased by 1.5, 2, 4, 5 and 6 times in the subsequent problems starting from Problem 7 and the values are presented in Table C.5 of Appendix C. The other parameters are taken from Problem 7 and the corresponding parameter values are shown in Figure C.1. The optimum solutions of the problems are given in Table C.6–C.8. The graphical representation of the sensitivity of decision variables is provided in Figures 4.11–4.17.

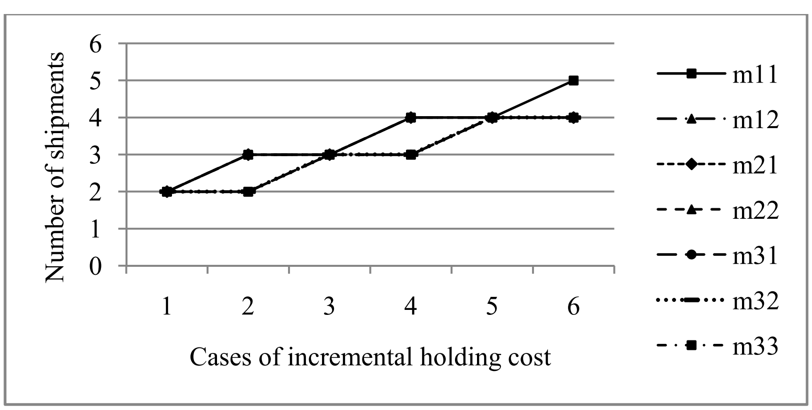

Figure 4.14 Sensitivity of number of shipments to holding costs

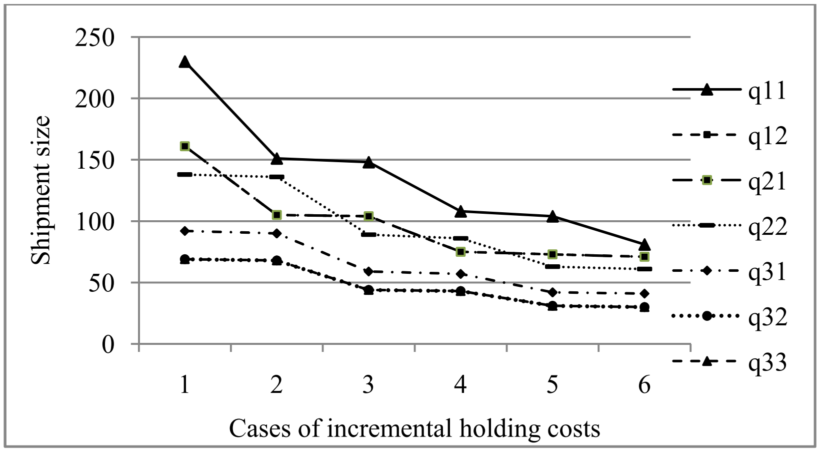

Figure 4.15 Sensitivity of shipment size to holding costs

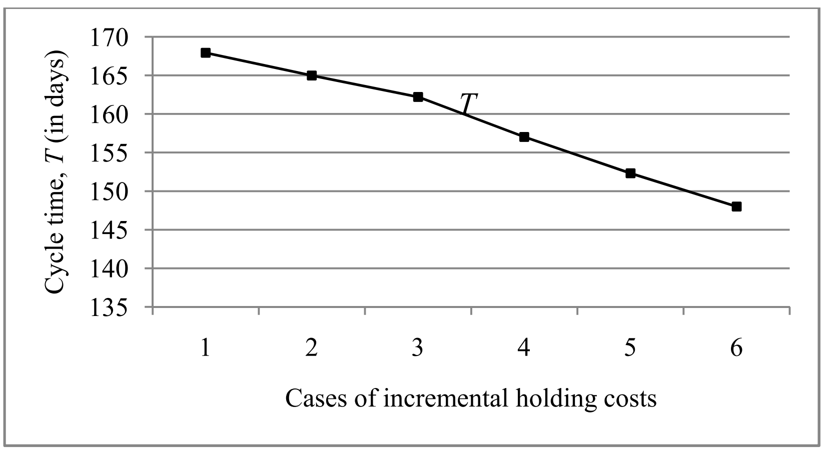

Figure 4.16 Sensitivity of cycle time to holding cost

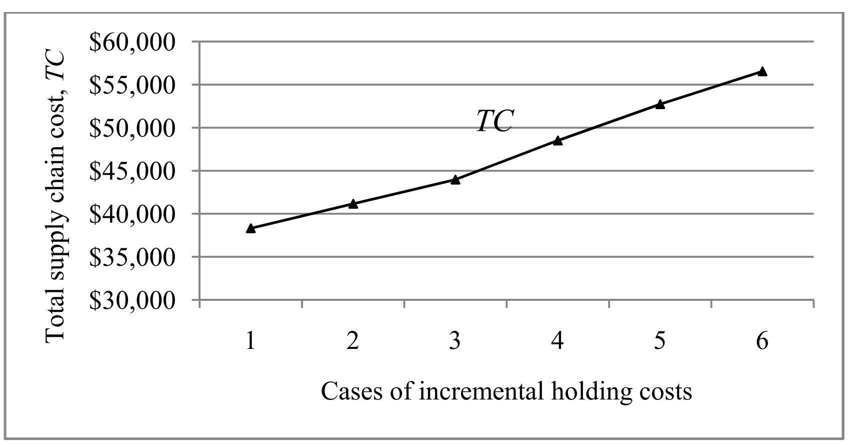

Figure 4.17 Sensitivity of total supply chain cost to holding cost

As the holding cost increases at all the echelon levels, the shipment size decreases and the number of shipments increases. This tendency is logical, since holding larger inventory in the upstream level is relatively less costly compared to holding larger inventory in the downstream level. Since there is no change in the shipment cost, increasing holding cost at all level also increases the total supply chain cost. The optimum cycle time decreases with increasing holding cost to reduce inventory holding time, which reduces the total holding costs.

Six other problems have been compared to analyze the sensitivity of the decision variables to the length of permissible delay period. The problems have different length of permissible delay period with equal value of all other parameters. Four new problems (13–16) have been solved along with Examples 4.1 and 4.3, having permissible delay period 60, 90, 150, 180, 30 and 120 days, respectively. The solutions are summarized in Tables C.9–C.11 in Appendix C.

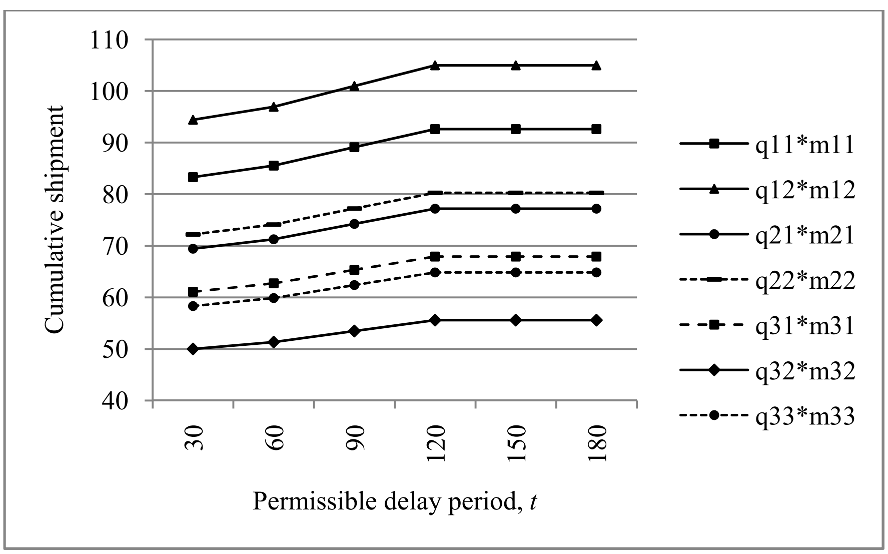

Figure 4.18 Sensitivity of ordering size of the retailers to permissible delay period

It is observed that, for the first three problems, cumulative shipment to the retailers from the distribution centers (the ordering quantity) increases to the subsequent problems as permissible delay period increases. This is a logical tendency since more ordering quantity with longer permissible delay period allows the retailer to get more cost savings from accumulated revenue and thus lowers down the overall supply chain cost for $T \geq t$. The mathematical expression of Equation (3.30) also verifies this fact. As t increases, G also increases and so the value of $m_{11}q_{11}$ should also be increased in order to minimize the supply chain cost. It should be noted that, since cycle time is equal for all subsystems, increase of $m_{11}q_{11}$ increasing all other $m_{ij}q_{ij}$. However, for $T < t$, t does not have any impact on $m_{11}q_{11}$ which is evident in Equation (3.31). Since the last three problems fall into the condition $T < t$, the ordering quantity of the retailers remains unchanged for all of the three problems. From the cost equations (3.30) and (3.31), it is obvious that with increasing permissible delay period the total supply chain reduces which is apparent in Figure 4.19.

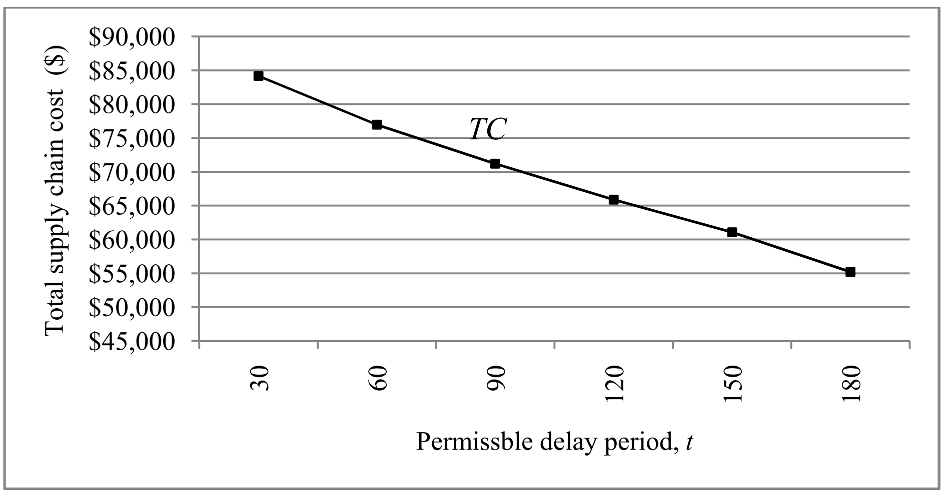

Figure 4.19 Sensitivity of total supply chain cost to permissible delay period

CHAPTER 5

OPERATIONAL SCHEDULE

In this chapter, the developed inventory model for multi-channel three-echelon supply chain system is demonstrated with numerical data to show the details of the developed inventory system based on the applied algorithm. This is sufficient to show the operational schedule of the multi-channel model only, since this is the generic model for the three-echelon system with known demands, whereas single-channel model is only a special case of multi-channel model where the number of channel at each echelon level is one. Here, the procedure followed to obtain the optimal solution is explained for Example 4.2 from Chapter 4.

5.1 Operational Schedule for Multi-channel, Multi-echelon Model

The operational schedule for this system is evaluated by determining the optimum number of shipments and shipment size to minimize the total supply chain cost. The schedule for each shipment is also evaluated. The operational schedule is obtained on the parameter values given in Table 4.3 and 4.4 in Chapter 4. Following algorithm 2, at first q_{11}^*, q_1^* and m_1^* are evaluated using Eqs (4.40), (4.41) and (4.42), respectively. The initial values were 45.02 units, 112.4 units and 1.58, respectively, that give optimum cycle time, $T \cong 101$ days and satisfy condition $T \geq t$, where t = 30 days. The optimum integer value of m_1^* is obtained as 2, based on the initial value of 1.58 and q_1^* is adjusted to 89 units. The other values of $q_{ij}^*, m_{ij}^*, q_i^*$ and m_i^* are initially evaluated by using equations (4.19) – (4.28). The number of shipments are made integers and the shipment sizes are readjusted. However, it is observed that the initial values of m_1^* and m_3^* violate the constraint equation and hence these values and consequently the values of q_1^* and q_3^* are readjusted. Based on these values the total cost of the supply chain obtained is $67,974.

Following the algorithm, q_{11}^{*}, q_{1}^{*} and m_{1}^{*} are again evaluated using Eqs (4.40), (4.41) and (4.43), respectively. The initial values were 45.02 units, 112.4 units and 1.76, respectively, which give optimum cycle time, $T \cong 113$ days. This violates the condition $T < t$. Hence, the optimum schedule of the model is obtained for $T \geq t$. The inventory model is shown in Figure 5.1.

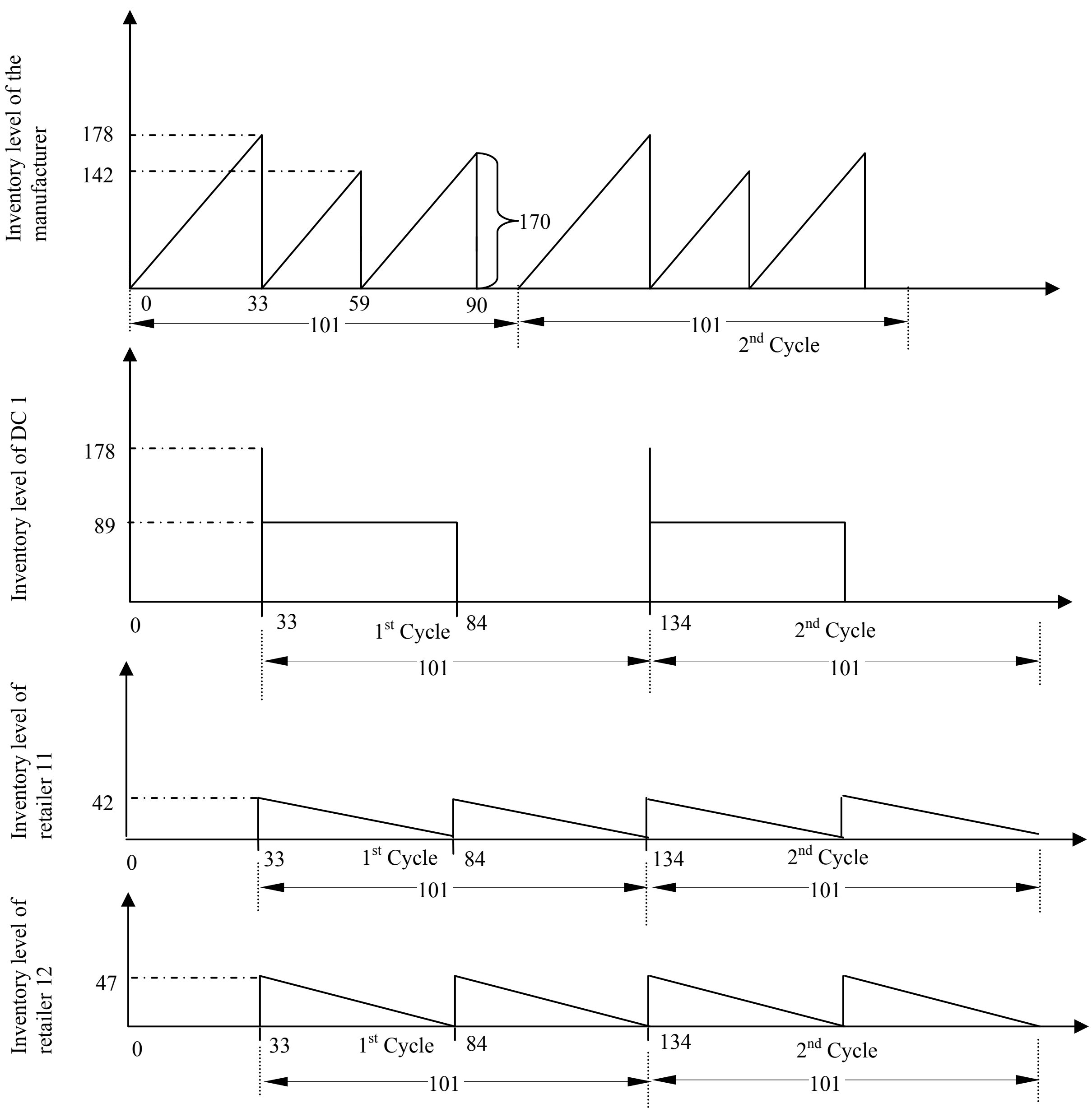

Figure 5.1 Operational Schedule of Multi-channel, Multi-echelon Supply Chain Model

Figure 5.1 shows the inventory model of the manufacturer, distribution center 1 and corresponding two retailers. It is shown that the optimum cycle time for the subsystems is approximately 101 days. Starting production from time 0, the manufacturer ships the first delivery to distribution center 1 as soon as it finishes producing the shipment size, 178 units of product, on 33rd day. The manufacturer makes it second delivery on 59th day, which is the first delivery to distribution center 2, as soon as it finishes producing 142 units of product. The third shipment from the manufacturer is made to the distribution center 3 on 90th day, when it just finishes producing the shipment size, 169 units of product. The next production cycle of the manufacturer starts from day 101.

Distribution center 1 gets its order in a single delivery of 178 units of product from the manufacturer on 33rd day. It is scheduled to make two deliveries on 33rd day, the first delivery to retailer 11 having a size of 42 units and the second delivery to retailer 12 having a size of 47 units. The distribution center carries the remaining 89 units of inventory for the next 51 days before it delivers the third and fourth shipment on 84th day. The third and fourth shipment is made to its retailers having size of 42 units and 47 units as before. The next cycle of the distribution center starts from day 134.

Retailer 11 gets its first shipment of size 42 from its distribution center on 33rd day as mentioned earlier. It gets the next shipment on 84th days, just when its inventory of 42 units is consumed in 51 days. When its inventory again is depleted on 134th day, it starts its second ordering cycle by placing an order of 89 units which is delivered in two shipments as of the first cycle. The delivery and ordering schedule is similar for retailer 12, except the ordering quantity is 94 with shipment size is 47 units.

The operational schedules of the rest of the subsystems are summarized in Table 5.1.

Table 5.1 Operational schedule for Example 4.2

From Subsystem	Delivery	To Subsystem	Delivery Time (day)	Shipment Size	Cumulative Delivery	Leftover Inventory
Manufacturer	1	DC 1	33	178	178	0
	2	DC 2	59	142	319	0
	3	DC 3	90	170	489	0
DC 1	1	Retailer 11	33	42	42	136
	2	Retailer 12	33	47	89	89
	3	Retailer 11	84	42	131	47
	4	Retailer 12	84	47	178	0
DC 2	1	Retailer 21	59	35	35	107
	2	Retailer 22	59	36	71	71
	3	Retailer 21	109	35	106	36
	4	Retailer 22	109	36	142	0
DC 3	1	Retailer 31	90	31	31	139
	2	Retailer 32	90	25	56	114
	3	Retailer 33	90	29	85	85
	4	Retailer 31	140	31	115	54
	5	Retailer 32	140	25	140	29
	6	Retailer 33	140	29	170	0

5.2 Practical Implication

The model presented here is more general and allows more flexibility of usage. However, for practical application, sometimes determining cost parameters for every single product can be costly. So, to simplify the model, some of the cost parameters which have relatively less impact on the overall supply chain cost, for example the flexibility loss cost, can be ignored. Moreover, for an enterprise having numerous product diversifications, applying this model for every product may not be practical. In such a case, such enterprises can be advised to perform ABC analysis to prioritize their products. This model can be applied on more expensive priority products (A) individually and on less expensive ones (B and C) as a product group, to get reduction of total supply chain cost in a more economic way.

CHAPTER 6

RESEARCH CONCLUSIONS

In this research, the inventory model of a collaborative three-echelon supply chain system has been developed. The research has been aimed at determining the optimum JIT delivery policy of a single product that minimizes the overall supply chain cost.

In this chapter, conclusion and significance of this research are discussed briefly, and finally, some suggestions are made for possible future research.

6.1 Conclusions

The study started with developing a single-channel three-echelon model having single subsystem in each echelon level. Then the model is extended to a more general multi-channel model having more than one subsystem in downstream echelon levels. In this study, the product belongs to a supply chain of three-echelon levels, a manufacturer, distribution centers and retailers, where the retailers enjoy permissible delay in payment to the distribution centers.

The developed inventory models allow multiple shipments per order, if optimum, based on the cost and other parameter values. Multiple small lot deliveries of an order reduces the holding cost of the downstream levels considerably compared to single large lot delivery per order. So, when the shipment cost is not high, compared to the holding cost, multiple shipment can reduce the supply chain cost.

An industry that consists of three-echelon delivery supply chain system, having trade credit benefit to the final buyer, can adapt the model to realize the savings of the overall supply chain cost.

6.2 Research Significance

This research presents a new perspective that recognizes an approach of obtaining an optimum just-in-time delivery policy of a three-echelon supply chain system with the objective of reducing the overall cost of the entire supply chain, where the retailer enjoys permissible delay period of payment. The model of single manufacturer, multiple distribution centers and multiple retailers, allows different shipment size and number of shipment per order for different channels of a particular echelon level; that is, different distribution centers might have different optimum shipment sizes and different number of shipments per order. Moreover, in the model there is no need to wait for the entire lot to be produced to feed the downstream level. Even though these assumptions enhance the computational burden of the model, they are logical and lead to further reduction of overall supply chain cost. However, the **lot-splitting** and **trade credit consideration** have not been addressed simultaneously in any of the three-echelon models in literature.

6.3 Possible Future Extensions

The inventory model developed here is limited to certain conditions which can be relaxed in future research. By relaxing some restrictions considered in this study, the problem will become even more complicated but it will be more realistic. Therefore, in order to enhance the supply chain system more, the following possible extensions are worthwhile to be examined:

(a) Different Cycle Time: The scope of the developed inventory model is limited only to equal cycle time for all the subsystems, and this reduces the flexibility of the model. Therefore, future research may be directed to relaxing this limitation by considering different cycle time, for example, multiple cycle time to each, which would provide more flexibility in selection of shipment size and number of shipments for all of the subsystems and thus might result in more reduction of supply chain cost.

(b) Random demand: In this research, the demand at the retailer is assumed to be constant; however, in reality, that might not be the case. Consequently, if a random demand rate is considered for the integrated inventory system instead of a constant one, the model will be closer to reality.

(c) Cross-transfer delivery: Since the demand is known in advance, this study considered only the upstream integrated delivery policy instead of cross-transfer delivery. When the demand is random, cross-transfer delivery should be assumed to meet the uncertainty of the demand, so that any retailer can get delivery from any of the distribution centers.

(d) Shortage: In this study, no shortage of inventory is allowed. However, when the demand is random, there might be a possibility of shortage. So in such a case, shortage should be considered.

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APPENDIX A

CALCULATION OF MANUFACTURER'S AVERAGE INVENTORY FOR SINGLE-CHANNEL, MULTI-ECHELON SUPPLY CHAIN SYSTEM

In Figure 3.4 the production time T_0T_e equates m_Dq_D/P, and the boundary $T_0ZA_nB_n$ represents the cumulative production by the manufacturer over a cycle with a cycle time T. Let $T_1, T_2, \cdots, T_n$ be the points when the first, second, $\cdots$, m_Dth orders of q_D units each are shipped to the distribution center. Hence, the distance $T_0T_1 = q_D/P$ and $T_1T_2 = T_2T_3 = \cdots = T_{n-1}T_n = q_D/D$. The step-ladder $T_0T_1A_1B_1A_2 \cdots B_{n-1}A_nB_n$ represents the cumulative quantity shipped by the manufacturer during cycle time T. As such, at any point, the manufacturer's inventory is the difference between the boundary $T_0ZA_nB_n$ and the step ladder $T_0T_1A_1B_1A_2 \cdots B_{n-1}A_nB_n$. Now the area of the triangle T_0ZT_e is given by $\frac{1}{2}(m_Dq_D)(\frac{m_Dq_D}{P})$ in Figure A.1.

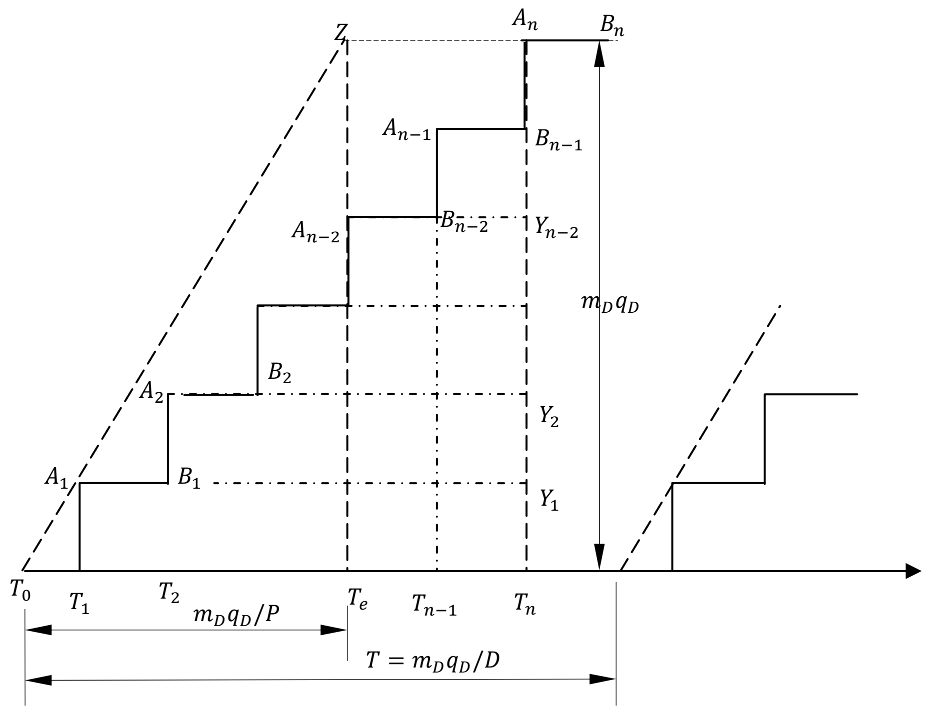


Figure A.1 Cumulative production and shipments of a manufacturer over a cycle

The area of the rectangle $T_e Z A_n T_n$ is given by

$$\Delta_1 = (m_D q_D)\left[\frac{(m_D-1)q_D}{D} - \frac{(m_D-1)q_D}{P}\right]. \qquad (A.1)$$

The sum of the areas of the rectangles $T_1A_1Y_1T_n, B_1A_2Y_2Y_1, \cdots B_{n-2}A_{n-1}B_{n-1}Y_{n-2}$ is given by

$$\Delta_2 = q_D\left[\frac{(m_D-1)q_D}{D} + \frac{(m_D-2)q_D}{D} + \cdots + \frac{q_D}{D}\right] = \frac{q_D^2}{D}\frac{(m_D-1)m_D}{2}. \qquad (A.2)$$

Hence, the manufacturer inventory is given by the net area,

$$\Delta_3 = \frac{m_D q_D^2}{2}\left[\frac{m_D-1}{D} - \frac{m_D-2}{P}\right]. \qquad (A.3)$$

The unit time average inventory is got by dividing the inventory of the manufacturer by the common cycle time, $T(=\frac{m_D q_D}{D})$, which is shown by the expression below:

$$I_M{}^S = \frac{m_D q_D}{2}\left(1 - \frac{D}{P} - \frac{1}{m_D} + \frac{2D}{m_D P}\right) \ \blacksquare \qquad (A.4)$$

APPENDIX B

CALCULATION OF MANUFACTURER'S AVERAGE INVENTORY FOR MULTI-CHANNEL, MULTI-ECHELON SUPPLY CHAIN SYSTEM

In Figure B.1, the area of KLO is given by

$$\nabla_1 = \frac{(\sum_{i=1}^{w} D_i T)^2}{2P}. \qquad (B.1)$$

The area of the rectangle $LMNO$ is given by

$$\nabla_2 = \sum_{i=1}^{w} D_i T \left[\frac{T}{m_1}(m_1 - 1) + \frac{TD_1}{m_1 P} - \frac{\sum_{i=1}^{w} D_i T}{P}\right]. \qquad (B.2)$$

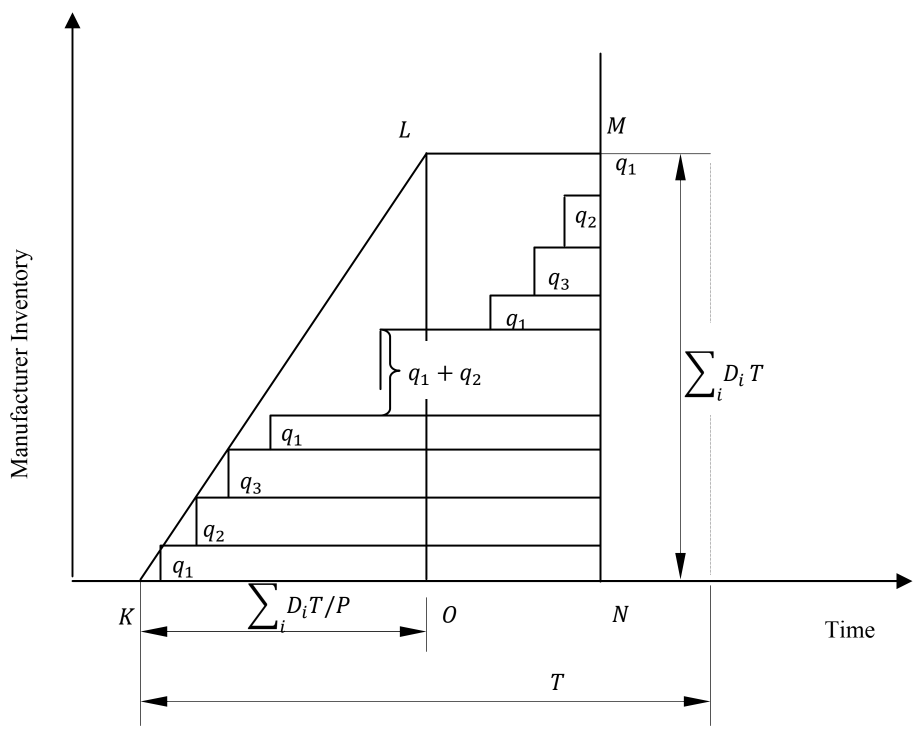


Figure B.1 The manufacturer's inventory in a upstream integrated supply chain system

The area of the rectangles (stairs) for each distribution center can be shown below.

For distribution center 1,

$$\frac{T}{m_1} D_1 \left[\frac{T}{m_1}(m_1 - 1) + \frac{T}{m_1}(m_1 - 2) + \cdots + \frac{T}{m_1}\right] = \frac{T^2}{m_1^2} D_1 \left[\frac{m_1(m_1 - 1)}{2}\right].$$

For distribution center 2,

$$\frac{T}{m_2} D_2 \left[\frac{T}{m_2}(m_2 - 1) + \frac{T}{m_2}(m_2 - 2) + \cdots + \frac{T}{m_2}\right] + m_2 \left\{\frac{T}{m_2} D_2 \left[\frac{T}{m_1}(m_1 - 1) - \frac{T}{m_2}(m_2 - 1) - \frac{D_2 T}{m_2 P}\right]\right\}.$$

For distribution center 3,

$$\frac{T}{m_3} D_3 \left[\frac{T}{m_3}(m_3 - 1) + \frac{T}{m_3}(m_3 - 1) + \cdots + \frac{T}{m_3}\right] + m_3 \left\{\frac{T}{m_3} D_3 \left[\frac{T}{m_1}(m_1 - 1) - \frac{T}{m_2}(m_2 - 1) - \frac{D_2 T}{m_2 P} - \frac{D_3 T}{m_3 P}\right]\right\}.$$

So in more generic terms, for distribution center i,

$$\frac{T}{m_i} D_i \left[\frac{T}{m_i}(m_i - 1) + \frac{T}{m_i}(m_i - 2) \cdots + \frac{T}{m_i}\right] + m_i \left\{\frac{T}{m_i} D_i \left[\frac{T}{m_1}(m_1 - 1) - \frac{T}{m_i}(m_i - 1) + \frac{D_1 T}{m_1 P} - \sum_{l=1}^{i} \frac{D_l T}{m_l P}\right]\right\}.$$

,which after some manipulation can be simplified to,

$$\frac{D_i T^2}{2P} \left\{\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - P m_1 (m_i - 1)}{m_1 m_i}\right\}.$$

Hence, the sum of the rectangles of all the distribution centers,

$$\frac{T^2}{2P} \left\{\sum_{i=1}^{\omega} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - P m_1 (m_i - 1)}{m_1 m_i}\right]\right\}.$$

Now the manufacturer's inventory is the difference between the boundary of the collective area of the triangle and the rectangle and the step ladder that is formed by rectangles corresponding to different distribution centers. So the inventory for the manufacturer is given by

$$\frac{(\sum_{i=1}^{w} D_i T)^2}{2P} + \frac{\sum_{i=1}^{w} D_i T^2}{P}\left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right]$$

$$- \frac{T^2}{2P}\left\{\sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\}.$$

Thus, the manufacturer's average inventory, I_M is obtained by diving the above expression by cycle time T which is given below

$$I_M{}^m$$

$$= \frac{T}{2P}\left\{\left(\sum_{i=1}^{w} D_i\right)^2 + 2\sum_{i=1}^{w} D_i \left[\frac{P}{m_1}\left(\frac{D_1}{P} + m_1 - 1\right) - \sum_{i=1}^{w} D_i\right]\right.$$

$$\left. - \sum_{i=1}^{w} D_i \left[\frac{2m_i\left[P(m_1 - 1) + D_1 - m_1 \sum_{l=1}^{i} D_l/m_l\right] - Pm_1(m_i - 1)}{m_1 m_i}\right]\right\} \quad \blacksquare \quad (B.3)$$

APPENDIX C

INPUT PARAMETERS AND SOLUTIONS OF PROBLEMS (1-16) FOR SENSITIVITY ANALYSIS (SECTION 4.6)

Table C.1 Varying transportation costs of problem 1-6

Problem	τ_{11}	τ_{12}	τ_{21}	τ_{22}	τ_{31}	τ_{32}	τ_{33}	τ_{D1}	τ_{D11}	τ_{D12}
1	150.00	120.00	135.00	165.00	180.00	150.00	135.00	324.00	525.00	540.00
2	100.00	80.00	90.00	110.00	120.00	100.00	90.00	216.00	350.00	360.00
3	75.00	60.00	67.50	82.50	90.00	75.00	67.50	162.00	262.50	270.00
4	50.00	40.00	45.00	55.00	60.00	50.00	45.00	108.00	175.00	180.00
6	37.50	30.00	33.75	41.25	45.00	37.50	33.75	81.00	131.25	135.00

Problem	τ_{D2}	τ_{D21}	τ_{D22}	τ_{D3}	τ_{D31}	τ_{D32}	τ_{D33}	τ_{M1}	τ_{M2}	τ_{M3}
1	330.00	600.00	630.00	315.00	585.00	600.00	570.00	1200.00	1275.00	1170.00
2	220.00	400.00	420.00	210.00	390.00	400.00	380.00	800.00	850.00	780.00
3	165.00	300.00	315.00	157.50	292.50	300.00	285.00	600.00	637.50	585.00
4	110.00	200.00	210.00	105.00	195.00	200.00	190.00	400.00	425.00	390.00
5	82.50	150.00	157.50	78.75	146.25	150.00	142.50	300.00	318.75	292.50
6	66.00	120.00	126.00	63.00	117.00	120.00	114.00	240.00	255.00	234.00

Table C.2 Optimum shipment sizes of problems 1-6

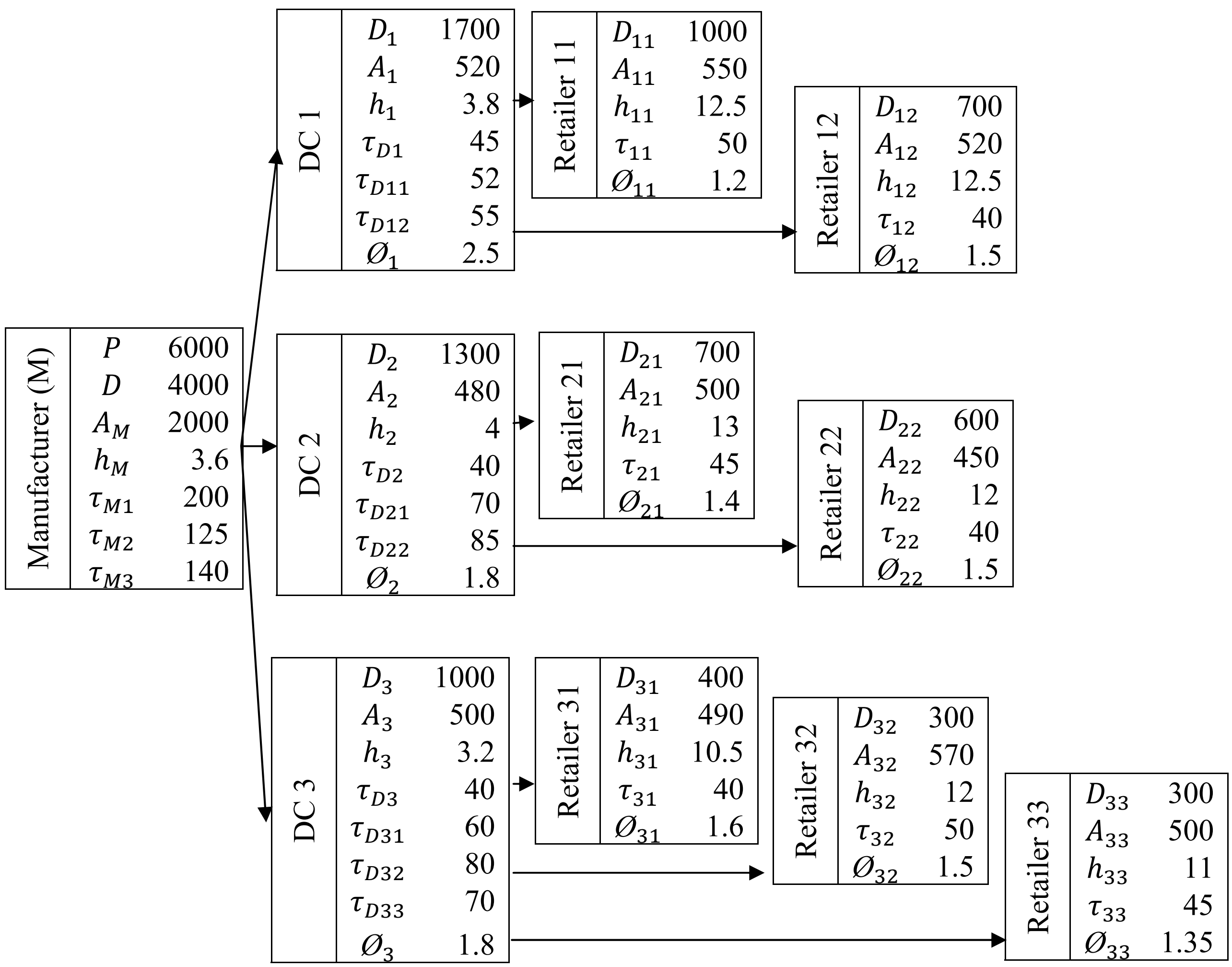
Problem	q_{11}	q_{12}	q_{21}	q_{22}	q_{31}	q_{32}	q_{33}	q_1	q_2	q_3
1	83	94	69	72	61	50	58	178	142	170
2	83	94	69	72	61	50	58	178	142	170
3	42	47	35	72	61	50	29	178	142	170
4	42	47	35	36	31	25	29	89	142	85
5	42	47	35	36	31	25	29	89	71	85
6	42	31	35	36	31	25	29	89	71	85

Table C.3 Optimum number of shipments of problems 1-6

Problem	m_{11}	m_{12}	m_{21}	m_{22}	m_{31}	m_{32}	m_{33}	m_1	m_2	m_3
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	2	2	2	1	1	1	2	1	1	1
4	2	2	2	2	2	2	2	2	1	2
5	2	2	2	2	2	2	2	2	2	2
6	2	3	2	2	2	2	2	2	2	2

Table C.4 Optimum cycle time and total cost of problems 1-6

Problem	Total supply chain cost, TC (\$)	Cycle time, T (days)	$T \geq t$ or $T < t$
1	95,824	101	$T \geq t$
2	84,182	101	$T \geq t$
3	76,756	101	$T \geq t$
4	69,212	101	$T \geq t$
5	64,375	101	$T \geq t$
6	61,092	101	$T \geq t$



In the example, c = \$60/unit, k = 0.05, I_p= 0.20, I_e= 0.13 and t = 30 days.

Figure C.1 Input parameters of Problem 7

Table C.5 Varying holding costs of problems 7–12

Problem	h_{11}	h_{12}	h_{21}	h_{22}	h_{31}	h_{32}	h_{33}	h_1	h_2	h_3	h_M
7	12.50	12.50	13.00	12.00	10.50	12.00	11.00	3.80	4.00	3.20	3.60
8	18.75	18.75	19.50	18.00	15.75	18.00	16.50	5.70	6.00	4.80	5.40
9	25.00	25.00	26.00	24.00	21.00	24.00	22.00	7.60	8.00	6.40	7.20
10	37.50	37.50	39.00	36.00	31.50	36.00	33.00	11.40	12.00	9.60	10.80
11	50.00	50.00	52.00	48.00	42.00	48.00	44.00	15.20	16.00	12.80	14.40
12	62.50	62.50	65.00	60.00	52.50	60.00	55.00	19.00	20.00	16.00	18.00

Table C.6 Optimum shipment sizes of problems 7–12

Problem	q_{11}	q_{12}	q_{21}	q_{22}	q_{31}	q_{32}	q_{33}	q_1	q_2	q_3
7	230	161	161	138	92	69	69	391	299	230
8	151	105	105	136	90	68	68	384	196	226
9	148	104	104	89	59	44	44	252	193	222
10	108	75	75	86	57	43	43	244	140	143
11	104	73	73	63	42	31	31	177	136	139
12	81	71	71	61	41	30	30	172	105	101

Table C.7 Optimum number of shipments of problems 7–12

Problem	m_{11}	m_{12}	m_{21}	m_{22}	m_{31}	m_{32}	m_{33}	m_1	m_2	m_3
7	2	2	2	2	2	2	2	2	2	2
8	3	3	3	2	2	2	2	2	3	2
9	3	3	3	3	3	3	3	3	3	2
10	4	4	4	3	3	3	3	3	4	3
11	4	4	4	4	4	4	4	4	4	3
12	5	4	4	4	4	4	4	4	5	4

Table C.8 Optimum cycle time and total cost of problems 7–12

Problem	Total supply chain cost, *TC* ($)	Cycle time, *T* (days)	$T \geq t$ or $T < t$
7	38,312	168	$T \geq t$
8	41,167	165	$T \geq t$
9	43,967	162	$T \geq t$
10	48,519	157	$T \geq t$
11	52,738	152	$T \geq t$
12	56,548	148	$T \geq t$

Table C.9 Optimum shipment sizes of problems 13–16 and Example 4.1 and 4.3

Problem	q_{11}	q_{12}	q_{21}	q_{22}	q_{31}	q_{32}	q_{33}	q_1	q_2	q_3
Ex 4.1	83	94	69	72	61	50	58	178	142	169
13	86	48	71	74	63	51	60	182	145	174
14	89	50	37	77	65	53	62	190	151	181
Ex 4.3	93	52	39	80	68	56	32	198	157	188
15	93	52	39	80	68	56	32	198	157	188
16	93	52	39	80	68	56	32	198	157	188

Table C.10 Optimum number of shipments of problems 13–16 and Example 4.1 and 4.3

Problem	m_{11}	m_{12}	m_{21}	m_{22}	m_{31}	m_{32}	m_{33}	m_1	m_2	m_3
Ex 4.1	1	1	1	1	1	1	1	1	1	1
13	1	2	1	1	1	1	1	1	1	1
14	1	2	2	1	1	1	1	1	1	1
Ex 4.3	1	2	2	1	1	1	2	1	1	1
15	1	2	2	1	1	1	2	1	1	1
16	1	2	2	1	1	1	2	1	1	1

Table C.11 Optimum cycle time and total cost of problems 13–16 and Example 4.1 and 4.3

Problem	Total supply chain cost, *TC* ($)	Cycle time, *T* (days)	$T \geq t$ or $T < t$
Ex 4.1	84,182	101	$T \geq t$
13	76,960	104	$T \geq t$
14	71,199	108	$T \geq t$
Ex 4.3	65,897	113	$T < t$
15	61,056	113	$T < t$
16	55,214	113	$T < t$

VITA

Farhana Rahman was born to Md Abdur Rahman Choudhury (father) and Sayeeda Rahman (mother) in Bogra, Bangladesh, in 1979. She earned a Bachelor of Science in Industrial and Production Engineering Degree with Honours from Bangladesh University of Engineering and Technology in 2003. She joined her *alma mater* as a lecturer in the same year. Farhana completed her Master of Science in Industrial and Production Engineering from the same university in 2006. Later, she came to the USA to pursue a graduate degree at Louisiana State University with an offer of graduate assistantship. She graduated in 2008 with a degree of Master of Science in Industrial Engineering. Now she is attending a doctoral program in the H. Milton Stewart School of Industrial and Systems Engineering at Georgia Institute of Technology.