

OPTICAL CONTROL PLANE: THEORY AND ALGORITHMS

A Dissertation

Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agricultural and Mechanical College  
in partial fulfillment of the  
requirements for the degree of  
Doctor of Philosophy

in

The Department of Electrical and Computer Engineering

by

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August 2006

# Acknowledgments

I would like to express my sincere gratitude to Dr. Ahmed El-Amawy for being my research mentor. I am thankful for all the guidance and advice I received from him throughout my PhD stage. The research philosophy and the approach for detailed examination I learned from him are invaluable assets that I will never forget. I am extremely grateful that I developed my inquisitive mind under his assistance.

I would also like to thank all members of my committee for the important contribution they had on the my development as a researcher. I am grateful to Dr. Sukhamay Kundu for accepting to be my minor advisor and for his continuous support. The occasional brainstorming sessions along with his precious comments helped me improve the quality of this dissertation considerably. I want to thank Dr. Jerry Trahan and Dr. J. Ramanujam for their helpful comments and pointers to inherent theoretical hurdles I encountered along the way. I also want to thank Dr. Hsiao-Chun Wu for his encouragement and often flattering comments.

Lastly, but most importantly, I would like to express my deep consideration to my parents. Without their love and constant support this thesis would not have been possible.

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# Abstract

In this thesis we propose a novel way to achieve global network information dissemination in which some wavelengths are reserved exclusively for global control information exchange.

We study the routing and wavelength assignment problem for the special communication pattern of non-blocking all-to-all broadcast in WDM optical networks. We provide efficient solutions to reduce the number of wavelengths needed for non-blocking all-to-all broadcast, in the absence of wavelength converters, for network information dissemination. We adopt an approach in which we consider all nodes to be tap-and-continue capable thus studying lighttrees rather than lightpaths. To the best of our knowledge, this thesis is the first to consider “tap-and-continue” capable nodes in the context of conflict-free all-to-all broadcast. The problem of all to-all broadcast using individual lightpaths has been proven to be an NP-complete problem [6]. We provide optimal RWA solutions for conflict-free all-to-all broadcast for some particular cases of regular topologies, namely the ring, the torus and the hypercube. We make an important contribution on hypercube decomposition into edge-disjoint structures. We also present near-optimal polynomial-time solutions for the general case of arbitrary topologies. Furthermore, we apply for the first time the “cactus” representation of all minimum edge-cuts of graphs with arbitrary topologies to the problem of all-to-all broadcast in optical networks. Using this representation recursively we obtain near-optimal results for the number of wavelengths needed by the non-blocking all-to-all broadcast.

The second part of this thesis focuses on the more practical case of multi-hop RWA for non-blocking all-to-all broadcast in the presence of Optical-Electrical-Optical conversion. We propose two simple but efficient multi-hop RWA models. In addition to reducing the number of wavelengths we also concentrate on reducing the number of optical receivers, another important optical resource. We analyze these models on the ring and the hypercube, as special cases of regular topologies. Lastly, we develop a good upper-bound on the number of wavelengths in the case of non-blocking multi-hop all-to-all broadcast on networks with arbitrary topologies and offer a heuristic algorithm to achieve it. We propose a novel network partitioning method based on “virtual perfect matching” for use in the RWA heuristic algorithm.

# Chapter 1

## Introduction

### 1.1 Optical Communications Essentials

Optical communications exhibit very attractive features in almost every category. The incredible bandwidth of about 40Gbps on a single wavelength [65], low signal attenuation, low signal distortion, low power requirement and the low cost [48] make optical networking an undisputable choice. Optical communication between a source and a destination starts at the transmitter's end, where the signal is converted from the electronic domain to the optical domain, transmitted over the optical medium and then converted back from optical to electronic form at the receiver's end, (Figure 1.1). Usually, the transmitter is a tunable laser, able to span over a large range of wavelengths.

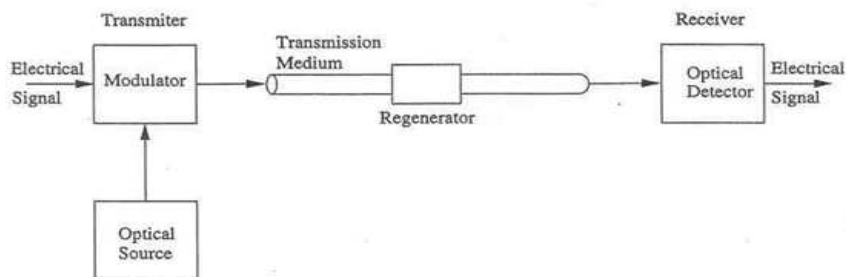


FIGURE 1.1. Example of optical transmission.

The main characteristics of a tunable laser are its tuning range, tuning time and the tuning type: continuous or discrete. “Continuously tunable” laser refers to a laser able to tune to all the wavelengths in its tuning range, whereas a discretely tunable laser refers to a laser that is tunable to only selected wavelengths.

The optical medium refers to the type of optical fiber used. There are two types of fibers: single mode and multimode fibers. A mode refers to the way an optical wave propagates through the fiber, which translates into a solution to Maxwell’s wave equation [48]. Single-mode fiber is more appropriate for long-haul optical transmission, while the multimode fiber is usually used in Local Area Networks (LANs) and Metro Area Networks (MANs) [33]. The impairments encountered in

the optical fiber can be classified into two principal types, signal attenuation and dispersion. Signal attenuation is a power loss over the length of the fiber. Dispersion refers to some distortion in the signal and can be further classified into modal, chromatic and polarization dispersion. Modal dispersion results from the different angles a wavelength may enter a fiber, thus is encountered only on multi-mode fibers. Chromatic dispersion is caused by different propagation speeds of different wavelengths. Lastly the polarization mode dispersion is due to non-uniformities in the fiber and results in different propagation delays for different polarizations. Thus, amplification and signal regeneration are required along the optical fiber. Usually this operation is called the 3-R operation (re-shaping, re-amplification and re-timing) and is required to regenerate the signal to its original form.

The receivers usually consist of photodetectors, in the form of ( $pn$ ) photodiodes and electronics for amplification and processing of the received signal. The interested reader is referred to [33] and [48] for in depth coverage of the optical transmission.

The underlying technology used in optical data transmission is Wavelength Division Multiplexing (WDM), which multiplexes multiple wavelengths (frequencies) carrying data independently, on the same optical fiber. Figure 1.2 shows an example of a WDM link. First the different wavelengths are multiplexed and transmitted along the fiber. Amplification is performed at the beginning and at the end of the link. In addition, in-line amplification is performed if necessary.

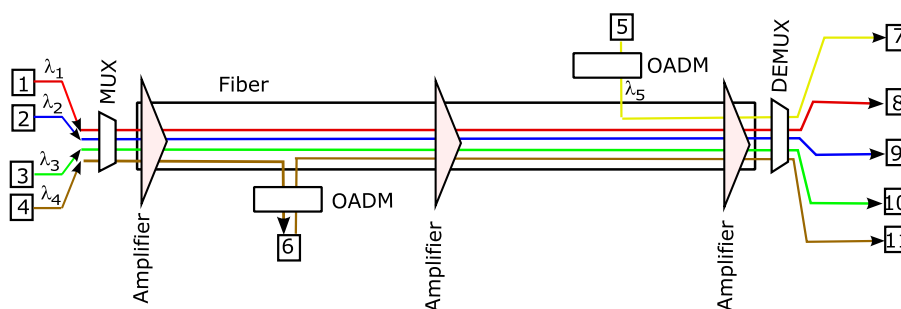


FIGURE 1.2. Example of a typical WDM link

Another optical device used in optical communications is the Optical Add-Drop Multiplexer (OADM, or simply ADM). The OADM consists of a MUX/DEMUX pair. It is used to terminate, or “drop”, a wavelength at an intermediate node along the path, and to inject, or “add” a new data stream using the same wavelength as the dropped signal. Please note in the example illustrated

in Figure 1.2 that node 6 terminates the connection used on  $\lambda_4$ , and initiates a new connection on the same wavelength. Moreover, a new data stream can be just added on the fiber, as node 5 uses  $\lambda_5$ . Please note that an OADM can add or drop more than one wavelength, at an intermediate node. At the end of the connection, the wavelengths are demultiplexed into individual signals, dropped at their respective receiver nodes, and converted to an electronic form.

A typical long haul optical network (also called wide area network, WAN) presently consists of optical cross-connects (OXC) interconnected using optical fibers in a mesh topology operating using WDM (or DWDM) technology. An optical crossconnect is defined as an OADM and an optical switch pair. The cross-connects offer access-points (also called Central Office points CO) to Metro Area Networks MANs. A MAN is presently deployed as an interconnection of optical rings, on SONET (synchronous optical network) technology. The nodes in the SONET rings offer access-points (also called points of presence POP) to local area networks (LANs). LANs are mainly IP (electronic) networks. Future optical networks envision a “fiber-to-the-home” (FTTH) all-optical network model, where LANs at the last network level are also optically deployed. This model is also known as the “last mile optical network”. A network infrastructure example is presented in Figure 1.3.

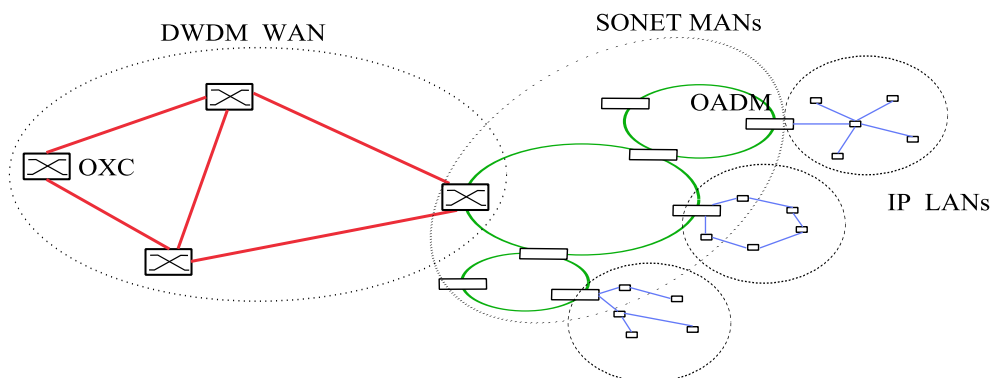


FIGURE 1.3. Example of network infrastructure.

Two other optical devices used in optical networks which will be referred to in this thesis are briefly discussed next.

An optical splitter is a device that splits the optical signal from a fiber on two or more fibers. The power of the optical signal divides equally among the number of splitting fibers. Thus, on

a two way split, as depicted in Figure 1.4 each outgoing fiber will get approximately 50% of the incoming power.

A “tap-and continue” crossconnect “taps” the information on a wavelength, without dropping the wavelength. This type of function is performed whenever the connection request is of the form “one-to-many”, such that there is a source and multiple receivers. Therefore, tap-and-continue is a very useful feature for multicast and broadcast connection types. A tap and continue crossconnect is similar to a local node split with the main difference being that the tapping function taps a small amount of power (much less than 50%, typically 3 – 5%). These two optical devices are illustrated in Figure 1.4.

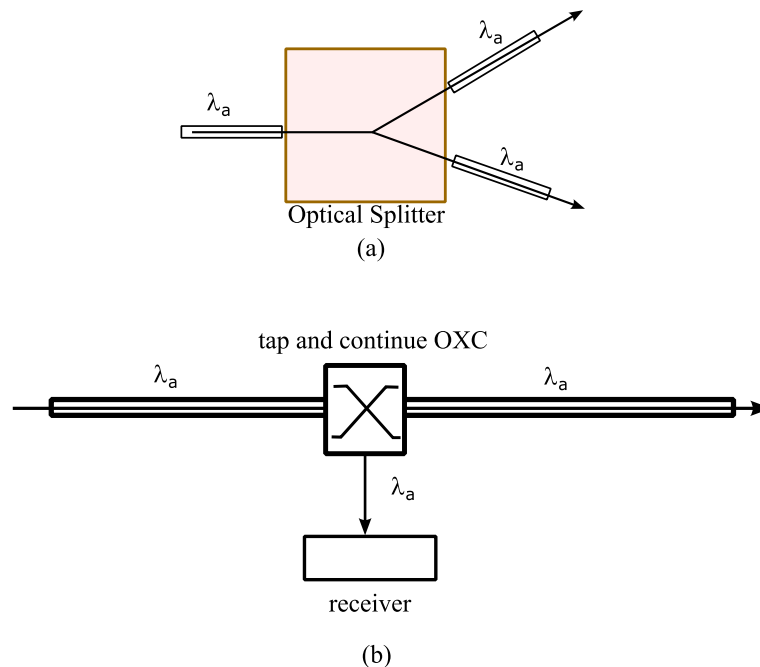


FIGURE 1.4. Example: (a) an optical splitter; (b) a tap and continue OXC.

Optical networking and WDM in particular introduced new problems, that do not exist in electrical networks. The major problems raised by optical networking are briefly addressed next.

The only switching technique implemented today in optical communications is circuit switching. Circuit switching reserves a path from source to destination for the duration of the entire communication. The path is released only after the entire message has been transmitted. Such a path is called a lightpath. In circuit switched optical networks the wavelength continuity constraint must be satisfied. The wavelength continuity constraint states that the same wavelength has to

be used on all links of a lightpath [28]. This is called the “full wavelength continuity constraint”. If the same wavelength is not available on every link of the switched path from the source to the destination, but other wavelengths are available on each link in the path, then the message will have to be converted from a wavelength to another at every node that cannot satisfy the wavelength continuity constraint. This is called the “partial wavelength continuity constraint”. There are two ways to do this. One is to convert the message from the optical to the electronic domain and to buffer it until the same wavelength or any other wavelength becomes available. Then an electrical-optical conversion takes place and the message is sent forward. This procedure is known as the optical-electric-optical (OEO) conversion. The second approach involves all-optical conversion using optical converters. Such converters are not yet practical. They are expected to be expensive and noisy. Therefore they are not very attractive.

Several researchers investigated packet switching in optical networks. Some researchers proposed a new switching technique that has both circuit and packet switching characteristics. It is called “optical burst switching” (OBS) [82]. It operates as follows. Packets destined to the same egress node are grouped together. The data packets follow the control setup request packet. A small delay is introduced between the control packet and the data packets. It is assumed that the delay will allow the control packet to configure all switches on the path before the data packets arrive. Once the burst of packets is received by the destination, the lightpath is released. Notice that the control message follows an approach similar to packet switching while the data burst has circuit switching characteristics. However, the OBS approach is still in its infancy and the message blocking problem doesn’t make it an attractive solution.

The original routing problem in traditional networks becomes the routing and wavelength assignment (RWA) problem in optical networks. Static RWA computes the routes and assigns the wavelengths off-line. The objective function attempts to minimize the resources used. Dynamic routing is performed on-line and uses global or partial network information. Also the routing problem can be taken separately from the wavelength assignment problem, or treated together as a single problem.

The main focus of our research is on one network control function, specifically the network information dissemination. The next section provides information on the optical control plane and briefly describes all control operations required to ensure fast and reliable optical data communication.

## 1.2 Control Plane

One main disadvantage in current practice is the difficulty of optical buffering with current technology. This makes it difficult to implement optical packet switching which gives rise to incompatibility with electronic IP networks. Another disadvantage is the high cost of some optical components, which makes optical networks an expensive choice. Nevertheless, it is expected that in the future the cost of optical components will drop significantly, and that soon any optical configuration will be more affordable [57].

An optical network is typically organized in two network planes [2], [20], [88], a data plane is intended to carry the data traffic and a control plane for managing the connections in the data plane (Figure 1.5).

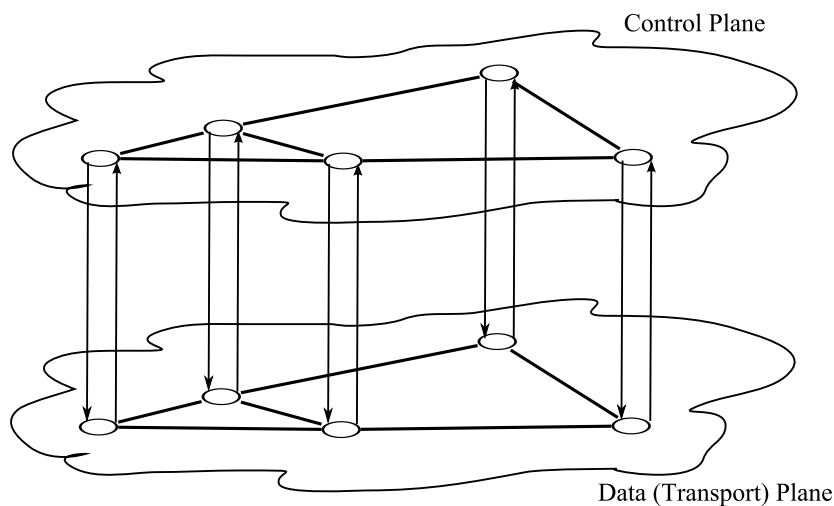


FIGURE 1.5. The two planes of an optical network.

The information exchanged between the two planes is crucial for proper running of an optical network. The optical node consists of an optical cross-connect (OXC) for data communication and a control module for exchanging the control messages (Figure 1.6).

The data plane transports user traffic among the network nodes. Nodes exchanging information are connected by lightpaths in a circuit switched manner. In current practice the control plane

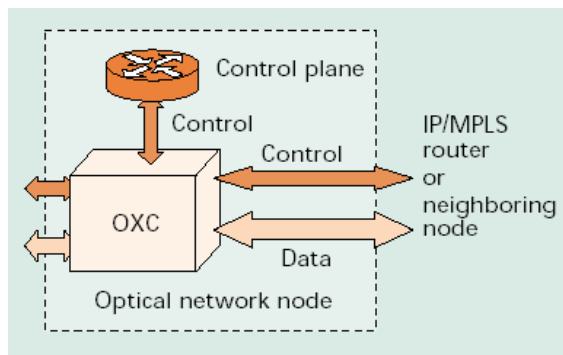


FIGURE 1.6. Optical Node. From reference [20].

is a different network, with appropriate software, that is used to control the vital functions of the data plane. The two planes are either operating on the same network topology or on two different network topologies. The main functions of a control plane and its characteristics are briefly explored next.

### 1.2.1 Control Plane Functions

Although the work on the control plane is still in the research phase [11], [15], [20], [21], [40], [60], [88], there is universal agreement on the main functions of the control plane.

#### - Topology and Resource Discovery

This function is vital for routing and wavelength selection decisions for establishing lightpaths. The topology discovery is a task performed periodically by the control plane to provide a complete network topology to all the nodes in the data plane. Resource discovery encompasses the discovery of all the information needed for establishing a lightpath, except for topology information. Some of this information may include: number of fibers on a link, optical fiber capacity, wavelength usage on each fiber, available ports at the optical switches, the number of transceivers for each node and wavelength converters availability. The control plane should deliver this information to all the data nodes in a fast and reliable manner, such that each node will have a global view of the network's present status. The process of collecting and distributing the necessary information is called "network information dissemination".

#### - Route Computation

Although this is generally performed at the call-originating node, it is still the control plane's responsibility to perform a route computation and to assign a wavelength for the lightpath. All

the information needed for establishing a lightpath should already be available at the data plane node. This would be accomplished by the first control plane function discussed previously, topology and resource discovery. Route computation involves special routing and wavelength assignment algorithms, as well as traffic engineering functions. Traffic engineering for optical networks is defined in [20] as following:

“Building an optical network that efficiently and reliably satisfies a diverse range of service requests involves the application of network and traffic engineering techniques to determine optimal or near optimal operating parameters for each of the following three components:

- A traffic demand - either measured or estimated, usually expressed as a traffic matrix.
- A set of constraints - such constraints include physical layer layout, link capacity, the OXCs, and other optical devices (including fiber amplifiers, wavelength converters, etc.) deployed.
- A control policy - consists of the network protocols, policies, and mechanisms implemented at the OXC control modules.”

Once the route and the wavelength to establish the lightpath are computed, the next step is to physically setup the lightpath. The third function of the control plane is lightpath management. Its major tasks are lightpath setup and teardown and protection switching in case of link or node failures. The failures are considered to be in the data plane.

### **1.2.2 Present Technologies: GMPLS**

The work on the optical network control is still under much research. Several organizations such as IETF (The Internet Engineering Task Force), OIF (Optical Engineering Forum) and ITU-T (International Telecommunication Union) offered solutions to the optical network control problem, and so far IETF’s GMPLS solution (Generalized Multiprotocol Label Switching) seems to be generally accepted.

MPLS (Multiprotocol Label Switching) was introduced by IETF to improve the classic connectionless IP with a virtual circuit-switching technology in the form of a label-switched path. The first step toward a control plane for optical networks was the IETF MPAS protocol (Multiprotocol Lambda Switching), suggested because of the isomorphic relationships between the features

required by the optical control plane and the MPLS functions. GMPLS was introduced to support multiple types of switching. Besides lambda switching, GMPLS inherited the IP-switching (packet switching) from MPLS, added ATM (datagram switching), SONET/SDH (time division multiplexed switching), and specifically for optical networks,  $\lambda$ -switching and fiber-switching.

GMPLS is a broad collection of protocols. References [2], [18], [21], [43], [67], [88], [89], [11] offer more information on GMPLS. In the following we will describe only the optical features of GMPLS suggested for performing the three main functions of the control plane mentioned above.

Based on GMPLS specifications, each optical node consists of an OXC (in the data plane) and a control node (in the control plane) Thus each node in the data plane is shadowed by a node in the control plane. The control plane is an IP-network, whose topology may or may not be the same as the data plane's topology. The IP-based control plane is used for transmission of control messages such as routing information or signaling.

- Topology and resource discovery

GMPLS uses a link management protocol (LMP), exchanging "hello" packets between neighboring nodes, to perform topology discovery and link health monitoring.

There are two main categories of routing protocols. One is "distance vector", and the second one is "link state". A link state protocol, such as OSPF (open shortest path first) is used by GMPLS for resource discovery and network state distribution. OSPF forms neighborhood adjacencies, floods the network with link state information along the adjacencies created and ensures that all nodes in the same adjacency area have the same topological database. There are current efforts to extend OSPF to support optical networks. Additional optical resource information has to be included in the link advertisement such as bandwidth, wavelength availability, wavelength converters availability, number of optical fibers on a link and other traffic engineering information. As opposed to electrical networks, where there may be a few links between two neighbors, in optical networks there may be thousands of links (in the form of wavelengths) between two neighboring nodes. This tremendous amount of link state information combined with the flooding nature of OSPF introduces an undesirable overhead to the link state information and considerably increases the size of the link state databases needed at each node. To solve this problem, IETF introduced

the concept of “link bundling” to reduce the amount of information advertised. A bundled link includes multiple wavelengths or even fibers with similar characteristics, with the restrictions that each wavelength or fiber included in the bundle has the same type and originates and terminates at the same node-pair. There are two major drawbacks for link bundling technology. First, it is difficult to select the wavelengths or the fibers to form a bundle, due to dynamic changes in the lightpaths. The second drawback is the loss of some link state information introduced by the technology. The missing information could be essential for routing and wavelength assignment.

- Route computation

There are many proposed routing algorithms discussed for optical networks [5], [14], [20], [41], [69], [81], [85], [87], [89], [90], [91], [92]. The routing can be static, dynamic or adaptive based on global information or local information. GMPLS uses the CSPF (constrained shortest path first) path computation algorithm. The algorithm is based on the topology and the link state database information and uses either Dijkstra’s or the distance vector technique. The constraints introduced by the optical information make the computation of a shortest path an NP-complete problem. That means that heuristics have to be used. For the wavelength assignment problem GMPLS has several alternatives. One is to take the wavelengths as a constraint in CSPF and select the route and the wavelength for the lightpath at the same time. Another alternative, which makes more sense in a dynamic environment, is to couple the wavelength selection with signaling and to select the wavelength using a forward or a backward reservation technique. On a lightpath setup, the setup message sent toward the destination collects all the available wavelengths along the path. For backward reservation the destination selects a wavelength and reserves the resources on the path back to the destination. In forward reservation, the destination sends an acknowledgment back to the source, containing the available resources on the selected path. Then the source selects a wavelength and reserves the resources along the path toward the destination.

- Lightpath management

For lightpath management, GMPLS uses two signaling protocols: RSVP-TE (resource reservation protocol-traffic engineering) and CR-LDP (constrained routing-label distribution protocol). These are back reservation protocols which are very similar to each other. The source node sends a

path setup message toward the destination node, with no reservation on the way. The destination checks the identifier and the parameters in the setup message and reserves the resources in the backward trip to the source node. The two protocols mainly differ in the messages sent, identifiers and parameters used, differences that have an impact on the scalability and setup time for each protocol.

In summary, GMPLS uses an IP-based network for the control plane, with node shadowing of the data plane. IP routing and switching algorithms are modified or enhanced to support the circuit-switched lightpaths. Link state advertisement is done by flooding the network and link bundling is used to reduce the size of the control messages and link state database. Lightpath management is carried out by two signaling protocols based on back reservation techniques.

### **1.3 Thesis Outline**

The remaining of the thesis is organized as follows. In Chapter 2 we propose our control function implementation, formulate our problem as a graph theoretical problem and review the existing relevant literature. Chapter 3 investigates the problem of one-hop RWA for non-blocking all-to-all broadcast. Chapter 4 explores two methods of RWA for the case of multi-hop all-to-all broadcast. In Chapter 5 we review our research contributions and identify new research directions.

# Chapter 2

## Problem Definition and Literature Survey

### 2.1 Motivation

The role of the control plane is to monitor and manage data communication in the transport plane. The ideal control plane would take fast and reliable actions whenever communication requests or topological changes occur in the transport plane. In this thesis we concentrate on one specific control function, namely network information dissemination in optical networks.

There has been a general agreement on the fact that, under the link state information routing protocol, fast and accurate global network information has to be available at all routers in the network. Reference [11] makes a very important observation about future all-optical networks, where the wavelength continuity constraint has to be taken into account by the routing protocols. OSPF and IS-IS protocols do not include information on the wavelength availability in their Link State Advertisements (LSAs). In current optical network control plane architectures, lighthpath provisioning is still done manually and usually the duration of an established lightpath spans from minutes to hours. Thus, current routing protocols update the LSAs with a frequency in the order of minutes. It has been pointed out in [11] and [20] much faster and accurate LSAs will be needed for the future all-optical networks.

In the early 2000 the interest in faster and more accurate LSAs started to grow. Several studies have shown the merits of accurate global information and the positive impact of fast LSA updates on the blocking probability in optical networks. The work in [66] studies QoS in IP networks, and shows the large impact link state updates have on the probability of successfully routing new connections. In [68], the authors show through simulation that outdated network information as well as high control message overhead negatively impact the blocking probability in dynamic optical networks. In [45] the authors analyze the three types of blocking probability, due to insufficient network capacity, due to inaccurate or outdated network information, and due to over-reservation. Their conclusion is that blocking due to inaccurate or outdated network information becomes

increasingly important with bursty traffic and heavy traffic loads. The study in [78] concludes, based on simulations, that the blocking probability in dynamic lightpath establishment is greatly affected by the frequency of link state information updates. Some other studies [44], [1], [42], [35] show the importance of having fast and accurate global network information dissemination.

Several studies on the blocking probability or lightpath provisioning in dynamic optical environments consider that global information on wavelength utilization is available for analysis, without considering the means of achieving it. References [34], [62], [31] are a few examples of such studies.

The research included in this dissertation offers a solution to the problem of efficiently advertising the global network information in a fast and accurate manner, where global exchange of information is performed in the optical layer. Next section provides details on our network information dissemination method.

## 2.2 Global Control Information Exchange Model

Our research explores the new concept of all-optical control plane. In this approach control for the optical data plane will be also deployed optically making use of the optical resources already available.

As mentioned in the previous chapter, the control plane can be deployed as a separate network (out-of-band) or can be deployed on the same data network (in-band). In the case of in-band control, the control plane may have the same topology as the data network. We consider the control plane topology given, as a set of optical OXCs interconnected by optical fibers. We model this network as an undirected graph  $G(V, E)$ , where the set of nodes  $V$  represents the collection of OXCs and the set of edges  $E$  represents the collection of fibers. For example consider the NSF WDM network illustrated in Figure 2.1(a). The resulting undirected graph is shown in Figure 2.1(b).

To achieve global network information dissemination, each node in the control network has to send its information to all other nodes in the network, such that the collective information contains the global state of the network. This is equivalent to an all-to-all broadcast operation. The global information exchange should be conflict free. Using optical communication for the all-to-all conflict-free broadcast, we guarantee the fastest possible delivery of information. For the rest of this thesis we will use the terms “non-blocking” and “conflict-free” interchangeably.

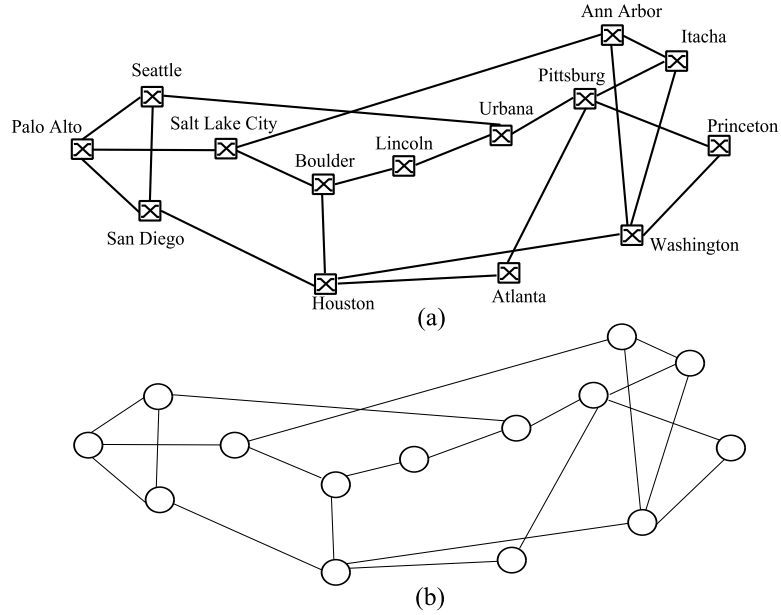


FIGURE 2.1. (a) NSF WDM network. (b) the undirected graph corresponding to the NSF network.

The task of global exchange of information will require some wavelengths from the network total available capacity be dedicated to that task. We aim to reduce the number of wavelengths needed for exchanging the control information. As mentioned in Chapter 1, we make use of the light splitting capability of optical nodes and also use tap and continue capable optical switches. Thus, the broadcast established by a source node is of the type “one-to-many”, and is represented by a lighttree, instead of a lightpath. We also consider the fibers to be bi-directional, such that the same wavelength can be used on the same fiber in opposite directions. A lighttree will then be a directed tree rooted at the broadcasting source spanning all other nodes in the network. The source may use different wavelengths for the links in different edge-disjoint subtrees. The wavelength used in any subtree is subject to the wavelength continuity constraint. Henceforth we consider the lighttrees to be directed and will use the terms lighttree and spanning tree interchangeably.

In Figure 2.2 node 1 is the source of a lighttree that spans all nodes in the network. Please note that all intermediate nodes “tap” the information from the respective wavelength, without dropping the wavelength. Nodes 6 and 9 will have to split the incoming signal. Figure 2.3 illustrates the lighttree established sourced at node 1.

Hence, the problem we have to solve can simply be stated as follows: “Find the off-line routing and wavelength assignment (RWA) for non-blocking all-to-all broadcast in a given optical network

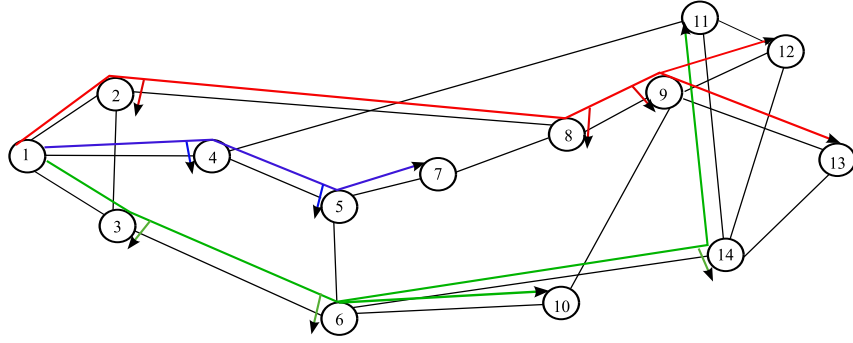


FIGURE 2.2. Example of a lighttree rooted at node 1. The lighttree uses 3 wavelengths, each denoted by a different color. Nodes 6 and 9 preform light splitting.

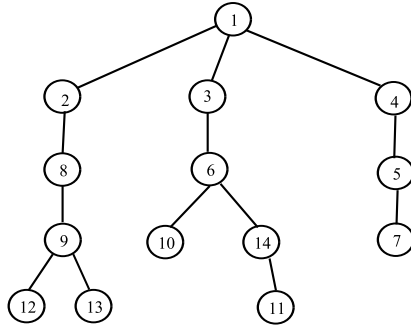


FIGURE 2.3. The spanning tree associated with the lighttree rooted at node 1.

that minimizes the number of wavelengths”. This problem reduces to an uncommon graph coloring problem. The problem formulation can be stated as follows:

Given an undirected graph  $G(V, E)$ , find  $V$  directed spanning trees, one rooted at each of the  $V$  nodes and assign a color to each tree branch. Find the  $V$  spanning trees that would minimize the number of colors such that no edge will have the same color in the same direction. Edges support the same color in opposite directions. Each spanning tree represents a broadcast tree and each color represents a wavelength. A spanning tree may use different colors for its subtrees, or use the same color for the entire spanning tree. Notice that the above formulation implies non-blocking global exchange of information. Figure 2.4 illustrates the case of 2 concurrent broadcasts rooted at nodes 1 and 2. The lighttree rooted at node 2 is dashed. Assume the lighttrees use the same wavelength. Figure 2.4 (a) shows the case of 2 non-blocking trees. Figure 2.4 (b) shows the case of 2 blocking broadcast trees. Notice that the directed link  $\langle 1, 3 \rangle$  is used by both lighttrees, in this latter case. We also study the all-to-all broadcast in the presence of optical-electronic-optical (OEO) conversions. This corresponds to the case of multi-hop routing, whereas each hop consists of one or more “one-to-one” or “one-to-many” connections that adhere to the wavelength continuity

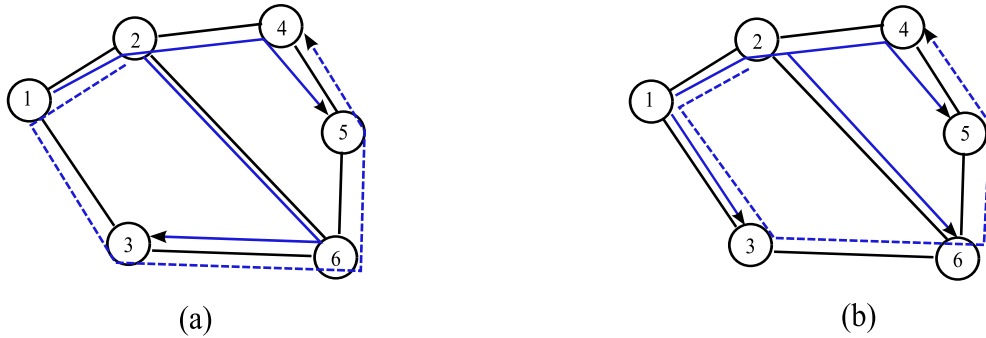


FIGURE 2.4. Example of two lighttrees. (a) represents the case of 2 concurrent non-blocking lighttrees; (b) represents 2 concurrent blocking trees

constraint. Thus, a broadcast tree is broken down to multiple lighttrees. The main motives behind the multihop approach are the physical optical constraints: the limited number of wavelengths on a fiber, and more importantly, the number of receivers at each destination node. It is a much more realistic scenario in which the number of wavelengths and the number of optical receivers can be drastically reduced. When the number of transmitters is equal to the number of receivers, we use the term “transceiver” to refer to a transmitter/receiver pair. The problem formulation remains the same, except that in this case we attempt to reduce both the number of wavelengths and the number of receivers per optical node.

## 2.3 Literature Review

A large amount of research has been done lately in the area of optical networks and RWA in particular. The all-to-all broadcast in the optical domain has received a fair amount of attention as well. In this section we review selected work carried out in this field and relevant to our problem. Specific references will also be provided at the beginning of each new section, as appropriate, in the following chapters.

Among the most interesting problems raised by the optical networking development is the problem of routing and wavelength assignment (RWA). Static RWA as well as dynamic RWA have been studied extensively [5], [6], [7], [17], [22], [23], [41], [65], [81], [90], [92].

An interesting theoretical approach on RWA online algorithms can be found in [5]. The authors study online RWA on trees, trees of rings and meshes, and use the “competitive ratio” as the performance measure. They define the competitive ratio as the “worst case ratio over all request sequences between the number of colors used by the on-line algorithm and the optimal number

of colors necessary on the same sequence”. The authors present very attractive RWA algorithms for the above mentioned topologies with a competitive ratio of  $O(\log N)$ , and propose an online algorithm for arbitrary topologies. The work in [41] presents for the first time an analytical model to compute the blocking probability, using a “Fixed Path - Least Congestion” (FPLC) online RWA algorithm. They also propose an RWA scheme using neighborhood information showing the increased network performance through simulation and analytical methods. A more practical approach of online routing is presented in [90]. The authors present in a unified way the challenges encountered by the optical control plane for robust online RWA. The authors review the main routing techniques proposed for use in GMPLS, like the Fixed-Alternate Routing and Least Congested Routing. They also review the benefits of adaptive routing based on network state information. They consider local information, neighborhood information and global information. They study the issues related to each technique. The wavelength assignment is considered as a separate problem. The approach used for wavelength assignment in this study is first-fit. Another common wavelength assignment strategy in online RWA is random wavelength assignment, where a wavelength is selected at random over the available set of wavelengths.

A comprehensive approach for the RWA problem, along with its mathematical formulation can be found in [6]. The authors review a collection of results and present some of their own. The RWA problem is defined as follows. Let  $G(V(G), A(G))$  be a digraph, and  $I$  be collection of requests instance. A request is a pair of nodes  $(x, y)$ , where  $x$  acts as the source and  $y$  as the destination. The RWA problem denoted  $(G, I)$  asks for a routing  $R$  for the instance  $I$  and assigning each request from  $I$  a wavelength, such that no two dipaths of  $R$  sharing an arc have the same wavelength. The authors define  $\vec{\pi}(G, I, R)$  to be the maximum load of an arc for the routing  $R$  for a given collection of requests instance  $I$  on a network modeled as a symmetric digraph  $G(V(G), A(G))$ .  $\vec{\pi}(G, I)$  is defined as the maximum load of an edge for all possible routings  $R$  for the given instance  $I$ .  $\vec{w}(G, I, R)$  is used for the smallest number of wavelengths needed for a given routing  $R$  and  $\vec{w}(G, I)$  is used for the smallest number of wavelengths over all possible routings. The authors used the notations  $w(G, I)$  and  $\vec{w}(G, I)$  to distinguish between undirected graphs and symmetrical digraphs, respectively.

Some important results are presented. The first result is that  $\vec{w}(G, I) \geq \vec{\pi}(G, I)$  for any given instance  $I$ , in any network  $G$ . Furthermore, the authors prove that determining  $\vec{\pi}(G, I)$  in the general case is *NP*-complete, correlating it to the integral multicommodity directed flow problem. Two special cases are identified to have polynomial time complexity. The first is finding RWA for any instance  $I$  when  $G$  is a tree. The second corresponds to the “one to many” single multicast type instances. The authors also define  $I_A$  to be the All-to-All request instance, and give the upper bound  $\vec{\pi}(G, I_A) \geq \frac{N}{2\beta(G)}$ , where  $\beta(G)$  represents the arc expansion. Interestingly,  $\vec{\pi}(G, I_A)$  is equivalent to the edge-forwarding index, studied at length in graph theory. The authors also study some regular topologies. An interesting result presented is that any permutation can be efficiently solved in the binary hypercube using no more than two wavelengths. Furthermore, the case of  $I_A$  instance in the hypercube is found to satisfy  $\vec{w}(G, I_A) = \vec{\pi}(G, I_A)$ .

In most studies the RWA problem has been divided into a routing problem and a wavelength assignment problem and solved as two separate problems. Since the RWA problem is computationally difficult, many solutions were presented in the form of ILP formulations. In recent studies, [83] offers a simple and efficient solution for the static RWA, where the routing problem and the wavelength assignment problem are considered jointly. The authors present a “Tabu Search” algorithm to solve the RWA problem and compare it against the solution provided by an ILP formulation. A tabu search is an iterative procedure that takes an initial solution and repeatedly constructs new solutions by searching in the neighborhoods of the current solution. While the ILP formulation takes more than one day to find the solution for a 50-node network, the authors claim that their approach takes no more than half an hour, and the results are very close to those produced by the ILP formulation.

Reference [16] considers the problem of dynamic RWA and proposes a novel routing scheme that assumes the existence of wavelength converters in the network. The work in [92] is one of the first to provide a solution to the static multihop RWA problem. The multihop concept is used in the context of virtual topologies, in which a connection request may span over multiple lightpaths. Thus, the nodes connecting two lightpaths are supposed to perform full OEO conversions. The authors solve the RWA problem using a mixed ILP formulation (MILP) on an auxiliary created

“connection graph”, based on given traffic matrix. They show through simulation the effectiveness of their solution.

This dissertation considers offline RWA, thus we are more interested in the static RWA algorithms.

As stated in Chapter 1, all-optical wavelength converters are expensive devices. Several research efforts have been made on minimizing the number of converters globally or on a given path. The impact of converters on the blocking probability was studied in [4], [36], [61], [64], [72], [73], [77], [79], [80]. The works in [36] and [73] offer analytical models for computing the blocking probability in all-optical networks with wavelength converters. The authors in [72] analyze the influence on the number of converters and their placement on the blocking probability and offer a dynamic programming algorithm for converter placement.

Special regular topological cases also received attention from an optical networking perspective. Some important RWA results have been offered for the ring [39], [46], [56], [65], torus [10], [27], and the binary hypercube [6], [7], [22], [59], [65], [74], [81], [93].

An interesting result on the ring can be found in [56]. We will make use of this result in Chapter 3. The authors present lower bounds for the edge-load  $\pi$  and the corresponding number of wavelengths  $w$  required by permutation on optical rings, for both directed and undirected topologies. They found that:

$$\begin{aligned}\vec{w}(C_n, I_1) &\leq \left\lceil \frac{n}{3} \right\rceil, \quad \vec{\pi}(C_n, I_1) \leq \left\lceil \frac{n}{4} \right\rceil, \\ w(C_n, I_1) &\leq \left\lceil \frac{n}{2} \right\rceil, \quad \pi(C_n, I_1) \leq \left\lceil \frac{n}{2} \right\rceil.\end{aligned}$$

Where  $C_n$  represents a ring of  $n$  nodes.

A comprehensive study on regular topologies was conducted in [65].  $l$ -uniform personalized communication is considered for the ring, 2D torus and the binary hypercube. Of interest to us are the results obtained for the static  $l$ -uniform traffic. The author defines  $l$ -uniform traffic to be static when each end node transmits  $l$  wavelengths to, and receives  $l$  wavelengths from each other end nodes. Each node in the regular topologies studied is considered to be an end node.  $W_{s,l}$  is used to represent the minimum number of wavelengths that will support  $l$ -uniform static

traffic. Please notice that the  $l = 1$  case corresponds to the case of all-to-all broadcast without the tap-and continue feature. The following upper bounds are derived. For the ring,

$$W_{s,l} = \begin{cases} \left\lceil \frac{l(N^2-1)}{8} \right\rceil, & \text{for } N \text{ odd} \\ \left\lceil \frac{lN^2}{8} \right\rceil, & \text{for } N \text{ even} \end{cases}$$

and for the binary hypercube,

$$W_{s,l} = \frac{lN}{2}.$$

where  $N$  represents the number of nodes in the ring, respectively the hypercube. In the same work, the author provides bounds on the number of wavelengths for arbitrary topologies, and offers solutions for the case of dynamic traffic [65].

Please recall that all references cited so far did not make use of the tap-and-continue capability of nodes.

Other problems of multicast, broadcast and gossiping have been investigated under various assumptions in the context of WDM optical networks [30], [10], [71], [65], [27], [8], [55] and [22]. We will use the terms gossiping and all-to-all broadcast interchangeably. In [71] the authors use Integer Linear Programming to solve the optimal RWA problem for multiple multicasts and their back-up trees. The study in [30] offers an on-line algorithm to route a multicast for the case of maximally edge-connected graphs. It is important to point out that the work in [30] considered Edmonds branching theorem in the online algorithm for constructing a multicast session. Again tap-and-continue was not assumed.

An extended coverage of broadcast and gossiping in optical networks can be found in [10]. The authors present bounds on the number of wavelengths needed for broadcast and gossiping under different conditions (one round, multiple rounds, one hop, and multi-hop) for optical networks with arbitrary topologies.

Without the tap-and-continue feature the following results have been reported and are of particular interest to us. The number of wavelengths for one-to-all communication pattern in arbitrary topologies is found to be bounded by [10]:

$$\left\lceil \frac{n-1}{d_{min}(G)} \right\rceil \leq W_o(G) \leq \left\lceil \frac{n-1}{k} \right\rceil.$$

For the case of maximally edge-connected graphs the bound is:

$$Wo(G) = \left\lceil \frac{n-1}{d_{min}(G)} \right\rceil,$$

where  $Wo$  represents the number of wavelengths needed for one broadcast (one-to-all),  $n$  represents the number of nodes in the network,  $k$  is the edge connectivity and  $d_{min}(G)$  is the minimum node degree [10].

The case of all-to-all broadcast is also evaluated for the arbitrary topologies case and the following results are derived [10]. The bound on the number of wavelengths is:

$$W_A(G) = O\left(\frac{n \log^2 n}{\beta^2(G)}\right),$$

where  $\beta(G)$  represents the edge-expansion and  $W_A$  represents the number of wavelengths needed for all-to-all broadcast. Because for small  $\beta(G)$  the above bound is weak, the authors give a better upper bound on the number of wavelengths for all-to-all broadcast as,

$$W_A(G) = \left\lceil \frac{n(n-1)}{k} \right\rceil,$$

Specific results for the cases of regular topologies are provided as follows [10]. For the ring,

$$W_A(C_n) = \left\lceil \frac{\pi(C_n)}{2} \right\rceil = \left\lceil \frac{1}{2} \left\lceil \frac{n^2}{4} \right\rceil \right\rceil.$$

and for the hypercube,

$$W_A(H_d) = \frac{\pi(H_d)}{2} = 2^{d-1}.$$

The authors also provide mathematically derived bounds on the number of wavelengths for multi-hop routing. Interested readers are referred to [10] for other specific results and the corresponding proofs.

Some specific topologies have also been studied in [65], [27], [8]. However, in these studies the tap-and-continue capability of nodes was not considered.

Most of the research on multi-hop optical networks considered the problem of virtual network design in order to optimize the optical resources for a given traffic matrix. In [49] multi-hop routing is used to design virtual topologies such as to optimize the link load and the communication delay. In [12], a virtual multi-hop topology based on the De-Bruijn graph is constructed, taking the

traffic matrix as input. Another topology design problem for a multi-hop optical network was solved in [37] using an ILP formulation. Along with the multi-hop problem the authors also took into consideration the traffic grooming problem. The work in [70] studied another interesting problem related to multi-hop optical networks, namely the maximum distance for a lightpath until the signal needs to be regenerated. The study in [29] addressed the problem of RWA for a single multicast in multi-hop optical networks. However, the tap-and continue feature was not considered. In [27] one interesting solution to the problem of all-to-all broadcast is given for the ring and torus (2D and 3D) optical network topologies. Again, this study does not take advantage of the tap-and continue feature.

The authors suggest two routing models, a simple model and a merge model. The difference between the models is that in the merge model the messages received at a node are merged together to be transmitted in the next hop on a single wavelength. The main results obtained by the authors is the ring partition into segments and then scheduling the communication for each hop to take place either within the segments or intra-segments. This partitioning is then used for multihop routing in the torus, which is considered an interconnection of rings. The authors mathematically developed the bounds for the number of wavelengths. The number of wavelengths needed for gossiping in  $k$  hops is of the order  $O(N^{1+\frac{1}{k}})$  for the ring,  $O(N^{1+\frac{1}{2k}})$  for the 2D torus and  $O(N^{1+\frac{1}{3k}})$  for the 3D torus, for the simple model [27]. The merge model is evaluated only for 2-hop routing, as it is more mathematically involved [27].

Many other interesting problems related to optical routing and optical switching are detailed in [6], [7], [9], [17], [20], [38] and [89].

Although not related to recent efforts in optical networking, it is important to mention the graph theoretical results in [24] and [32], as they play a major role in developing our results. Reference [24] offered a bound on the number of disjoint branchings in a directed graph, and [32] suggested a very efficient way to represent all the minimum edge-cuts in a given graph. Both results are discussed in detail in Chapter 3.

# Chapter 3

## One Hop Conflict-Free All-to-All Broadcast

### 3.1 Introduction and Problem Definition

In this chapter we consider the problem of one hop, conflict-free, all-to-all broadcast in WDM networks. As mentioned in Chapter 2, this particular communication pattern is of great interest from a control plane perspective. Next, we briefly review the literature in regard to the problem of all-to-all broadcast in optical networks and introduce the problem definition.

In a broadcast operation a single node (called source) sends one message to all other nodes. In an all-to-all broadcast operation all nodes perform broadcasts concurrently, i.e., every node performs a broadcast operation to all other nodes. An optical hop represents a continuous lightpath, or light-tree, with no converters involved. This requires adherence to the wavelength continuity constraint. Conflict-free routing necessitates that no two lightpaths, or light-trees, use the same wavelength, on the same link, in the same direction.

In this thesis we make use of the tap-and-continue feature of optical nodes. Tap-and-continue nodes have recently been utilized in the literature. As stated in [47] such nodes are “an alternative to fully multicast-capable switches” and “can be implemented with only a very modest addition in hardware complexity”. This makes tap-and-continue a very attractive feature for multicast and broadcast type communications. Here we present original solutions to the RWA gossiping problem for different cases of network topologies.

In this chapter we aim to reduce the number of wavelengths needed for conflict-free all-to-all broadcast. We investigate the relationship between the number of wavelengths needed and the network topological properties that may impact the minimum number of wavelengths. We also give routing and wavelength assignment (RWA) algorithms for each case studied. Furthermore we prove the optimality of our solutions for special classes of topologies.

The rest of the chapter is organized as follows. Section 3.2 presents the assumptions and the notations used throughout Chapter 3. Section 3.3 examines special cases of regular topologies, specif-

ically we study the ring, the torus, the binary hypercube and the k-ary n-cube. Section 3.4 considers the case of general arbitrary topologies. Specifically, the case of maximally-edge-connected topologies is studied in Section 3.4.2 and the more involved case of non maximally edge-connected topologies is studied in Section 3.4.3.

## 3.2 Notations and Assumptions

We consider a WDM network with  $N$  nodes. The network is modeled as an undirected graph  $G(V, E)$ , where  $V$  is the set of all vertices in the network and  $E$  is the set of all links. We use  $N = |V|$  to denote the number of vertices in the network. We will use the terms node and vertex interchangeably, unless a supergraph exists. In case of a supergraph we will always use the term vertex for the nodes of the original network graph  $G(V, E)$ , and supernode for a node of a supergraph.

We assume a circuit switched environment without any wavelength converters. This necessitates adherence to the wavelength continuity constraint. We consider bi-directional links such that a link can be used by the same wavelength in the two different directions at the same time. Each wavelength on a unidirectional link between two adjacent nodes will also be referred to as an optical channel. It is further assumed that the intermediate nodes on a lightpath between a source and a destination can tune their receivers to the same wavelength and receive the same message. We make use of the tap-and-continue node capability. Therefore, a node can send the same information to multiple nodes on the same lightpath using the same wavelength.

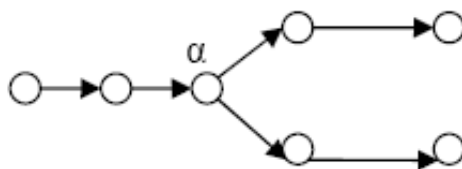


FIGURE 3.1. Split Example at node  $\alpha$ .

We also assume that all nodes are split capable such that a lightpath using a specific wavelength can split the signal at the splitting node into two disjoint sub-paths as shown in Figure 3.1 at node  $\alpha$ . In this case we have a lighttree that uses the same wavelength.

## 3.3 Case Studies: Common Regular Topologies

### 3.3.1 Ring

The ring is the simplest connected regular structure, where node degree is two for all nodes. The ring has been extensively studied in optical networks as mentioned in Chapter 2. SONET (synchronous optical network) mainly uses optical rings in Metropolitan Area Networks. Although the results for the ring are relatively simple, we will see later that these results are very beneficial in solving for other topologies. Next we present the main result on conflict-free all-to-all broadcast in the optical ring.

**Lemma 3.1.** *The total number of wavelengths needed to perform non-blocking all-to-all broadcast in a ring with  $N$  nodes,  $N \geq 4$ , is no greater than  $\lceil \frac{N}{2} \rceil$ . Furthermore,  $\lceil \frac{N}{2} \rceil$  is a tight bound.*

*Proof.* The cases where  $N = 2$  and  $3$  are trivial. For these cases it is easy to see that only one wavelength is needed. Now consider the general case of  $N > 3$ . In [56] it has been shown that the lower bound on the number of wavelengths to achieve non-blocking communications for any permutation in an optical ring with  $N$  nodes and bidirectional links is  $\lceil \frac{N}{2} \rceil$ .

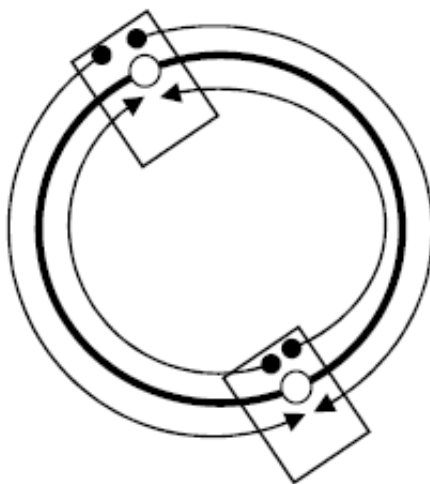


FIGURE 3.2. Two nodes at distance  $\lfloor \frac{N}{2} \rfloor$  using the same wavelength to broadcast to all other nodes.

We now show that the same bound applies for non-blocking all-to-all broadcast. Consider two nodes at distance  $\lfloor \frac{N}{2} \rfloor$ . These two nodes can broadcast using only one wavelength as shown in Figure 3.2. Recall that we assume bi-directional links. Therefore an edge can be used by the same wavelength in opposite directions. If two nodes can broadcast using only one wavelength then

$N$  nodes can broadcast using  $\lceil \frac{N}{2} \rceil$  wavelengths. This can be done by pairing nodes at distance  $\lfloor \frac{N}{2} \rfloor$  and assigning them the same wavelength. Since there is a maximum of  $\lfloor \frac{N}{2} \rfloor$  such pairs and possibly one unpaired node (which would be assigned an additional wavelength), a total of  $\lceil \frac{N}{2} \rceil$  wavelengths are needed. So, no more than  $\lceil \frac{N}{2} \rceil$  wavelengths are needed for non-blocking all-to-all broadcast in the  $N$ -node WDM optical ring with bi-directional links. Since a permutation is a subset of all-to-all communication and since  $\lceil \frac{N}{2} \rceil$  is the lower bound [56], it follows that  $\lceil \frac{N}{2} \rceil$  is also the lower bound for conflict-free all-to-all broadcast.  $\square$

Figure 3.3 shows an example of all-to-all broadcast in a 4-node ring using 2 wavelengths.

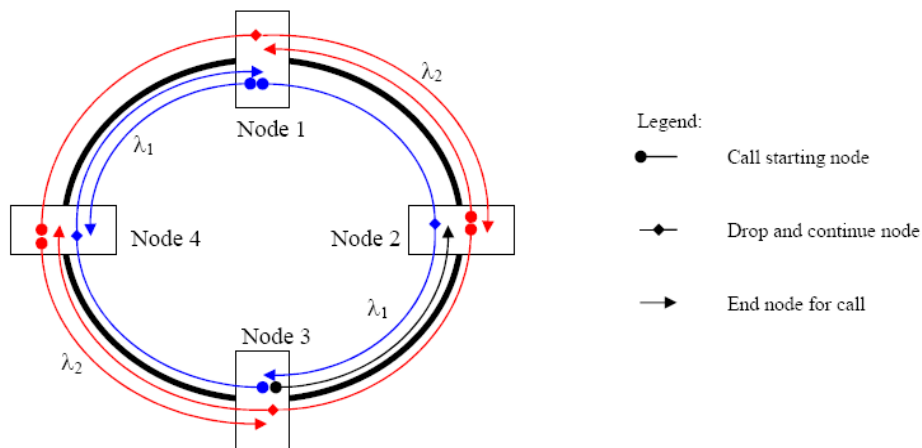


FIGURE 3.3. A 4-node ring performing all-to-all broadcast with 2 wavelengths, nodes 1 and 3 use  $\lambda_1$ , nodes 2 and 4 use  $\lambda_2$ .

Remarks: Please note that pairing diametrically opposite nodes is necessary only for using shortest paths. However, the number of wavelengths remains the same with other pairings if shortest paths are not essential. Please also observe that in the remainder of the chapter some RWA solutions may be implicit in the proofs of our results and thus may not be explicitly given.

**Corollary 3.2.** *Let  $R$  be a subset of nodes in the ring. Also let  $|R| = r$ . The number of wavelengths needed such that each of the  $r$  nodes concurrently broadcasts to all other  $N-1$  nodes without conflict in the ring is  $\lceil \frac{r}{2} \rceil$ .*

**Lemma 3.3.** *In an  $N$ -node WDM ring with  $k$  parallel bidirectional links,  $\lceil \frac{N}{2*k} \rceil$  is a tight bound on the number of wavelengths needed to perform non-blocking all-to-all broadcast.*

*Proof.* Let  $SN$  be the set of  $N$  nodes. Let a ring of  $N$  nodes with one link per edge be called a “simple  $N$ -node ring”. Then one can visualize an  $N$ -node ring with  $k$  parallel links per edge as  $k$  edge-disjoint simple  $N$ -node rings. For each simple  $ring(i)$ ,  $1 < i < k$ , consider selecting a distinct subset of no more than nodes  $SS(i)$  such that:

$$(1) \quad |SS(i)| \leq \lceil \frac{N}{k} \rceil, \quad 0 < i \leq k$$

$$(2) \quad \bigcup_{i=1}^k SS(i) = SN.$$

In each simple ring, a node-pair  $(u, v)$  will require a single wavelength to broadcast to all other  $N - 1$  nodes (see Figure 3.2). Hence for each simple ring, no more than  $\lceil \frac{N}{2 * k} \rceil$  wavelengths will be needed for the nodes in  $SS(i)$  to broadcast to all  $N$  nodes in the ring. Since the simple rings are edge disjoint, it follows that we can make use of wavelength reuse. Therefore the same set of wavelengths can be utilized on all simple rings without conflicts. Thus no more than

$$\lambda = \lceil \frac{N}{2 * k} \rceil$$

wavelengths are needed for all-to-all broadcast on the  $N$ -node ring with  $k$  parallel edges.  $\square$

### 3.3.2 Torus

The 2-dimensional torus (Figure 3.4) can be defined as a  $k$ -ary 2-cube, or a  $k$  by  $k$  mesh with wraparound connections. Thus, the  $k$ -ary torus can be viewed as an interconnection of  $2k$  rings,  $k$  horizontal and  $k$  vertical edge disjoint rings of  $k$  nodes each. Typically the directions on the vertical rings are referred to as the N and S (North and South) directions, and the directions in horizontal rings are referred to as the E and W (East and West).

Next we present a straightforward result, similar to the one for the ring.

**Theorem 3.4.** *The total number of wavelengths needed to perform non-blocking all-to-all broadcast in a Torus with  $N = k^2$  nodes is no greater than  $\lceil \frac{N}{4} \rceil$ . Furthermore, this bound is tight.*

*Proof.* It has been shown in [3] that a 2-dimensional torus can be decomposed into 2 edge-disjoint Hamiltonian cycles. Thus we can see the 2D torus as an  $N$ -node ring with 2 parallel edges. Based on Lemma 3.3, we need  $\lceil \frac{N}{4} \rceil$  wavelengths to perform non-conflict all-to-all broadcast.

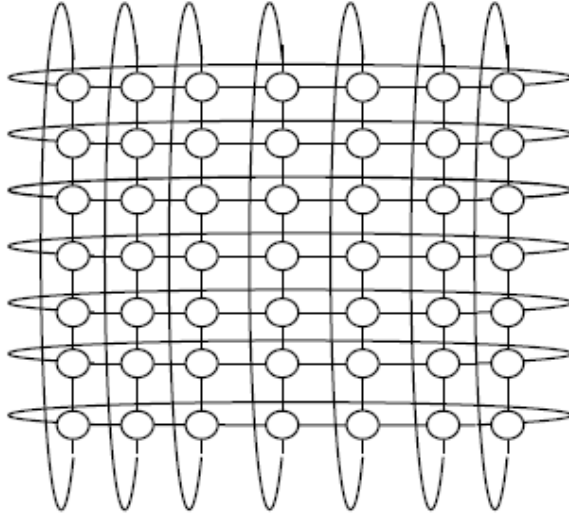


FIGURE 3.4. A 2-dimensional torus topology with  $k = 7$  nodes in each dimension.

We use contradiction to prove that this bound is tight. Assume that one could use  $\lceil \frac{N}{4} \rceil - 1$  wavelengths for conflict-free all-to-all broadcast.

Case a)  $N \bmod 4 = 0$ . In this case  $\lceil \frac{N}{4} \rceil - 1 = \frac{N}{4} - 1$ . The total number of unidirectional links (optical channels) in the torus is  $4 * N = 4 * k^2$ . The total number of optical channels used by a single broadcast is  $N - 1$ . Thus, a number of  $N * (N - 1)$  optical channels are needed for all-to-all broadcast. Using exactly  $\frac{N}{4} - 1$  wavelengths per physical link we get a total of  $(\frac{N}{4} - 1) * (4 * N) = N * (N - 4)$  optical channels, which is less than the number of optical channels needed for performing non-blocking all-to-all broadcast,  $N * (N - 4) < N * (N - 1)$ . Hence, using  $\lceil \frac{N}{4} \rceil - 1$  wavelengths is not possible without blocking.

Case b)  $N \bmod 4 \neq 0$ . Again assume a number that  $\lceil \frac{N}{4} \rceil - 1$  wavelengths are adequate. In this case  $\lceil \frac{N}{4} \rceil - 1 = \lfloor \frac{N}{4} \rfloor$ . All nodes in the torus have a node degree of 4. Thus, any node can receive no more than 4 messages at the same time, on the same wavelength. The floor function  $\lfloor \frac{N}{4} \rfloor$  implies that there will be at least one group of 5 nodes or more, broadcasting using the same wavelength. Thus, this case is also impossible.  $\square$

The RWA algorithm will be simple in this case: Decompose the torus into 2 edge-disjoint rings. Select groups of 4 nodes, 2 in each ring, to broadcast using the same wavelength.

Notice that the routes in the above RWA solution do not follow shortest paths. Therefore, next we concentrate on establishing a bound on the number of wavelengths needed when it is necessary

to use shortest paths in the torus. We will use the N, S and E,W direction notations described above.

We use  $(x, y)$  to denote a node in the torus positioned at the intersection between horizontal ring  $x$ , and vertical ring  $y$ ,  $1 \leq x, y \leq k$ . The minimum distance between two arbitrary nodes  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by:  $d((x_1, y_1), (x_2, y_2)) = \min(|x_2 - x_1|, k - |x_2 - x_1|) + \min(|y_2 - y_1|, k - |y_2 - y_1|)$ . Thus the route from node  $(x_1, y_1)$  to node  $(x_2, y_2)$  following a shortest path will take at least one  $y$  hop, if  $x_1 = x_2$  and  $y_1 \neq y_2$ , at least one  $x$  hop, if  $y_1 = y_2$  and  $x_1 \neq x_2$ , and at least one  $x$  hop and one  $y$  hop if  $x_1 \neq x_2$  and  $y_1 \neq y_2$ .

The following result gives an upper bound on the number of wavelengths for conflict-free all-to-all broadcasting following shortest paths in the  $2D$  torus.

**Theorem 3.5.** *An upper bound on the number of wavelengths for the conflict free all-to-all broadcast in the  $2D$  torus following shortest paths is  $\lceil \frac{N}{3} \rceil$ .*

*Proof.* We prove this theorem by showing that we can select disjoint groups of 3 nodes to broadcast using the same wavelength. Choose 3 nodes  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  satisfying the following three conditions:

$$\begin{aligned} (1) \quad & x_1 = x_2 \\ (2) \quad & d((x_1, y_1), (x_2, y_2)) = \left\lfloor \frac{k}{2} \right\rfloor \\ (3) \quad & d((x_1, y_3), (x_3, y_3)) = \left\lfloor \frac{k}{2} \right\rfloor \end{aligned}$$

The first condition states that the first two nodes  $(x_1, y_1)$ , and  $(x_2, y_2)$  belong to the same horizontal ring. The second condition states that the first 2 nodes are diametrically opposite in the horizontal ring. The third and last condition states that the horizontal ring containing the third node  $(x_3, y_3)$  is at distance  $\lfloor \frac{k}{2} \rfloor$  from the horizontal ring containing the first two nodes. In the following we describe a simple RWA for these three nodes following shortest paths. Please refer to Figure 3.5 for illustration. Node  $(x_1, y_1)$  and node  $(x_2, y_2)$  will use N (S) first and E (W) second in an  $y$ - $x$  routing manner following shortest paths, to broadcast to all nodes outside horizontal ring  $x_1 (= x_2)$ , and except nodes in horizontal ring  $x_3$ . To reach the nodes on horizontal ring  $x_3$ ,

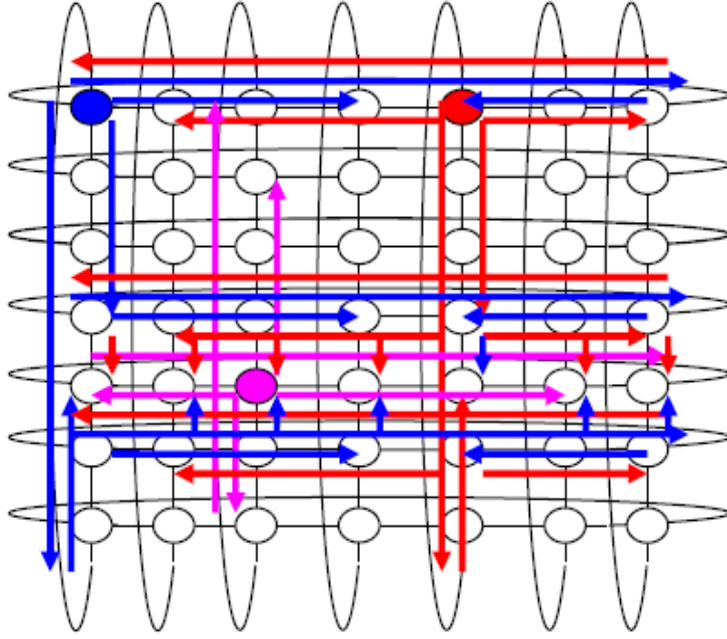


FIGURE 3.5. The blue and red nodes are diametrically opposite on the same horizontal ring. The pink node is vertically diametrically opposite to the blue and red nodes.

the unused links  $\langle (x_3 - 1, y), (x_3, y) \rangle$  and  $\langle (x_3 + 1, y), (x_3, y) \rangle$  are used. Notice that node  $(x_3, y_3)$  can now broadcast with no conflict using the same wavelength following x-y routing, using E (W) links first, and N (S) links second. Thus we can choose groups of 3 nodes to broadcast conflict-free with only one wavelength for a total number of wavelengths of  $\lceil \frac{N}{3} \rceil$ .  $\square$

In summary, we have shown that  $\lceil \frac{N}{4} \rceil$  is a tight bound, when non-shortest paths are allowed. For the case where shortest paths are required, we provided an RWA method that requires no more than  $\lceil \frac{N}{3} \rceil$  wavelengths. An interesting open problem would be to find a tight bound on number of wavelengths for no-conflict all-to-all broadcast following shortest paths,  $\lambda_{sp}$ , such that  $\lceil \frac{N}{4} \rceil \leq \lambda_{sp} \leq \lceil \frac{N}{3} \rceil$ . Please note that the results obtained above remain the same, in the case of meshes with wrap-around connections, and different number of rows and columns,  $k_1 \neq k_2$ ,  $N = k_1 * k_2$ .

In the next section we turn our attention to the binary hypercube, which is a more complex topology in the class of regular topologies.

### 3.3.3 Hypercube

The binary hypercube is an attractive topology that was thoroughly studied in the area of interconnection networks. The  $n$ -dimensional binary hypercube topology is defined a connected topology

with  $2^n$  nodes, where each node is labeled with an  $n$ -bit binary label. Two nodes are directly connected if and only if their labels differ in exactly one bit. Figure 3.6 illustrates a binary hypercube of dimension 4. Thus, the  $n$ -dimensional binary hypercube is another special case of a regular topology where each node has degree  $n$ .

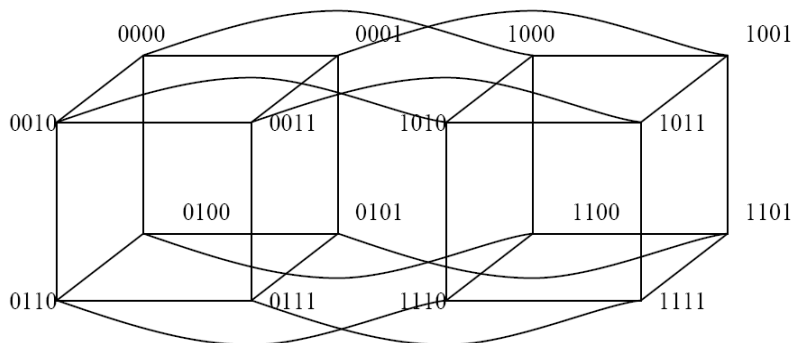


FIGURE 3.6. A 4D Hypercube

For the analysis of all-to-all broadcast in the WDM hypercube, we will use a similar approach to that used for the torus. We start by providing the lower bound on the number of wavelengths required for non-blocking all-to-all broadcast in the binary hypercube along with an elegant RWA algorithm. This section establishes the following important result. Any  $n$ -dimensional hypercube of odd dimensionality,  $n = 2 * k + 1$ , can be decomposed into  $k - 1$  edge disjoint Hamiltonian cycles and an additional edge-disjoint 3-regular structure. Our RWA algorithm makes use of this result. Next we consider the case where using shortest paths is required. We derive the minimum number of wavelengths required for non-blocking all-to-all broadcast in the hypercube using shortest paths, based on the approach presented in [23].

### 3.3.3.1 RWA for Non-Blocking All-to-All Broadcast Using Unrestricted Length Paths

This section solves the RWA problem and establishes a new tight bound on the number of wavelengths needed for all-to-all, non-blocking, broadcast in an  $N = 2^n$  node WDM hypercube. As expected, in order to achieve the minimum number of wavelengths, the routes need not follow shortest paths. In this subsection we will assume that route lengths are not necessarily minimum. The newly found bound is  $\lceil \frac{N}{n} \rceil$  for all  $n > 3$ . To prove this tight bound we treat the cases of

$n$ =even and  $n$ =odd separately. The  $n$ =odd case is more involved than the  $n$ =even case, but both cases share the same exact bound as we will show.

The following result is from reference [54] and is restated here for convenience. Interested readers can find the proof in [54].

**Theorem 3.6.** *An  $N$ -node binary hypercube, where  $N = 2^n$ , can be decomposed into  $\lfloor \frac{n}{2} \rfloor$  edge disjoint  $N$ -node rings.*

Note that according to the above theorem a Hamiltonian cycle in the hypercube is an  $N$ -node ring. Thus there are  $\lfloor \frac{n}{2} \rfloor$  edge-disjoint  $N$ -node rings in an  $N$ -node hypercube. Figure 3.7 illustrates a 16-node Hypercube decomposed into two 16-node disjoint rings. The following result establishes

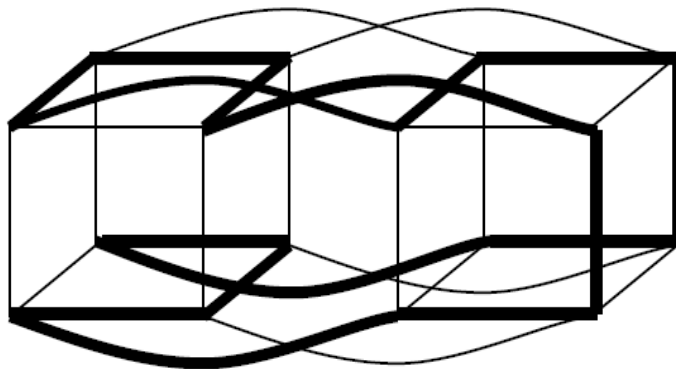


FIGURE 3.7. A 16-node binary hypercube decomposed in 2 edge-disjoint 16-node rings. One ring is shown in bold and one in non-bold.

the general bound applicable to hypercubes of any dimensionality.

**Theorem 3.7.** *The number of wavelengths needed for non-blocking all-to-all broadcast in the  $N$ -node binary hypercube,  $N = 2^n$ , is at most:*

$$\lambda = \left\lceil \frac{N}{2 * \lfloor \frac{n}{2} \rfloor} \right\rceil.$$

*Proof.* Based on Theorem 3.6, an  $N$ -node hypercube can basically be viewed as an  $N$ -node ring with  $\lfloor \frac{n}{2} \rfloor$  parallel bidirectional links. Based on Lemma 3.3,  $\lceil \frac{N}{2*k} \rceil$  wavelengths are enough to perform non-blocking all-to-all broadcast in an  $N$ -node WDM ring with  $k$  parallel links. If  $k = \lfloor \frac{n}{2} \rfloor$  then it follows that the number of wavelengths is at most  $\lambda = \left\lceil \frac{N}{2 * \lfloor \frac{n}{2} \rfloor} \right\rceil$ .  $\square$

### Hypercube of even dimensionality

In each  $N$ -node simple ring we will select a subset of nodes to broadcast to all other nodes in the ring such that the union of nodes selected in all rings is the set of all  $N$  nodes in the hypercube. Thus, the union of all broadcast operations on all rings will result in an all-to-all broadcast pattern for the hypercube. Therefore, for a hypercube of even  $n = 2 * k$ , the newly found bound reduces to:

$$\lambda = \left\lceil \frac{N}{2 * \lfloor \frac{n}{2} \rfloor} \right\rceil = \left\lceil \frac{N}{2 * \lfloor \frac{2*k}{2} \rfloor} \right\rceil = \left\lceil \frac{N}{n} \right\rceil.$$

### Hypercube of odd dimensionality

Please note that for an odd  $n = 2 * k + 1$  the bound would be:

$$\lambda = \left\lceil \frac{N}{2 * \lfloor \frac{n}{2*k+1} \rfloor} \right\rceil = \left\lceil \frac{N}{2 * k} \right\rceil = \left\lceil \frac{N}{n-1} \right\rceil.$$

Now we show that this bound can be improved upon. In this subsection we prove that the bound on the number of wavelengths for non-blocking all-to-all broadcast in an odd dimensional hypercube is also

$$\left\lceil \frac{N}{n} \right\rceil.$$

Consider a hypercube of odd dimensionality  $n = 2 * k + 1$ . It is known [54] that no more than  $k$  edge-disjoint Hamiltonian cycles can be embedded in this  $n$  dimensional hypercube. An odd  $n$  dimensional hypercube can be constructed by connecting two even,  $n - 1$  dimensional hypercubes by links along the  $n$ th dimension. By Theorem 3.6 there are  $k$  edge-disjoint Hamiltonian cycles in each of the two  $n - 1$  dimensional hypercubes. We assume that the Hamiltonian cycles have been already found, based on [8]. Each Hamiltonian cycle (ring) in an  $(n - 1)$  dimensional hypercube will have a counterpart in the other  $(n - 1)$  dimensional hypercube. Let a given cycle (ring) with “0” in bit position  $n$  be called a “top” ring and the corresponding ring (with “1” in bit  $n$ ) be called a “bottom” ring. Therefore, using the edge-disjoint Hamiltonian cycles in the two  $n - 1$  dimensional hypercubes, the odd hypercube of dimension  $n$  can be seen as a collection of

$k$  “cylinders”, where the top  $k$  rings, as well as the bottom  $k$  rings are edge disjoint. For each “cylinder”, the top and bottom rings are connected by the same (non-disjoint) set of edges of dimension  $n$  (see Figure 3.8). Notice that each cylinder will contain two isomorphic rings. A node in the top ring will be connected to a node in the bottom ring which differs from it in only bit  $n$  (the most significant bit).

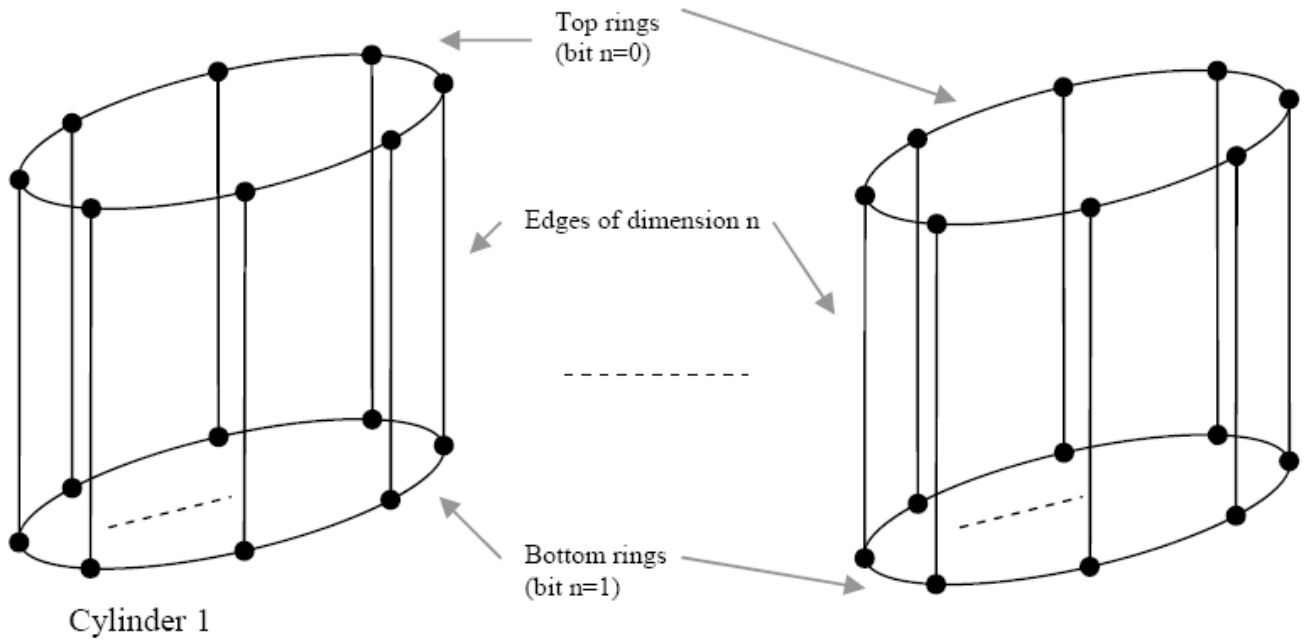


FIGURE 3.8.  $k$  cylinders with  $2^n$  nodes. Top and bottom rings are 2 Hamiltonian cycles in the hypercube of dimension  $n - 1$ . Vertical edges are hypercube edges of dimension  $n$ .

In the following we give a simple method to generate different edge-disjoint Hamiltonian cycles in the  $n$  dimensional hypercube, for the odd  $n$  case. Although a maximum of  $k$  edge-disjoint Hamiltonian cycles can be found, we will use only  $k - 1$  such cycles.

Take the top and bottom rings from one cylinder, say cylinder  $i$ . Denote any two neighboring nodes in the top ring as  $x_i$  and  $y_i$ . Let's say that these nodes are connected to nodes  $x'_i$  and  $y'_i$  in the bottom ring. Because the nodes in the top and bottom rings correspond one to one, it follows that the nodes  $x'_i$  and  $y'_i$  are also neighbors. It is easy to see that by removing the edges  $\langle x_i, y_i \rangle$  and  $\langle x'_i, y'_i \rangle$  and adding the edges  $\langle x_i, x'_i \rangle$  and  $\langle y_i, y'_i \rangle$  we generate a Hamiltonian cycle of  $2^n$  nodes. By doing this to any  $k - 1$  cylinders, such that the  $x_i$  and  $y_i$  node pairs are distinct

in different Hamiltonian cycles, i.e.:  $x_i \neq x_j \neq y_i \neq y_j, \forall 1 \leq i, j \leq k - 1$ , we obtain  $k - 1$  edge disjoint Hamiltonian cycles of  $2n$  nodes (see Figure 3.9).

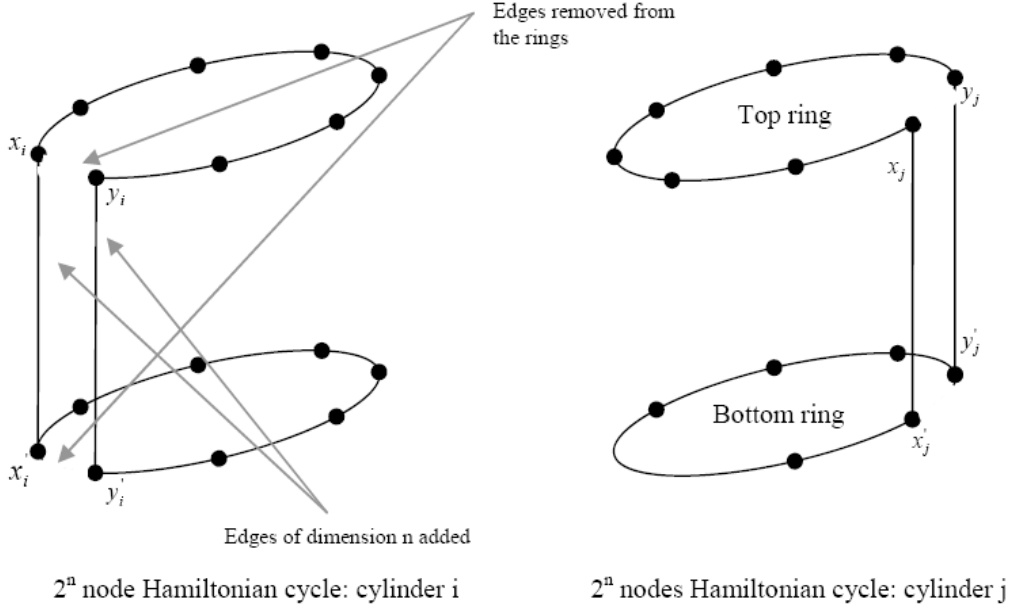


FIGURE 3.9. Edge-disjoint Hamiltonian cycles created from cylinders  $i$  and  $j$ .

Now remove from the hypercube all edges used by the  $k - 1$  Hamiltonian cycles. Each node in the hypercube had an initial node degree of  $n = 2 * k + 1$ . The node degree in a ring just described is 2 (each Hamiltonian cycle is nothing but an  $N$ -node ring). Thus the total number of edges removed at each node is  $2 * k - 2$ . It follows that the resulting node degree for all nodes after removing the edges in the  $k - 1$  Hamiltonian cycles is:

$$n - 2 * (k - 1) = 2 * k + 1 - 2 * k + 2 = 3.$$

Therefore what remains is a topology that is 3-regular and connected. For the remainder of the section we will refer to this topology as the “3-regular structure”.

Based on the above, the following result can be stated.

**Theorem 3.8.** *An  $n$ -dimensional hypercube of odd dimensionality,  $n = 2 * k + 1$ , can be decomposed into  $k - 1$  edge disjoint Hamiltonian cycles plus an additional 3-regular structure with  $N = 2^n$  nodes, which is also edge-disjoint with respect to all other  $k - 1$  Hamiltonian cycles.*

*Proof.* The proof follows directly from the decomposition described above. □

It is interesting to note that the 3-regular structure can be described as follows: The structure contains two rings of  $2^{n-1}$  nodes each. The two rings correspond on one to one basis. Additionally, a node in a given ring is either:

1. connected to its corresponding node on the other ring by a dimension  $n$  edge; or
2. connected to another node in the same ring by an “internal edge”;

An “internal edge” is defined as an edge that connects two nodes in the same ring that are non-adjacent in the ring but are neighbors in the hypercube. The number of nodes in category (1) above is  $2^{n-1} - 2 * (k - 1)$ , whereas the number of nodes in category (2) is  $k - 1$ . Figure 3.10 illustrates the composition of the 3-regular structure.

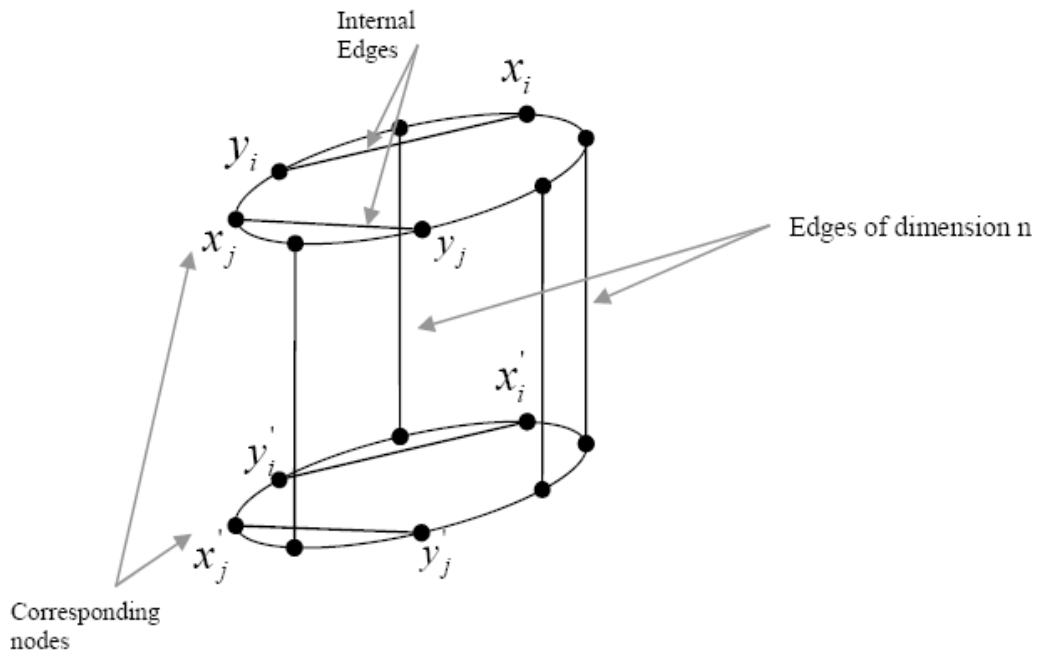


FIGURE 3.10. A schematic for a 3-regular structure, with edges  $\langle x_i, x'_i \rangle$  and  $\langle y_i, y'_i \rangle$  removed and edges  $\langle x_i, y_i \rangle$  and  $\langle x'_i, y'_i \rangle$  added.

Next we utilize this newly discovered property of the hypercube to devise a method for non-blocking all-to-all broadcast in the odd dimensional hypercube using these  $k$  edge-disjoint topologies.

We can select disjoint subsets of nodes to perform broadcast in each of the  $k$  topologies such that the union of all node subsets is the set of all nodes, thus resulting in an all-to-all broadcast in the hypercube. Based on Lemma 3.1, two nodes can broadcast in a ring using only one wavelength.

Assume, for now, that we can select 3 nodes in the 3-regular structure to broadcast using only one wavelength. To minimize the total number of wavelengths, the number of wavelengths used in the  $k - 1$  edge-disjoint rings should be equal to the number of wavelengths used in the 3-regular structure. Let that number of wavelengths be denoted by  $\lambda$ . The number of nodes selected in any one of the  $k - 1$  rings will be  $2 * \lambda$ . Using disjoint node subsets we will have  $2 * \lambda * (k - 1)$  nodes selected in  $k - 1$  rings. The number of nodes selected in the 3-regular structure will be  $3 * \lambda$ . Since we desire an all-to-all broadcast, the union of all broadcasting nodes should equal the total number of nodes in the hypercube, or:

$$2 * \lambda * (k - 1) + 3 * \lambda = 2^n, \text{ where } k = \frac{n - 1}{2}. \text{ Thus,}$$

$$2 * \lambda * \left( \frac{n - 3}{2} \right) + 3 * \lambda = 2^n$$

The resulting number of wavelengths is  $\lambda = \lceil \frac{2^n}{n} \rceil$  which is the desired bound. Based on Corollary 3.2 we know that in a ring of  $N$  nodes, a subset of nodes  $\gamma$  will use  $\lceil \frac{r}{2} \rceil$  wavelengths to perform broadcast, where  $|\gamma| = r$ . Thus the problem for the  $k - 1$  rings is solved. In the following we will focus on the remaining 3-regular structure.

### Algorithm Outline

Our approach for establishing the bounds will be based on selecting  $\lambda$  disjoint groups of 3 nodes each. Each group will use only one wavelength. We will show that such disjoint 3-node groups can always be found and that each of the selected 3-node groups can broadcast in a non-blocking manner using only one wavelength.

Two nodes connected by an internal edge in a ring will be denoted by  $i_r$  and  $j_r$ , for  $1 \leq r \leq k - 1$ , where  $k - 1$  is the total number of internal edges in the ring. The corresponding nodes in the second ring will be denoted by  $i'_r$  and  $j'_r$ .

### **Choosing a 3 – node group.**

For each 3-node group choose two nodes  $S_i$  and  $S'_i$  that are connected by an edge of dimension  $n$ , and any other node that has not been previously selected.

**Theorem 3.9.** *It is always possible to find and select  $\lambda$  disjoint 3-node groups in the 3-regular structure using the node selection procedure presented above, where  $\lambda = \lceil \frac{2^n}{n} \rceil$ .*

*Proof.* As we select an edge of dimension  $n$  per group, it follows that we need at least  $\lambda$  different edges of dimension  $n$ . But because the third node in a group can be selected at random, and this third node could also have an outgoing edge of dimension  $n$ , we will need at most 2 edges of dimension  $n$  per group, for a total of  $2 * \lambda$  edges of dimension  $n$  in the 3-regular structure.

As mentioned above, there will be  $k - 1$  internal edges in each ring belonging to the 3-regular structure, corresponding to the  $k - 1$  edge disjoint Hamiltonian cycles previously constructed. Thus, the total number of available edges of dimension  $n$  in the 3-regular structure is given by:

$$2^{n-1} - 2 * (k - 1) = 2^{n-1} - (n - 3),$$

where  $2^{n-1}$  is the number of dimension  $n$  edges in an  $n$ -dimensional hypercube. But we desire  $\lambda$  to be  $\lceil \frac{2^n}{n} \rceil$ . We would want to show that the number of edges of dimension  $n$  available in the 3-regular structure is greater than the number of edges of dimension  $n$  needed to select  $\lambda$  disjoint groups of 3 nodes. Thus we have to show that the following inequality is always satisfied for  $n \geq 5$ :

$$\frac{2^{n-1} - (n - 3)}{2} \geq \left\lceil \frac{2^n}{n} \right\rceil \quad (3.1)$$

We now prove that inequality 3.1 is true by induction on  $n$ .

For the base case  $n = 5$  we obtain  $7 \geq 7$ .

Assume inequality 3.1 is true for  $n$ . This implies that:

$$\begin{aligned} \frac{2^{n-1} - (n - 3)}{2} &\geq \frac{2^n}{n} + 1. \text{ Thus,} \\ n * 2^{n-1} - n * (n - 3) &\geq 2^{n+1} + 2 * n \end{aligned} \quad (3.2)$$

For  $n + 1$  we obtain:

$$\begin{aligned} (n + 1) * 2^n - (n + 1) * (n - 2) &\geq 2^{n+2} + 2 * (n + 1), \text{ or, using } 2^n = 2^{n-1} + 2^{n-1}, \\ n * 2^{n-1} + n * 2^{n-1} + 2^n - n * (n - 3) - n - (n - 2) &\geq 2^{n+1} + 2^{n+1} + 2 * n + 2 \end{aligned}$$

Adding  $2 * n - n * (n - 3)$  to both sides and rearranging the terms we obtain:

$$n * 2^{n-1} - n * (n - 3) + n * 2^{n-1} - n * (n - 3) + 2^n - n - (n - 2) + 2 * n \geq$$

$$\geq 2^{n+1} + 2 * n + 2^{n+1} + 2 * n + 2 - n * (n - 3) \quad (3.3)$$

Based on 3.2, inequality 3.3 reduces to:

$$2^n - n - (n - 2) + 2 * n \geq 2 - n * (n - 3), \text{ or } 2^n \geq 5 * n - n^2, \text{ which is always true.}$$

Hence the proof. □

Thus, we have shown that it is always possible to select  $\lambda$  disjoint groups of 3 nodes in the 3-regular structure, satisfying our selection criterion when  $\lambda = \lceil \frac{2^n}{n} \rceil$ . Next we present a conflict-free routing strategy for the 3 nodes in any group.

### Routing.

As stated earlier we will follow the approach of selecting groups of 3 nodes. We will then have the three nodes in each group perform concurrent conflict-free broadcasts to all other nodes using a single wavelength. With  $\lambda$  such groups we will collectively have  $3 * \lambda$  nodes perform broadcast to all other nodes using  $\lambda$  wavelengths. To be able to follow the routing procedure we introduce some notation.

As stated earlier the ring whose nodes have '0' in their bit 'n' position is called the "top ring" and the other ring is called the "bottom ring". A node on the top ring will have a corresponding node on the bottom ring. Other than ring connections, a node on the top (bottom) ring is either connected to its corresponding node in the bottom (top) ring or is connected to another node on the top (bottom) ring by an internal link (edge). Each group (g) will consist of 3 nodes: node  $S_{ig}$  on the top ring, its corresponding node  $S'_{ig}$  on the bottom ring (which is connected to it by an edge of dimension  $n$ ) and a third node on the top ring which we call  $S_{3g}$ . For clarity we will omit the  $g$  subscript in the subsequent discussion.

Refer to Figure 3.11. Starting at node  $S_i$ , we traverse the ring in the clockwise direction until we encounter the first node  $i_r$  at which there is an internal edge  $\langle i_r, j_r \rangle$ . Denote the node on the top ring just before  $j_r$  in the clockwise direction as  $x_{j_r}$ . Let the nodes on the bottom ring corresponding to  $i_r$ ,  $j_r$  and  $x_{j_r}$  be denoted as  $i'_r$ ,  $j'_r$  and  $x'_{j_r}$ , respectively. We will use  $i_r$  and  $j_r$  notation to generically denote nodes at the ends of internal edges and  $x_{j_r}$  to denote the node just before  $j_r$  in the clockwise direction. The reader should take  $i_r$ ,  $j_r$  and  $x_{j_r}$  as node types and not

as individual nodes. When we say nodes of type  $i_r$  we mean all nodes on the top ring which are at the start of an internal edge. Similar meaning applies to  $j_r$  and  $x_{j_r}$ .

We now construct the three broadcast trees starting with the one for  $S_i$ . Because of symmetry the broadcast tree for  $S'_i$  will be similar as we elaborate later.

For source  $S_i$  start routing on the top ring in the clockwise direction.

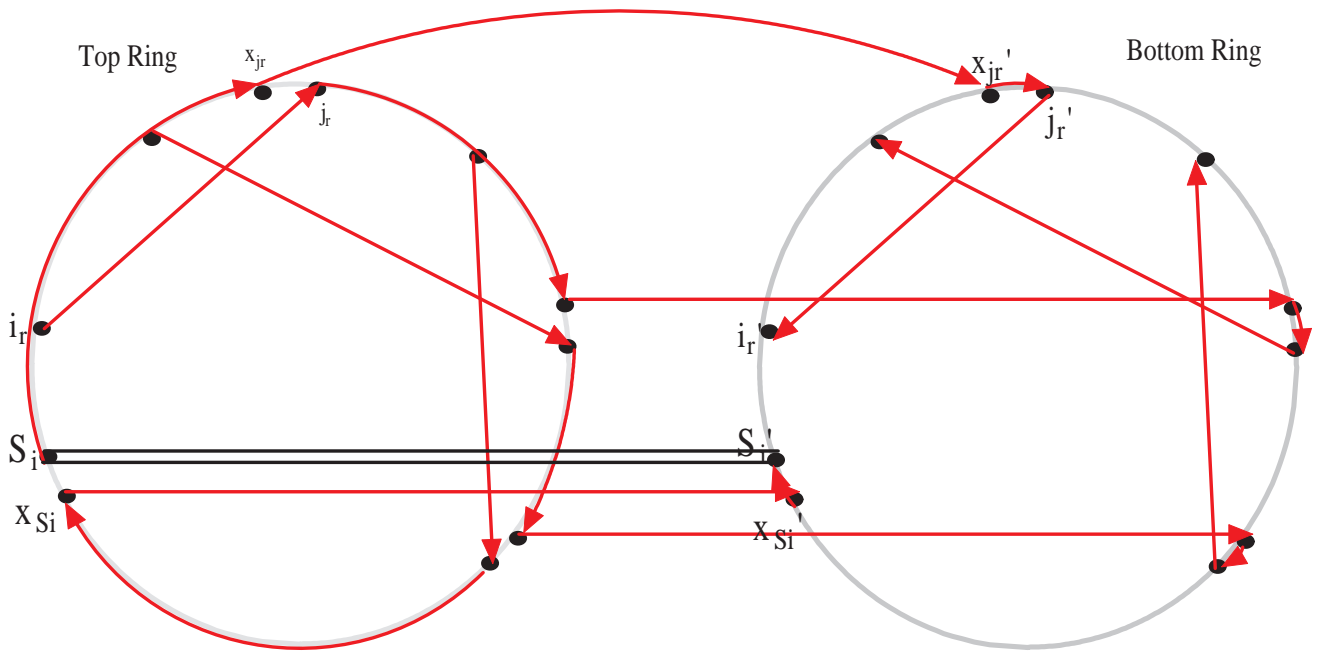
- Use all the edges in the top ring except edges of type  $\langle x_{j_r}, j_r \rangle$  and the edge  $\langle x_{S_i}, S_i \rangle$ .
- Whenever an edge of type  $\langle i_r, j_r \rangle$  is encountered on the top ring that edge is traversed.
- Use all edges of dimension  $n$  except edge  $\langle S_i, S'_i \rangle$  to send the message to nodes on the bottom ring. Notice that nodes  $i'_r, j'_r$  and  $S'_i$  of the bottom ring will not receive the message. To reach those nodes use edges of type  $\langle x'_{j_r}, j'_r \rangle$  to reach nodes  $j'_r$  and edges of type  $\langle j'_r, i'_r \rangle$  to reach nodes of type  $i'_r$ . Finally we use edge  $\langle x'_{S'_i}, S'_i \rangle$  to reach node  $S'_i$ . This completes the construction of the broadcast rooted at  $S_i$ .

For source node  $S'_i$ , apply the same routing strategy as that for node starting on the bottom ring, in the clockwise direction, as illustrated in Figure 3.11. For source node  $S_3$ , choose another node on the top or (bottom) ring, start routing counter-clockwise until all nodes in the first ring are covered. Use edge  $\langle S_i, S'_i \rangle$  (edge  $\langle S'_i, S_i \rangle$ ) to split the message to the other ring. On the second ring starting with node  $S'_i$  (node  $S_i$ ), that just received the message, route the message counter-clockwise until all nodes in the second ring are covered. This completes the broadcast tree for the third node. Figure 3.11 shows such a group of 3 nodes in a general 3-regular topology with 3 internal edges.

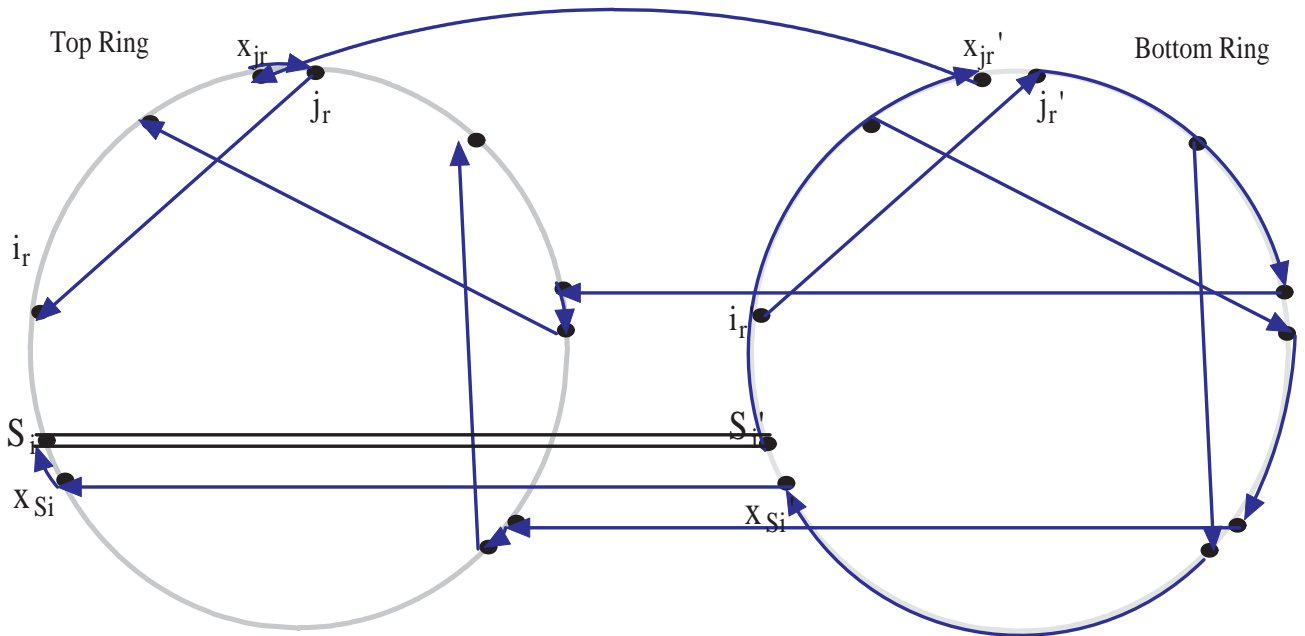
Please note that one may equally well choose to route counter-clockwise from nodes  $S_i$  and  $S'_i$ , and clockwise from the 3rd node.

**Theorem 3.10.** *The routing algorithm presented above is correct. Only one wavelength is needed for each 3-node group (selected using our procedure) to perform non-blocking concurrent broadcasts in the 3-regular structure. Furthermore there will be no conflict between groups.*

*Proof.* Assume the broadcast tree for node  $S_i$  has been constructed using the algorithm presented above. The edges used are:



Broadcast from  $S_i$ . Edge  $S_i S_i'$  not used. All other edges of dimension  $n$  are used, direction from top ring to bottom ring.



Broadcast from  $S_i'$ . Edge  $S_i' S_i$  not used. All other edges of dimension  $n$  are used, direction from bottom ring to top ring.

FIGURE 3.11. Shown are the broadcast trees for  $S_i$  (in red) and  $S_i'$  (in blue). The counter-clockwise direction is available on both rings, and edge  $\langle S_i, S_i' \rangle$  is unused for the third node broadcast.

- all the edges on the top ring in the clockwise direction, except edges of type  $\langle x_{j_r}, j_r \rangle$  and edge  $\langle x_{S_i}, S_i \rangle$ ;
- edges of type  $\langle i_r, j_r \rangle$ ;
- all edges of dimension  $n$ , except edge  $\langle S_i, S'_i \rangle$ ; in the direction from top to bottom;
- edges of type  $\langle x'_{j_r}, j'_r \rangle$ ,  $\langle j'_r, i'_r \rangle$  and edge  $\langle x'_{S'_i}, S'_i \rangle$  on the bottom ring.

Now assume that the broadcast tree for node  $S'_i$  is constructed. The edges used are:

- all the edges on the bottom ring in the clockwise direction, except edges of type  $\langle x'_{j_r}, j'_r \rangle$  and edge  $\langle x'_{S'_i}, S'_i \rangle$ ;
- edges of type  $\langle i'_r, j'_r \rangle$ ;
- all edges of dimension  $n$ , except edge  $\langle S'_i, S_i \rangle$ ; in the direction from bottom to top;
- edges of type  $\langle x_{j_r}, j_r \rangle$ ,  $\langle j_r, i_r \rangle$  and edge  $\langle x_{S_i}, S_i \rangle$  on the top ring.

It is easy to see now that the first 2 nodes selected,  $S_i$  and  $S'_i$  use all the edges on the rings in the clockwise direction, all the edges of dimension  $n$  except edge  $\langle S_i, S'_i \rangle$  and edge  $\langle S'_i, S_i \rangle$  and all ring internal edges in both directions. The 3rd node, chosen as any other node from the top (bottom) ring routes counter-clockwise on the ring edges and uses the unused  $n$  dimension edge  $\langle S_i, S'_i \rangle$  ( $\langle S'_i, S_i \rangle$ ) to split the message from one ring to the other. It uses ring edges in the opposite directions with respect to the direction used by  $S_i$  and  $S'_i$ , and one unused dimension  $n$  edge. Thus the selected 3 nodes can perform non-blocking broadcast concurrently using a single wavelength such that each edge is used at most 2 times but no more than once in each direction. Furthermore, each group of 3 nodes so defined uses its own distinct wavelength. Thus there will be no conflict between groups. Therefore the algorithm presented above is correct.  $\square$

### Optimality.

In the following we prove that the bound of  $\lambda = \lceil \frac{2^n}{n} \rceil$  is also the lower bound. Since it is also achievable, as we have shown, the bound is tight.

**Theorem 3.11.**  $\lambda = \lceil \frac{2^n}{n} \rceil$  is the lower bound on the number of wavelengths to perform conflict-free all-to-all broadcast in a hypercube of dimension  $n$ .

*Proof.* The total number of nodes in the hypercube is  $N = 2^n$ . All we have to show is that it is impossible to perform all-to-all broadcast in an  $N$ -node hypercube with less than  $\lceil \frac{2^n}{n} \rceil$  wavelengths. We prove the theorem by contradiction.

Suppose that we can perform non-blocking all-to-all broadcast using  $\lceil \frac{2^n}{n} \rceil - 1$  wavelengths. The total number of bi-directional links in the hypercube is  $\frac{N*n}{2}$ . If we consider each bi-directional link as two unidirectional links we will have a total of  $N * n$  unidirectional links in the hypercube. The number of optical channels used by a single node to broadcast using drop and continue (regardless of the algorithm used) is exactly  $N - 1$  (since the message must reach  $N - 1$  distinct nodes). Therefore the minimum number of optical channels needed to perform all-to-all broadcast in the hypercube is  $N * (N - 1)$ . To use the assumed maximum of  $\lceil \frac{2^n}{n} \rceil - 1$  wavelengths on each physical link will result in a maximum of  $(\lceil \frac{2^n}{n} \rceil - 1) * (N * n)$  optical channels. However,

$\lceil \frac{2^n}{n} \rceil$  can be expressed as:

$$\lceil \frac{2^n}{n} \rceil = \begin{cases} \frac{N - N \bmod n}{n} + 1 & \text{for } N \bmod n \neq 0 \\ \frac{N}{n} & \text{for } N \bmod n = 0 \end{cases}$$

Case a)  $N \bmod n \neq 0$ . This implies that the maximum number of optical channels used is

$$\frac{N - N \bmod n}{n} * (N * n) = N^2 - N \bmod n * N = N * (N - N \bmod n)$$

But the minimum number of optical channels needed is  $N * (N - 1)$ . For  $N \bmod n \geq 1$ ,  $N * (N - N \bmod n) < N * (N - 1)$ . It follows that the maximum number of optical channels in this case would not be adequate to perform non-blocking all-to-all broadcast in the hypercube.

Case b)  $N \bmod n = 0$ . In this case the maximum number of optical channels is

$$\left( \frac{N}{n} - 1 \right) (N * n) = N * (N - n).$$

But  $N * (N - n) < N * (N - 1)$ , for  $n > 1$ . Again, here the total number of optical channels is not adequate to perform non-blocking all-to-all broadcast in the hypercube. Since  $n$  is always greater

than 1 by assumption it follows that it is not possible to use fewer than  $\lceil \frac{2^n}{n} \rceil$  wavelengths. Hence the proof.  $\square$

### **3.3.3.2 Bound on the Number of Wavelengths for a Shortest Paths RWA Method**

In this subsection we derive the minimum number of wavelengths required by global conflict-free information exchange in the binary hypercube following shortest paths. We start with a brief description of the results reported in [23], and then we derive the minimum number of wavelengths required by this method.

The work in [23] presented a very attractive method for non-blocking all-to-all broadcast in the binary hypercube, following shortest paths. First, a common assumption in the literature is that the same broadcast tree is to be used by all nodes. Based on this assumption, the following very important necessary condition for conflict-free routing is established: “In cases where the same structure of broadcast tree is used by all nodes, no dimension can be used more than once by any lighttree, if non-blocking shortest paths routing is to be achieved in binary hypercubes”. The authors also provide the necessary framework for constructing a “universally used broadcast tree (UUBT)” that is to be used by all nodes, and also present the main steps for an RWA algorithm. In the following we review some of the notations used in [23] and will be needed here. The authors use the following notations. The broadcast tree constructed by the algorithm is called a special broadcast tree (SBT). An MST (major subtree) is defined to be a lighttree contained in the SBT. Recall that the same SBT has to be used by all nodes.

We restate for convenience the main conditions that need to be satisfied in order to achieve non-blocking all-to-all broadcast. Interested readers are referred to [23], for a complete theoretical description of the method.

1. “No dimension will be repeated in any major subtree (MST).
2. The collection of major subtrees will cover all the nodes except the source (i.e. total of  $N-1$  nodes).

3. Each major subtree MST will use a distinct wavelength to deliver the message to all the nodes in that MST.
4. The same SBT is to be used by all nodes.”

Next we derive the minimum number of wavelengths required by this method.

### **Bound on the number of wavelengths**

Condition 1 ensures that routing is done using shortest paths only. Condition 2 ensures that the operation results in a complete broadcast. Condition 3 ensures that any given MST is a lighttree. All four conditions combined ensure that all-to-all broadcast is non-blocking.

We use  $B_n$  to denote the lower bound on the number of MSTs.

**Problem formulation:** Find  $B_n$  such that  $2^n - 1$  distinct nodes are covered by  $B_n$  MSTs.

Remark 1. Each MST covers at most  $n$  nodes since no dimension is to be repeated in any MST.

Remark 2. In an  $n$ -dimensional hypercube, the source of a broadcast will have  $\binom{n}{1}$  nodes at distance 1,  $\binom{n}{2}$  nodes at distance 2, ...,  $\binom{n}{k}$  nodes at distance  $k$ , and  $\binom{n}{n} = 1$  node at distance  $n$ . Please recall that, for a given SBT, we refer to the nodes at distance  $k$  from the source as the nodes in “level  $k$ ”.

Case (a):  $n^2 > 2^n - 1$

Based on Remark 1, a maximum of  $n^2$  nodes can be covered using  $n$  MSTs. Based on Remark 2, there are  $n$  nodes at distance one from the source (at level one). Thus,  $n^2$  distinct nodes can be covered using  $n$  MSTs. Therefore  $B_n$  is given by:

$$B_n = \left\lceil \frac{N - 1}{n} \right\rceil.$$

This is true for the cases of  $n=2, 3$  and  $4$ . Notice that  $B_n = n$  for  $n=2, 3$  and  $4$ . This is true because each subtree will reach up to  $n$  distinct nodes and  $n$  subtrees are possible. Thus,  $2n - 1$  nodes can be reached using  $n$  subtrees. Please also notice that this is also the bound we found in the previous sections. Thus we conclude that conflict-free all-to-all broadcast in the hypercubes of dimensionality  $n=2, 3$  and  $4$ , can be achieved using an optimal number of wavelengths.

Case (b):  $n^2 < 2^n - 1$

This is the case where  $n$  MSTs cannot cover all nodes in the hypercube for a complete broadcast. First suppose that we need  $n + \delta$  MSTs. The nodes covered by these MSTs will be no more than  $(n + \delta) * n$  based on Remark 1. However, based on Remark 2, there are only  $n$  distinct nodes at level 1 (distance one). Thus, any additional MST, will have to repeat one of the  $n$  nodes at level 1. In turn, the maximum number of nodes covered does not equal the maximum number of distinct nodes covered.

Consider for example the first dimensional hypercube where nodes will have to be repeated, which is 5 dimensional ( $n = 5$  or  $N = 32$ ). In this case the first five MSTs will cover  $5 \times 5 = 25$  distinct nodes (obviously  $< 31$ ). Therefore we must repeat some nodes at level 1.

We follow a simple recursive reasoning to determine the minimum number of MSTs for the general case of  $n$ -dimensional hypercube,  $n \geq 5$ .

Let  $X_k$  be defined as the maximum number of distinct nodes that can be covered by  $\binom{n}{k}$  (MSTs).

- $\binom{n}{1}$  MSTs can cover a maximum of  $\binom{n}{1} * n = n * n$  distinct nodes. Thus,  $X_1 = n * n$ .
- The maximum number on distinct nodes covered by  $\binom{n}{2}$  MSTs,  $X_2$  can be found as follows. The first  $\binom{n}{1}$  MSTs can cover  $X_1$  distinct nodes. Each of the  $\binom{n}{2} - \binom{n}{1}$  remaining MSTs can cover a maximum of  $(n - 1)$  distinct nodes. Thus  $X_2$ , the maximum number of distinct nodes covered by  $\binom{n}{2}$  MSTs is given by:

$$\begin{aligned} X_2 &= \left[ \binom{n}{2} - \binom{n}{1} \right] * (n - 1) + X_1 = \left[ \binom{n}{2} - n \right] (n - 1) + n * n = \\ &= \binom{n}{2} * (n - 1) + \binom{n}{1} \end{aligned}$$

- To find  $X_3$  we follow a similar approach. Notice that any additional MST, added to the  $\binom{n}{2}$  MSTs, will have to repeat nodes at level 2, since there are only distinct nodes at level 2, as stated in Remark 2. Thus, to find  $X_3$  we can say that: the first  $\binom{n}{2}$  MSTs will cover a maximum of  $X_2$  distinct nodes. The remaining  $\binom{n}{3} - \binom{n}{2}$  MSTs can cover a maximum of  $(n - 2)$  new and distinct nodes. Thus  $X_3$  is given by:

$$x_3 = \left[ \binom{n}{3} - \binom{n}{2} \right] * (n - 2) + X_2 = \dots$$

$$= \left[ \binom{n}{3} - \binom{n}{2} \right] * (n-2) + \binom{n}{2} * (n-1) + \binom{n}{1} = \binom{n}{3} * (n-2) + \binom{n}{2} + \binom{n}{1}$$

- Generalizing, with  $\binom{n}{k}$  branches and no repeated nodes at level  $k$ , we can cover a maximum of  $X_k$  nodes, where:

$$X_k = \left[ \binom{n}{k} - \binom{n}{k-1} \right] * (n-k+1) + X_{k-1}$$

Based on a similar logic as above,

$$X_k = \binom{n}{k} * (n-k+1) + \binom{n}{k-1} + \binom{n}{k-1} + \dots + \binom{n}{2} + \binom{n}{1} \text{ or}$$

$$X_k = \binom{n}{k} (n-k+1) + \sum_{i=1}^{k-1} \binom{n}{i} \quad (3.4)$$

Now assume that for a given  $k$  we get:

$$X_{k-1} < 2^n - 1 \leq X_k \quad (3.5)$$

The relation in 3.5 represents the case where  $\binom{n}{k-1}$  MSTs cannot cover all the nodes in the hypercube, whereas  $\binom{n}{k}$  MSTs would suffice. Thus, it is obvious that the minimum number of MSTs (therefore the minimum number of wavelengths) needed is:

$$B_n = \binom{n}{k-1} + \left\lceil \frac{(2^n - 1) - X_{k-1}}{n - k + 1} \right\rceil \quad (3.6)$$

Where the first term represents the first  $\binom{n}{k-1}$  MSTs and the second term represents the additional MSTs needed to cover the remaining nodes.

Thus,  $B_n$  can be determined as follows.

First, find level  $k$  for which relation 3.5 is satisfied. Then by substituting 3.4 in 3.6,  $B_n$  can be obtained as:

$$B_n = \binom{n}{k-1} + \left\lceil \frac{(2^n - 1) - \left[ \binom{n}{k-1} (n-k+2) + \sum_{i=1}^{k-2} \binom{n}{i} \right]}{n - k + 1} \right\rceil, \quad (3.7)$$

where  $B_n$  represents the minimum number of wavelengths (MSTs) such that all-to-all conflict free broadcast is achieved.

## Case studies

Case  $n = 5$

First consider the case of constructing a broadcast tree (SBT) for the  $n = 5$  dimensional hypercube. We have to satisfy the 4 conditions mentioned above. With 5 MSTs we can cover a maximum of 25 distinct nodes. Thus, the rest of  $31-25=6$  distinct nodes have to be placed in new MSTs. The number of distinct nodes at level 2 from a source in a 5 dimensional hypercube is  $\binom{5}{2} = 10$ . Thus, each of the next  $\binom{5}{2} - \binom{5}{1} = 5$  MSTs can cover a maximum of 4 new distinct nodes. It follows that the 6 remaining nodes have to be placed in  $\lceil \frac{6}{4} \rceil = 2$  new MSTs. Thus, for the 5 dimensional hypercube, the number of MSTs is 7. Recall that each MST uses a different wavelength. Figure 3.12 shows an example of a special broadcast tree for the 5 dimensional hypercube.

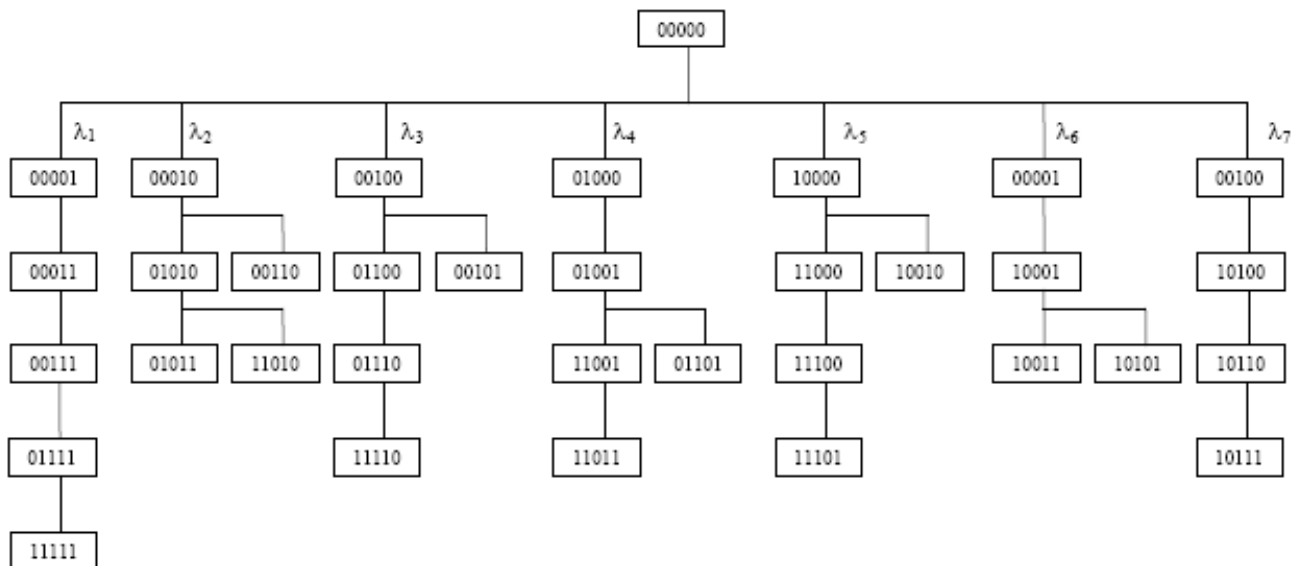


FIGURE 3.12. Special Broadcast Tree for a 5 dimensional hypercube, rooted at node with binary label “0”.

Please notice that this SBT is rooted at the node with the binary label “0” (zero). Each box represents the binary label of that node. Condition 4 above requires the same SBT to be used by all other source nodes. Figure 3.13 shows the same SBT rooted at a generic source “S”. The node boxes represent in this case the dimensions that have to be traversed from the source to reach that specific node.

Case  $n = 6$

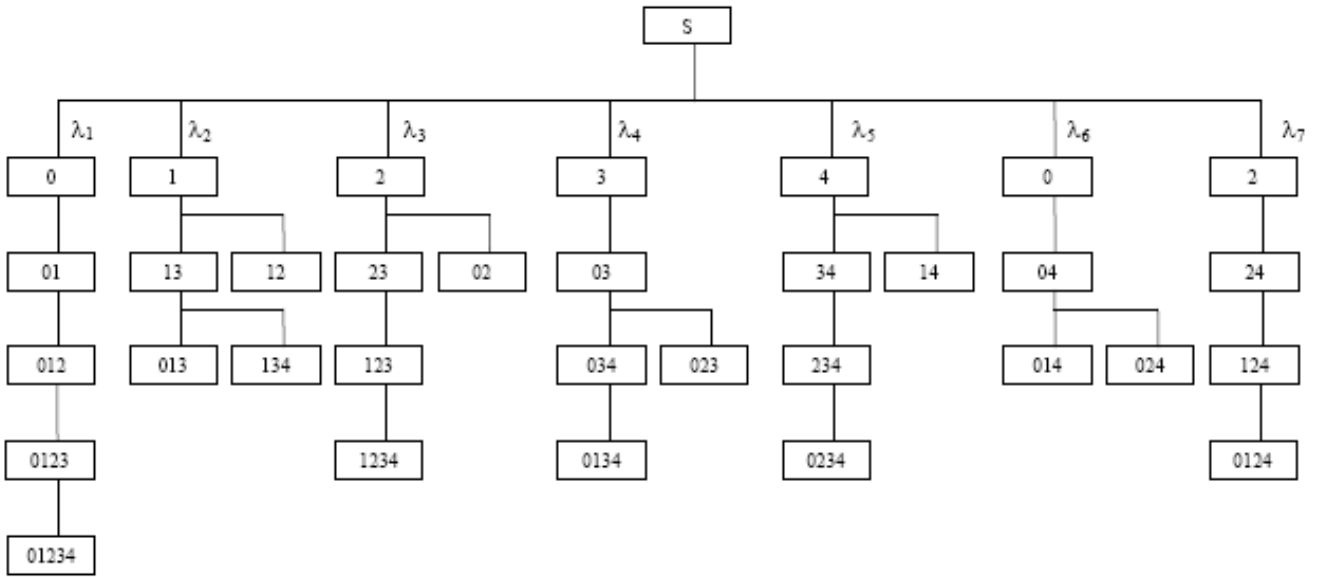


FIGURE 3.13. Special Broadcast Tree for a 5 dimensional hypercube routed at a generic source S.

Now please consider the more evolved case of a 6 dimensional hypercube. With 6 MSTs we cover a maximum on 36 distinct nodes. The number of distinct nodes at distance 2 from the source is  $\binom{6}{2} = 15$ . Thus we can use a maximum of  $15-6=9$  MSTs, each able to cover 5 distinct nodes, until repeating nodes at level 2. The number of nodes left to cover is  $63-36=27$ . Thus a number of  $\lceil \frac{27}{5} \rceil = 6$  additional MSTs are required. Thus a total of 12 MSTs (wavelengths) are required for constructing an SBT for the hypercube of dimension 6. An example of an SBT for the 6 dimensional hypercube is depicted in Figure 3.14. Again, for convenience this SBT shows the binary labels of the nodes.

Please notice, that several other SBTs can be found for each dimensional hypercube. However, once an SBT is found, based on condition 4 this SBT has to be used by all other source nodes.

The case of a 7 dimensional hypercube is an extreme case. There are 127 nodes to be covered. Using the above reasoning, 49 nodes are covered by the first 7 MSTs. There are  $\binom{7}{2} = 21$  distinct nodes at distance 2.  $B_n$  formula gives 20 MSTs needed, thus no nodes will be repeated at level 2. We can easily observe that  $127-49=78$  distinct nodes have to be covered by 13 new MSTs, each able to cover at most 6 new distinct nodes. Thus, all 20 MSTs will cover the maximum number of possible distinct nodes. The interested reader can find an example of an SBT for a 7 dimensional hypercube in reference [23].

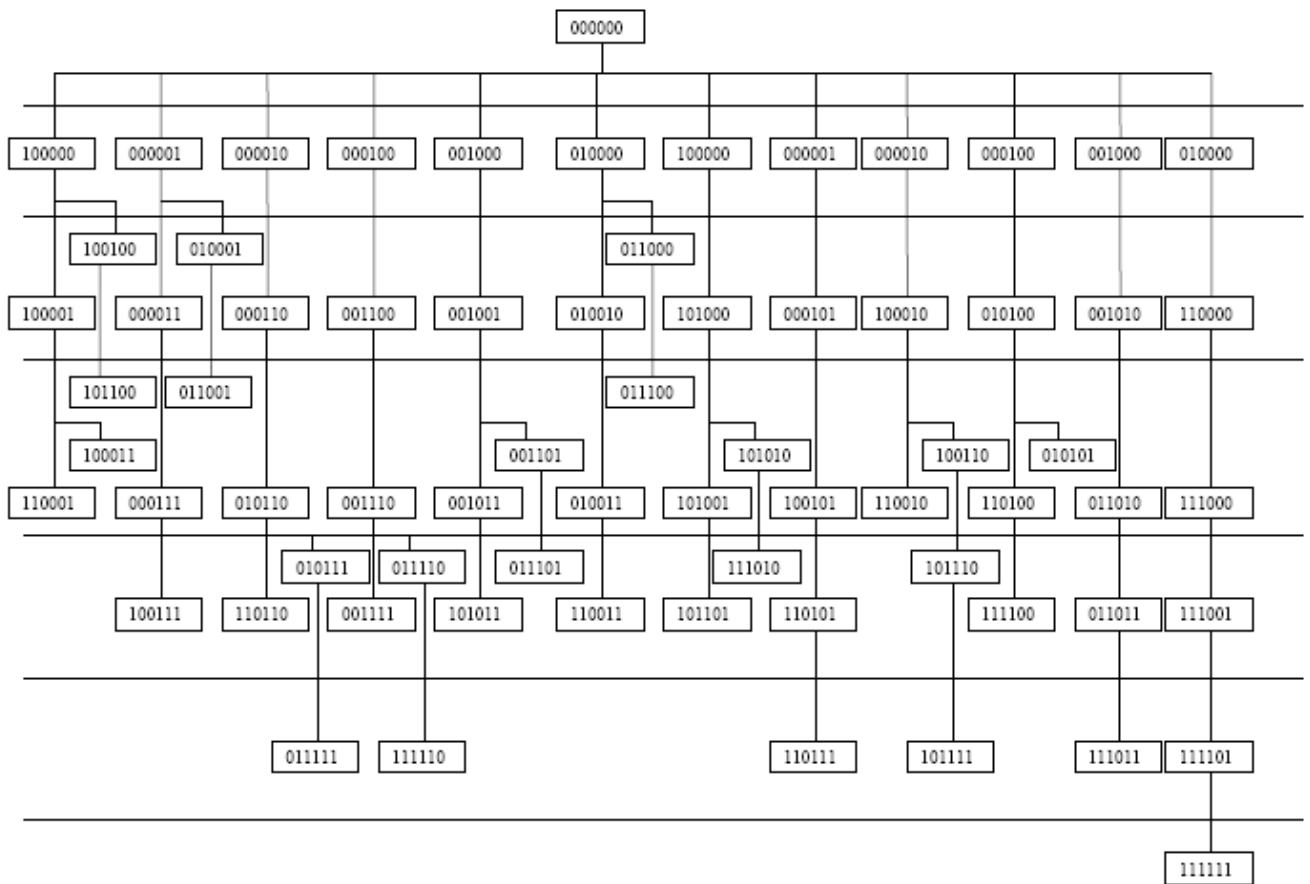


FIGURE 3.14. Special Broadcast Tree for a 6 dimensional hypercube, rooted at node with binary label “0”.

### 3.3.4 $k$ -ary $n$ -cube Discussion

The  $k$ -ary  $n$ -cube is a generalization of the regular topologies discussed thus far. It can be best described as an  $n$ -dimensional hypercube, with  $k$  nodes on each dimensional edge, with wrap-around connections. Thus, the  $N$ -node ring is nothing but a  $N$ -ary 1-cube; the 2D Torus is a  $k$ -ary 2-cube, and the  $n$ -dimensional binary hypercube is a 2-ary  $n$ -cube with no wrap-around connections.

Interestingly, the existence of the wrap-around connections makes the node degrees in the  $k$ -ary  $n$ -cube to be always even. A unified result for the conflict-free all-to-all broadcast on the  $k$ -ary  $n$ -cube would incorporate all results found so far for the ring, torus and hypercube. Following the pattern, we would assume that the tight bound on the number of wavelengths for conflict free all-to-all broadcast for the  $k$ -ary  $n$ -cube is

$$\lambda = \left\lceil \frac{N}{2 * n} \right\rceil,$$

where  $N$  is the total number of nodes in the  $k$ -ary  $n$ -cube,  $N = k^n$ . To prove it, and to follow a simple and elegant RWA as we did previously, we would also assume that there are  $n$  edge-disjoint  $N$ -node rings in the  $k$ -ary  $n$ -cube. To the best of our knowledge there is no such result concerning the  $k$ -ary  $n$ -cube. One particular result can be found in [3], where the special case of the  $k$ -ary  $n$ -cube, with  $n = 2^r$  a power of two, is shown to have  $n$  disjoint hamiltonian cycles. However, in Section 3.4 we will see that  $\lambda = \lceil \frac{N}{2*n} \rceil$  is indeed a tight bound for the  $k$ -ary  $n$ -cube, but there is no systematic way to determine the RWA, as for the ring, torus and hypercube cases.

We also assume that this bound is not achievable when the routes are constrained to follow shortest paths. An interesting open problem would be to find a tight (or upper) bound on the number of wavelengths for conflict-free all-to-all broadcast in the  $k$ -ary  $n$ -cube, following shortest paths.

Next section generalizes the results obtained thus far and proposes a novel way to solve the problem of conflict-free all-to-all broadcast in general topologies. For specific topology cases we will obtain a tight bound on the number of wavelengths.

## 3.4 General Arbitrary Topologies

In this section we consider the problem of conflict-free all-to-all broadcast (gossiping) in optical networks with general arbitrary topologies, and no wavelength converters.

First we present some preliminaries used in developing the bound. Next we present a tight bound on the number of wavelengths for the maximally edge-connected graphs. Then, a method to find the minimum number of wavelengths for maximally edge-connected graphs is presented. Subsection 3.4.3.2 revisits the method of cactus decomposition of a graph.

### 3.4.1 Definitions, Notations and Preliminaries

A broadcast operation consists of a single message issued by a source node and sent to all other nodes in the network. We define the “all-to-all broadcast” (gossiping) operation as a set of concurrent broadcasts from all nodes in the network. We consider a circuit switched environment and must therefore satisfy the wavelength continuity constraint [28]. We also consider bi-directional links such that the same wavelength can be used on the same link in opposite directions. Nodes are assumed to be tap-and-continue capable [47] such that intermediate nodes on a lightpath can also

receive the message. Furthermore, we assume each node to be split-capable such that a lightpath from an incoming link can be split into multiple outgoing links at the respective node.

We model the network as an undirected graph  $G(V, E)$ , where  $V$  is the collection of all vertices in the network and  $E$  is the collection of all links. We use  $N = |V|$ , to denote the number of vertices in the network. Each bi-directional link between two vertices  $x$  and  $y$  can be replaced by 2 directed links in opposite directions denoted as:  $\langle x, y \rangle$ , and  $\langle y, x \rangle$ . For the resulting digraph we use the notation  $G(V, Ed)$ , where  $|Ed| = 2 * |E|$ .

We assume that the connectivity of the network is known. For digraphs, network connectivity  $k$  corresponds to the minimum number of links whose deletion will disconnect the network into 2 strongly connected components [84].

An  $r$ -branching in a directed graph is defined as a rooted tree “branching out” from vertex  $r$ . Vertex  $r$  has indegree of 0, all other vertices have indegree of 1, and all other vertices are reachable from  $r$  [84]. We define  $\kappa(r; G)$ , the local connectivity of a vertex  $r$ , to be the minimum number of edges whose deletion makes some vertex unreachable from  $r$ . For a given vertex  $r$ , we use the notation  $d_{out}$  and  $d_{in}$  for the out-degree and in-degree, respectively. Notice that when converting an undirected graph to a directed one, the in-degree of any given vertex will equal the out-degree of that vertex. Thus for any vertex  $r$ ,  $d_{out} = d_{in}$ . We also use  $\delta$  to denote the minimum vertex degree, and  $\Delta$  to denote the maximum vertex degree.

Next we review some results from the literature on which we will rely in establishing the lower bound on the number of wavelengths. The next lemma is presented as a “remark” in [26].

**Lemma 3.12.** *If the connectivity of  $G(V, E)$  is  $k$ , then the connectivity of  $G(V, Ed)$  is also  $k$ .*

Our approach is based on extending a result by Edmonds [24] to the broadcast in optical networks. This result is restated in the following theorem.

**Theorem 3.13.** *For a vertex  $r$  in a digraph  $G$ , the maximum number of pair-wise edge-disjoint  $r$ -branchings in  $G$  is  $\kappa(r; G)$ .*

Based on the definition of  $r$ -branching, it follows that  $\kappa(r; G)$  is the maximum number of edge-disjoint spanning trees rooted at  $r$  [84].

The proof of the theorem can be converted to an algorithm for finding the  $\kappa(r; G)$  pair-wise disjoint spanning trees [84]. Another algorithm for finding  $\kappa(r; G)$  spanning trees was given by Tarjan in 1975 [75].

We will also make use of a result in [84] that states that:  $k \leq \kappa(r; G) \leq \delta$ . We derive the following result, which will prove useful.

**Lemma 3.14.** *Consider the construction of maximum number of edge-disjoint spanning trees rooted at a given vertex  $r$ . If  $d_{out}(r) = \kappa(r; G)$ , vertex  $r$  will have only one descendant vertex (child) in each of the  $\kappa(r; G)$  edge-disjoint spanning trees.*

*Proof.* We prove this lemma by contradiction. Assume that at least one of the  $\kappa(r; G)$  edge-disjoint spanning trees rooted at  $r$  will have 2 descendant vertices from  $r$ . In this case the number of distinct outgoing edges would have to be at least  $2 + \kappa(r; G) - 1 = \kappa(r; G) + 1$ . However, this contradicts the assumption that  $d_{out}(r) = \kappa(r; G)$ .  $\square$

The above result does not affect the generality of our approach, as shall be seen. We will augment the network graph with a single virtual vertex that satisfies Lemma 3.14, which will later be removed. We now develop the following result which will be essential in using our RWA method for conflict-free gossiping.

**Theorem 3.15.** *Consider an arbitrary directed graph  $G^d(V, Ed)$  with  $N$  vertices and connectivity  $k$ . Choose any arbitrary set  $T$  of vertices from  $V$ ,  $|T| \leq k$ . We can find  $|T|$  edge-disjoint spanning trees rooted at the nodes in the set  $T$ .*

*Proof.* We prove this theorem by construction. Take the graph  $G^d(V, Ed)$ . We know that network connectivity is  $k$ . Select any set  $T = \{x_1, x_2, x_3 \dots x_\gamma\}$  of vertices  $|T| = \gamma \leq k$ , in the  $N$ -node network. Add a new vertex  $S$  to the graph and connect it to the  $\gamma$  selected vertices using newly added directed edges  $\langle S, x_1 \rangle, \dots \langle S, x_\gamma \rangle$ . The newly obtained graph is  $G_S^d(V \cup S, Ed \cup (\langle S, x_1 \rangle, \dots \langle S, x_\gamma \rangle))$ . Because the connectivity of  $G$  is  $k \geq \gamma$ , and the newly added vertex  $S$  has  $\gamma$  outgoing edges, it follows that  $\kappa(S; G_S^d) = \gamma$ . Based on Edmonds' Theorem 3.13 we can find  $\gamma$  pair-wise edge-disjoint spanning trees rooted at  $S$ . We also know that  $d_{out}(S) = \gamma = \kappa(S; G_S^d)$ .

Based on Lemma 3.14,  $S$  will have only one descendant vertex in each spanning tree, and these descendants are  $x_1 \dots x_\gamma$ . Denote these spanning trees as  $ST_1, ST_2, \dots, ST_\gamma$ , respectively. If we remove the edge  $\langle S, x_1 \rangle$  from  $ST_1$ , we obtain a tree rooted at  $x_1$ . This is a spanning tree rooted at  $x_1$  in the original graph  $G^d(V, Ed)$ . By doing this to all spanning trees  $ST_i, 1 \leq i \leq \gamma$  rooted at  $S$  we obtain  $\gamma$  edge-disjoint spanning trees rooted at  $x_1, x_2, \dots, x_\gamma$  in  $G^d(V, Ed)$ .  $\square$

Figure 3.15 shows the application of the previous theorem on a 6-node directed graph with connectivity  $k = 2$ . refer to Figure 3.15. Part (a) represents the original graph. In part (b) a virtual node “ $S$ ” has been added. In part (c) there are two edge-disjoint spanning trees rooted at  $S$ . In part (d) the virtual node and the virtual edges have been removed. There are 2 edge-disjoint spanning trees rooted at nodes  $x_1$  and  $x_2$ .

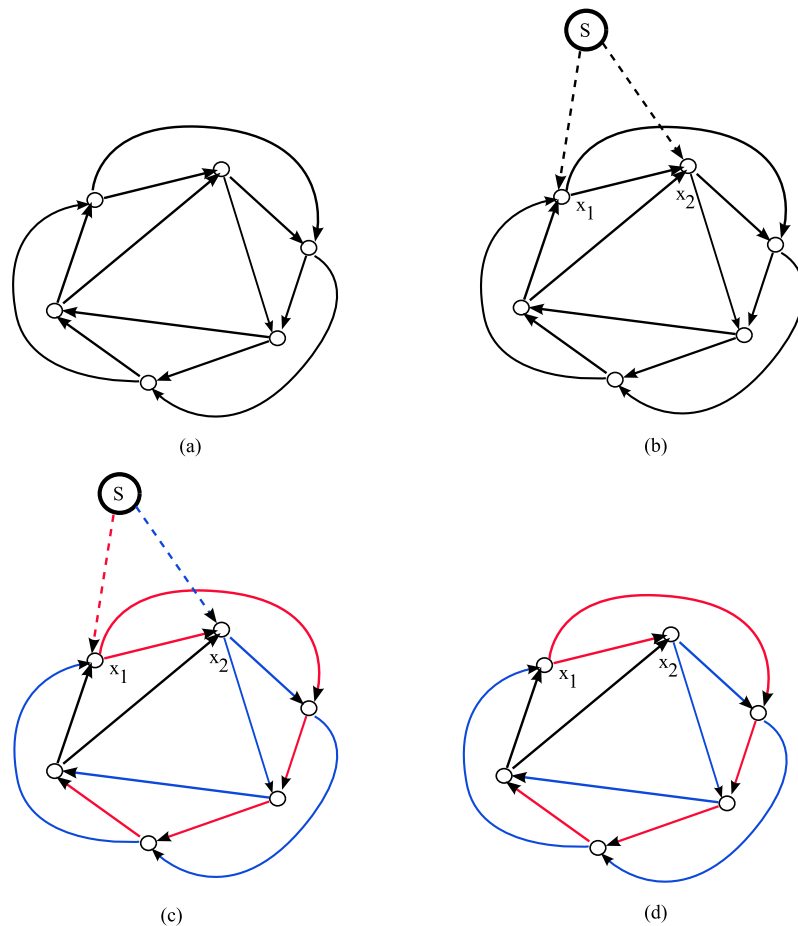


FIGURE 3.15. An example showing how to find 2 edge-disjoint trees. (a) the original graph; (b) the graph augmented with a virtual node and 2 virtual edges; (c) two edge-disjoint spanning trees rooted at  $S$ ; (d) two edge-disjoint spanning trees rooted at  $x_1$  and  $x_2$  after removing the virtual node.

**Theorem 3.16.** *Consider an optical network  $G(V, E)$  with connectivity  $k$  and  $N > k$  vertices. We can choose any  $k$  nodes to perform concurrent non-blocking broadcast, using only one wavelength.*

*Proof.* Convert the optical network  $G(V, E)$  to a directed topology  $G(V, Ed)$ . Based on Lemma 3.12, the connectivity is also  $k$ . Thus  $G(V, Ed)$  is nothing but a special case of an arbitrary directed graph  $G^d(V, Ed)$ . Based on Theorem 3.15, we can choose any subset of  $k$  nodes to construct  $k$  edge-disjoint spanning-trees. Consider each spanning tree to be a lighttree and all lighttrees to use the same wavelength. The theorem follows.  $\square$

### 3.4.2 Case of Maximally Edge-Connected Topologies

Theorem 3.16 leads to the straightforward result of RWA for conflict-free gossiping in optical networks presented next. In the following we use  $V_\delta = \{V_i \in V | d(V_i) = \delta\}$  to denote the set of all vertices of minimum degree.

**Theorem 3.17.** *Conflict-free all-to-all broadcast in an  $N$ -node optical network with connectivity  $k$  can be realized in one hop using no more than*

$$\lambda = \left\lceil \frac{N}{k} \right\rceil \quad (3.8)$$

*wavelengths.*

*Proof.* Based on Theorem 3.16 we can select any  $k$  nodes to broadcast with one wavelength. Thus, selecting disjoint groups of  $k$  nodes each we get the stated bound.  $\square$

Next we show that the above bound on the number of wavelengths is a tight bound in the case of maximally edge-connected topologies, for all cases except one special topological case. Consider the special topological case (STC) which satisfies:

- (a)  $|V_\delta| \leq k + 1$ .
- (b) Given virtual node  $S$ , and  $k + 1$  virtual directed edges  $\langle S, v_i \rangle$ ,  $v_i \in V_\delta$  such that all nodes in  $V_\delta$  are connected to  $S$ . Then  $\kappa(S; G_S) = k + 1$ .
- (c)  $k$  divides  $(N - 1)$ .

We consider the special topological case to exist if all (a), (b) and (c) are satisfied. The next result gives the lower bound on the number of wavelengths for the case of maximally edge-connected topologies.

**Theorem 3.18.** *The lower bound on the number of wavelengths for conflict-free all-to-all broadcast (gossiping) in an  $N$ -node maximally edge-connected optical network with connectivity  $k=\delta$ , is:*

$$\lambda = \left\lceil \frac{N}{k} \right\rceil - 1, \text{ for special topological case (STC)} \quad (3.9)$$

$$\lambda = \left\lceil \frac{N}{k} \right\rceil, \text{ otherwise} \quad (3.10)$$

*Proof.* Based on Theorem 3.17, equation 3.10 is satisfied. If  $|V_\delta| > k + 1$ , then we cannot select any node-group with more than  $k$  nodes to broadcast with one wavelength. So if (a) and (b) are satisfied, then based on Theorem 3.16 we can select  $k + 1$  nodes to broadcast with one wavelength. Please note that this would be the only group that has more than  $k$  nodes. If the number of remaining  $N - k - 1$  nodes is a multiple of  $k$ , then (c) is satisfied. Thus the total number of node groups (wavelengths) is

$$\begin{aligned} 1 + \left\lceil \frac{N - k - 1}{k} \right\rceil &= 1 + \left\lceil \frac{N - 1}{k} \right\rceil - 1 = \\ &= \left\lceil \frac{N - 1}{k} \right\rceil = \left\lceil \frac{N}{k} \right\rceil - 1. \end{aligned}$$

which is exactly equation 3.9. □

Theorem 3.18 provided a very interesting result in equation 3.9 above. If the virtual node  $S$  satisfies some specific conditions, then one group of more than  $k$  vertices can be selected, such that all vertices within the group broadcast using one wavelength. We will make extensive use of this property in the next subsection.

### 3.4.3 Case on Non-Maximally Edge-Connected Topologies: $\delta > k$

#### 3.4.3.1 Preliminaries

The proofs for Theorem 3.16 and Theorem 3.17 can be easily used to find the Routing and Wavelength Assignment for all-to-all broadcasts. Each group of  $k$  nodes is assigned a wavelength. The routes (broadcast trees) for the vertices in each group are given in [75] using Theorem 3.16. Thus the lower bound developed in Theorem 3.18 becomes a tight bound, for the case of maximally

edge-connected topologies. As we shall see in the following, the case of non-maximally edge-connected topologies is more involved. We will refer to this case as the  $\delta > k$  case. Our approach is based on a similar concept to the “cactus” representation of all minimum cuts in a graph first proposed in [32]. Before describing our RWA method for the  $\delta > k$  case, we present some preliminary results. Then we restate the main properties of all minimum edge-cuts and the principle of the “cactus” representation.

Consider the simple network topology example presented in Figure 3.16 (a). The network has  $N = 15$  nodes, connectivity  $k = 2$ , and the minimum degree  $\delta = 4$ . Based on Theorem 3.17, no more than  $\lambda = \lceil \frac{N}{k} \rceil = 8$  wavelengths are needed to perform non-blocking all-to-all broadcast. Theorem 3.17 also gives the lower bound on the number of wavelengths for the  $\delta = k$  case. But in this case, we shall see that only 4 wavelengths are necessary to perform conflict-free all-to-all broadcast.

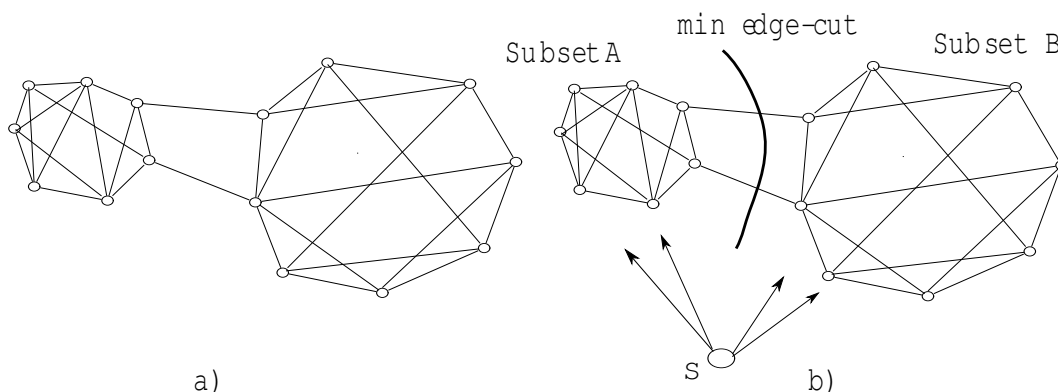


FIGURE 3.16. A  $\delta > k$  network example with 15 nodes, where connectivity is 2, and only 4 wavelengths are required for conflict-free all-to-all broadcast

Figure 3.16.(b) shows that there is only one minimum edge-cut of  $k = 2$  that partitions the set of  $V$  vertices into the subsets, say ‘Subset A’ and ‘Subset B’. It is easy to observe that the addition of a virtual supernode  $S$  connected to any 2 nodes in ‘Subset A’ and any 2 nodes in ‘Subset B’ will make the local connectivity of the virtual supernode  $S$ ,  $\kappa(S, G_S) = 4$ . Thus, based on Theorem 3.13 (Edmond’s), there are 4 edge-disjoint spanning trees starting at  $S$ . Based on Lemma 3.14 and Theorem 3.16, the four nodes connected to  $S$  can broadcast using only one wavelength. Because we can choose “any” 2 nodes on each side, it turns out that there will be a

total of four groups of nodes: three groups of 4 nodes and one group of 3 nodes, nodes within each group broadcasting with one wavelength, for a total of 4 wavelengths.

The previous example is very simple with only one minimum edge-cut. The network was partitioned into 2 sub-networks, each subnetwork being almost a complete graph. Other factors that can influence the minimum number of wavelengths may include: the number of minimum edge-cuts, the number of nodes in each partition and the “internal” connectivity of each partition; a term that will be defined later. A very good tool to tackle this problem is a structure called the cactus representation of all minimum edge-cuts. This structure and its properties are briefly described next.

### 3.4.3.2 All Minimum Edge-Cuts and the Cactus Representation

The cactus representation has received little publicity and is relatively unknown in the western world, as pointed out in [52]. The first attempt to characterize all minimum edge-cuts in a cactus representation structure is given in [19]. However, the article has not been translated to English, to the best of our knowledge. [32] provided the first algorithm to find the cactus representation in polynomial time. A very good description of the work in [19] and [32] can be found in [13] and [63].

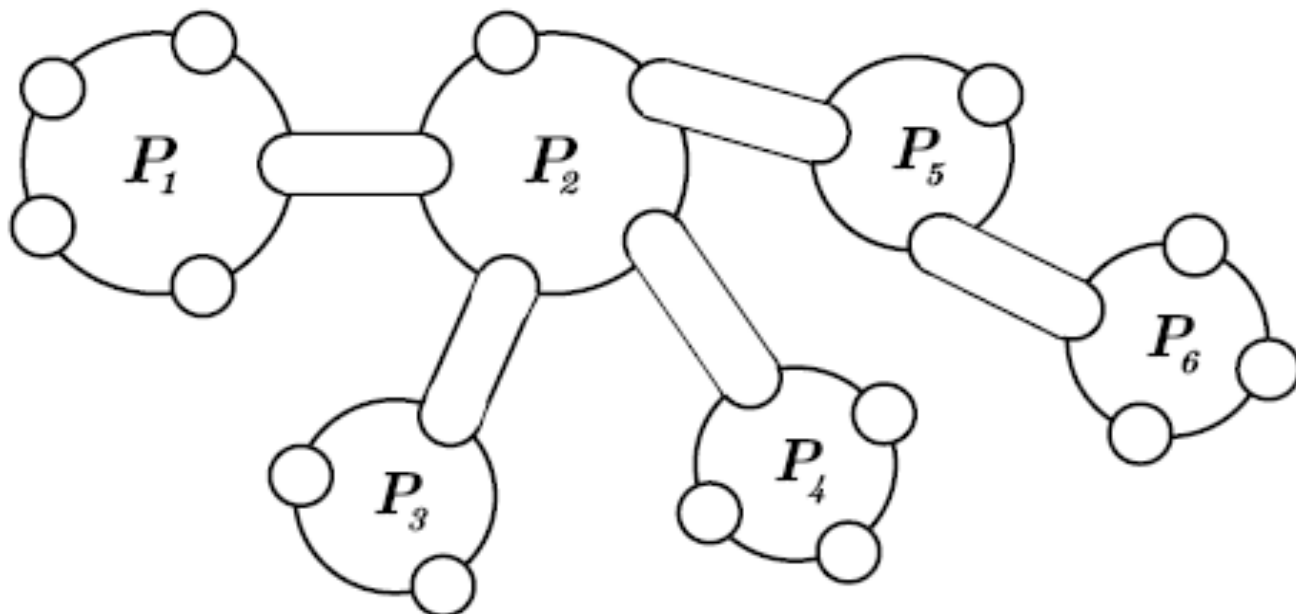


FIGURE 3.17. A cactus representing the circular partition cuts of 6 circular partitions

A cactus representation of a graph  $G(V, E)$  is a graph structure  $\mathcal{H}(G)$ . We will use the term “supernode” to refer to the nodes in  $\mathcal{H}(G)$ , and the term vertex to refer to a node in the set  $V$  of

$G(V, E)$ . Each vertex in  $V$  is mapped to a unique supernode of  $\mathcal{H}(G)$ . Each supernode in  $\mathcal{H}(G)$  is either a collection of connected vertices from  $V$ , or an empty set. Each minimum cut in  $\mathcal{H}(G)$  represents a minimum cut in  $G(V, E)$ .  $\mathcal{H}(G)$  is either a tree, or a tree of rings [13], if there are circular partitions. Figure 3.17 gives an example of a cactus structure (reproduced from [13]).

The edges in the cactus representation are either tree edges, or ring edges. Each edge in  $\mathcal{H}(G)$  is part of at most one cycle. The edges in  $\mathcal{H}(G)$  have well-defined weights such that a tree edge has weight  $k$  and a ring edge has weight  $\frac{k}{2}$ . The maximum number of nodes in  $\mathcal{H}(G)$  is  $2|V| - 1$ . Any minimum edge-cut in  $G$  can be found by removing a tree edge, or a pair of ring edges from the same ring, in  $\mathcal{H}(G)$ .

The cactus representation of any connected and undirected graph  $G$  always exists [53], [19]. A graph  $G$  may have several distinct cactus representations. A unique cactus representation is obtained if more constraints are imposed on the empty nodes of the cactus. Such a cactus is called “canonical” and has been proven to be unique [50], [51]. An example of a canonical cactus is given in Figure 3.18 (reproduced from [63]). Please notice that this example illustrates a maximally edge-connected graph with  $k = \delta = 4$ .

As we shall see, any constraints on the empty nodes of the cactus will not affect the generality of the RWA method, we present later.

### 3.4.3.3 Routing and Wavelength Assignment (RWA) for $\delta > k$ Case

We are now ready to describe our RWA method for conflict-free all-to-all broadcast in one hop, for the case where  $\delta > k$ . Our goal is to reduce the number of wavelengths. The method we describe is recursive and is based on the cactus representation. We start by providing some preliminary definitions and results.

**Property 3.19.** *Any circular partition of  $p$  supernodes in  $\mathcal{H}(G)$  can be reduced to a star graph with  $p$  supernodes connected to a central virtual (empty) node. The weight for a star edge is double the weight of a circular edge.*

The above property is valid only for our purpose of optical conflict free all-to-all broadcast using drop and continue, in the sense that both structures require the same number of wavelengths. Thus a ring of  $N$  nodes and unary edge weights can be viewed as a star with  $N$  nodes and one central

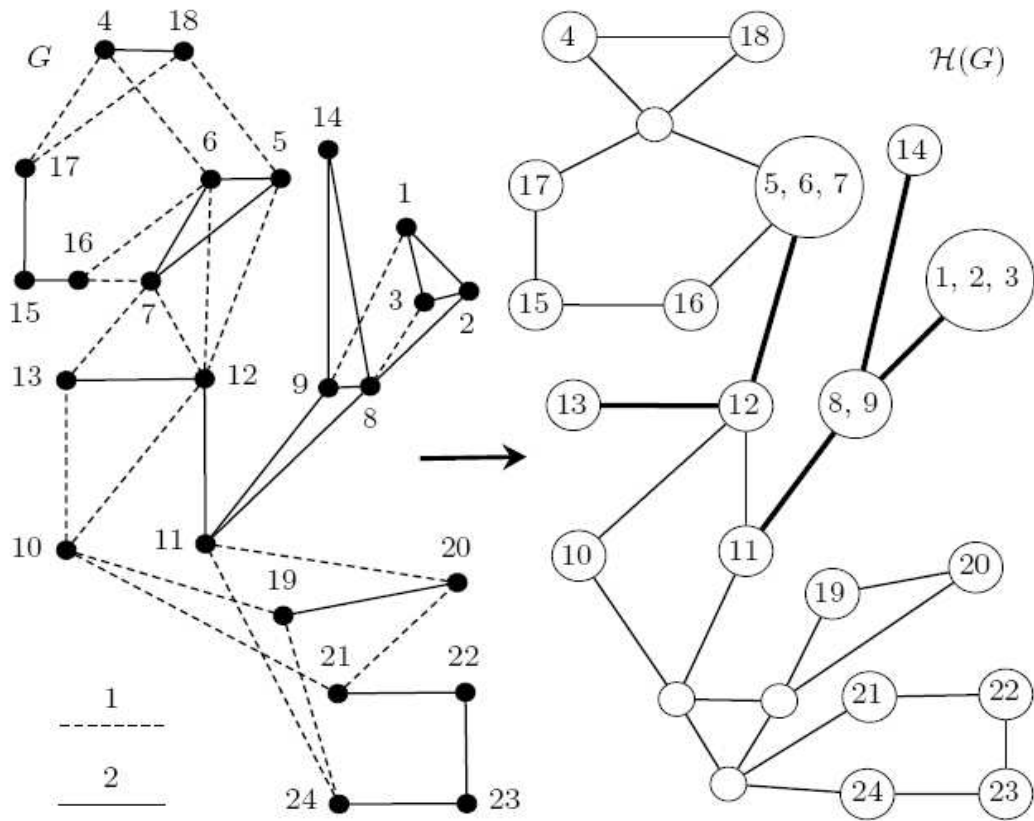


FIGURE 3.18. A graph  $G$  with 24 vertices and its canonical cactus  $\mathcal{H}(G)$  with 23 nodes. The connectivity  $k = 4$ . The continuous edges in  $G$  have weight 2, the dashed edges have weight 1.

empty node, and edges of weight 2 (see Figure 3.19). Please observe that the contraction of ring edges into star edges, indeed doubles their weight. Although the routing paths are different for a star, the two structures require the same number of  $\frac{N}{2*k}$  wavelengths, where  $N$  is the number of vertices contained in all the supernodes, and  $k$  is the minimum edge-cut.

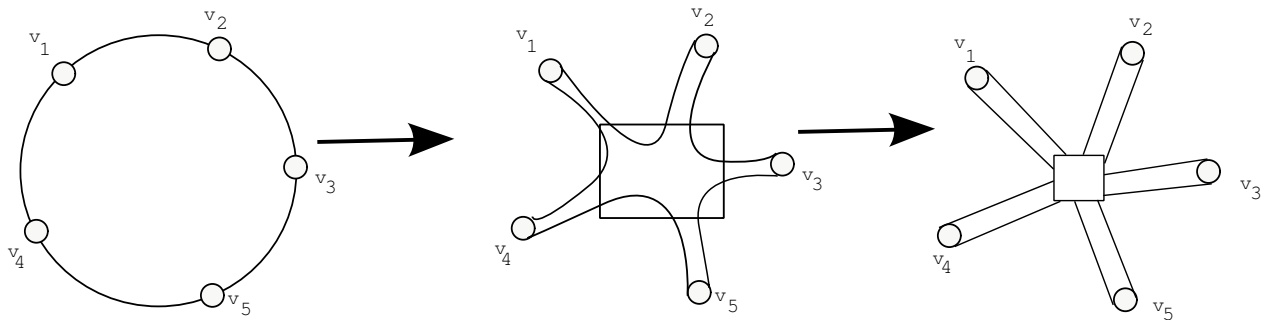


FIGURE 3.19. Example of a ring transformed into a star

**Property 3.20.** Any canonical cactus  $\mathcal{H}(G)$  can be translated, based on Property 3.19, to a tree of supernodes. Each edge in the tree will represent a minimum cut in  $G(V, E)$ .

An example of this translation is presented in Figure 3.20.

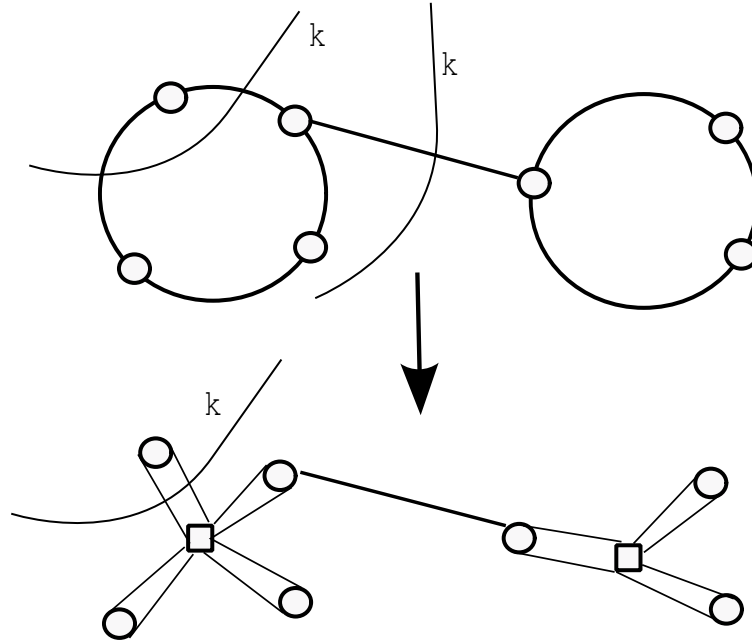


FIGURE 3.20. Example of a tree of 2 rings translated into a tree.

**Lemma 3.21.** *An undirected graph  $G(V, E)$  with odd connectivity  $k$  and unary edge weights has no circular partitions.*

*Proof.* Based on the well-defined edge weights of  $\mathcal{H}(G)$ , the weight of an edge in a circular partition is  $\frac{k}{2}$ . The weight of an edge in  $\mathcal{H}(G)$  corresponds to the number of edges in  $G(V, E)$  involved in a minimum edge cut. Because  $k$  is odd, there are no circular partitions.  $\square$

**Definition 3.22.** *A “supergraph” is a canonical cactus representation of a graph where all circular partitions have been replaced by stars.*

Henceforth we will use  $SG^k$  to denote the tree supergraph of connectivity  $k$ .

**Remark 3.23.** *The supergraph  $SG^k$  is unique.*

Because  $\mathcal{H}(G)$  is unique, it follows that  $SG^k$  is also unique. Based on [63], [50] and [51] we can find the supergraph of a graph  $G$  in polynomial time. We will use  $SG^k$  to denote the supergraph with minimum cut of size  $k$ . Please notice that  $SG^k$  does not represent all minimum cuts. It does however represent all smallest vertex-set minimum cuts. The term smallest vertex sets implies that the supernodes do not include any other minimum edge cuts. As we will see shortly, we use the

structure of  $SG^k$  only for the purpose of finding edge-disjoint lighttrees, and not for RWA. RWA is to be done on  $G(V, E)$ .

Next we present some properties of the supergraph structure we are going to use. We start by defining the “internal” connectivity of a supernode in  $SG^k$ . We use  $C_x$  to refer to a supernode.

**Definition 3.24.** *Let  $ES_x$  be the set of edges incident on supernode  $C_x$  in the supergraph. The internal connectivity of a supernode  $C_x$ , represented by  $k_x$ , is defined as the number of edges in the set of edges  $IS_x$  (edges connecting the vertices within  $C_x$ ), which is a minimum cut in  $G(V, E)$  such that  $IS_x \cap ES_x = \emptyset$ .*

In other words the internal connectivity of a supernode is the minimum number of “internal” edges  $IS_x$  that will disconnect  $G(V, E)$  in two connected components. Please notice that in some cases removal of all  $IS_x$  edges will not disconnect  $G(V, E)$ . In such cases we state that an internal connectivity does not exist (does not apply).

**Theorem 3.25.** *Given  $SG^k$  for  $G(V, E)$ , with  $\delta > k$ . The internal connectivity  $k_x$  of any supernode  $C_x$  is at least  $k_x = k + 1$ , if it exists.*

*Proof.* We prove this result by contradiction. Assume that  $|IS_x| \leq k$ .

Case (a).  $k_x = |IS_x| < k$ . This is impossible, since in that case  $k_x$  would have been the connectivity of  $G(V, E)$ .

Case (b).  $k_x = |IS_x| = k$ . The cactus decomposition finds all edges from all edge cuts. Since  $IS_x \cap ES_x = \emptyset$ , then  $IS_x$  is an edge-cut of degree  $k$ , not found by the cactus decomposition. This is a contradiction. □

**Definition 3.26.** *A supernode  $C_x$  in  $SG^k$  is “critical” if  $d(C_x) = k$ . All other supernodes with  $d(C_x) > k$  will thus be “non-critical”.*

In the following we present a step by step approach to find the minimum number of wavelengths needed for non-blocking all-to-all broadcast, for the case of arbitrary topologies with  $\delta > k$ . The first step of our procedure finds all minimum edge-cuts and constructs the tree supergraph where each edge represents a minimum cut. Henceforth we will concentrate on this tree supergraph, unless otherwise stated. Figure 3.21 shows an example of  $SG^k$ .

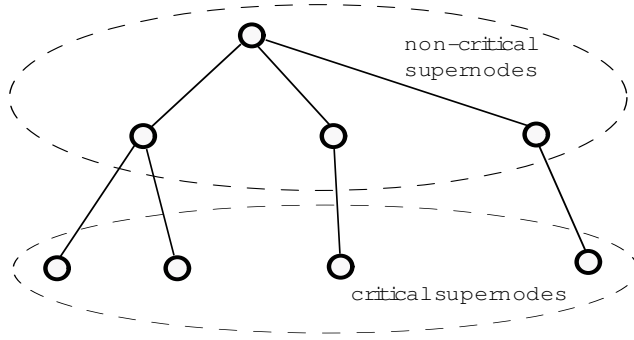


FIGURE 3.21. Example of  $SG^k$ . Each superedge has  $k$  edges.

We use the following notations for the supergraph  $SG^k$ . Let  $SN^k$  be the set of supernodes of the supergraph. Let  $|SN^k| = S^k$ . Let  $SN_i^k$  denote a specific super-node;  $1 \leq i \leq S^k$ .  $SN_i^k \in SN^k$ . We use superscript  $k$  for minimum edge-cut  $k$ , to distinguish it from other minimum cuts that will later be used. Let  $CN^k$  be the set of critical supernodes. Also let  $|CN^k| = C^k$  and let  $CN_i^k$  denote a specific critical supernode;  $1 \leq i \leq C^k$ ,  $CN_i^k \in CN^k$ . Similarly, let  $NCN^k$  be the set of non-critical nodes,  $|NCN^k| = NC^k$ , and  $NCN_j^k \in NCN^k$ ;  $1 \leq j \leq NC^k$ .

Note that for  $SG^k$ :  $CN^k \subseteq SN^k$ ,  $NCN^k \subset SN^k$ ,  $CN^k \cup NCN^k = SG^k$ . In other words,  $SG^k$  may have only critical nodes but it may not have only non-critical nodes.

We start with the constraint under which the bound on the number of wavelengths  $\lceil \frac{N}{k} \rceil$  can be improved upon.

**Lemma 3.27.** *For the case  $C^k > k + 1$ , the bound on the number of wavelengths remains*

$$\lambda = \left\lceil \frac{N}{k} \right\rceil$$

*which is tight.*

*Proof.* We prove this result by contradiction. Assume that we can find  $k + 1$  vertices to broadcast with one wavelength. Based on Theorem 3.16, adding a virtual node  $S$  and connecting it to  $k+1$  vertices, we get  $\kappa(S, G_d) = k + 1$ . For any  $k + 1$  vertices we choose, there will be at least one critical supernode  $CN_x^k$  with no vertices selected. Based on the critical supernode definition,  $d(CN_x^k) = k$  and the set of  $k$  edges incident on  $CN_x^k$  represent a minimum edge-cut. However the local connectivity for  $\kappa(S, G_d)$  is supposed to be  $k + 1$  which results in a contradiction.  $\square$

Thus we can conclude that if  $C^k < k + 1$ , then more than  $k$  vertices can be selected to broadcast with one wavelength. This will reduce the number of groups of vertices and consequently the number of wavelengths. So from now on we will concentrate on the case where  $C^k < k + 1$ . The question is: what is the maximum number of vertices that can be selected to broadcast with only one wavelength? Let  $M$  denote the maximum number of vertices that can be selected in a group. Next we give an upper bound on  $M$ .

**Theorem 3.28.** *M is upper-bounded by*

$$\left\lfloor \frac{C^k * k}{C^k - 1} \right\rfloor \quad (3.11)$$

*Proof.* We want to find the upper-bound on  $M$ . Thus for this proof we do not consider the internal connectivity of the supernodes, or the number of vertices inside each supernode.

Based on the proof to Lemma 3.27, a virtual node  $S$  has to connect to one vertex inside each critical supernode using  $C^k$  virtual edges of weight 1, and to any other  $k + 1 - C^k$  vertices, within any other supernodes, in order to have  $\kappa(S, G_d) = k + 1 = d(S)$ . At the same time, connecting  $S$  to exactly  $t$ ,  $t < k$ , vertices inside each critical supernode, and to any other  $k + t - t * C^k$  vertices, makes its local connectivity  $\kappa(S, G_d) = k + t = d(S)$ . Thus, it is easy to see that for  $M$  vertices to broadcast using only one wavelength, the vertices must be uniformly distributed among the critical supernodes. We can conclude that:

$$t = \left\lceil \frac{M}{C^k} \right\rceil,$$

where  $t$  represents the number of vertices to be selected from each supernode.

Assume that the edges of  $SG^k$  in Figure 3.21 have weight 1. If we choose one vertex from each critical supernode  $C^x$  to broadcast, then  $C^x - 1$  wavelengths are needed for  $C^x$  concurrent broadcasts. If we choose  $t$  vertices from each critical node to broadcast then  $t * (C^x - 1)$  wavelengths are needed. Since we want only one wavelength we get:

$$t * (C^x - 1) = 1,$$

If we consider each edge to have weight  $k$ , then:

$$\frac{t * (C^x - 1)}{k} = 1,$$

but we concluded that  $t$  should be of the form:  $t = \lceil \frac{M}{C^k} \rceil$ . Thus,

$$\left\lceil \frac{M * (C^x - 1)}{C^k * k} \right\rceil = 1$$

It follows that  $M$  is:

$$M = \left\lfloor \frac{C^k * k}{C^k - 1} \right\rfloor,$$

which is what expression 3.11 claims. Please note that  $M$  is maximum if the vertices are selected only from the critical supernodes.  $\square$

$M$  is also upper-bounded by  $2 * k$  as, physically, this would exhaust all available optical channels in both directions on a minimum edge-cut. Furthermore, Theorem 3.18 also states that  $M$  should be upper-bounded by  $\delta$  (Theorem 3.18 considers  $\delta = k$ ). Thus  $M$  is bounded by

$$M = \min \left( \left\lfloor \frac{C^k * k}{C^k - 1} \right\rfloor, 2 * k, \delta \right). \quad (3.12)$$

We can conclude that a lower bound on the number of wavelengths for conflict-free all-to-all broadcast in optical networks with arbitrary topologies, where  $\delta > k$ , is:

$$\lambda_l = \left\lceil \frac{N}{M} \right\rceil. \quad (3.13)$$

Thus we can state that the optimal  $\lambda$  satisfies:

$$\left\lceil \frac{N}{M} \right\rceil \leq \lambda \leq \left\lceil \frac{N}{k} \right\rceil.$$

Please note that in the majority of cases this lower bound is not reachable. Thus, in the following we will try to find a feasible  $M$ , so as to reduce the number of vertex groups, and consequently the number of wavelengths needed.

In Theorem 3.28 we have not taken into account the internal connectivity of the supernodes or the number of vertices within each supernode. Obviously  $M$  will depend on the internal connectivity of the supernodes. Next we find a construction within the canonical cactus representation that is useful in determining the relationship between the internal connectivity of the supernodes and  $M$ . Please recall that the obtained supergraph  $SG^k$  has  $S^k$  supernodes. Because this structure

is a tree we can conclude that there are exactly  $S^k - 1$  superedges in  $SG^k$ , with each superedge consisting of a minimum edge-cut of  $k$  edges.

**Lemma 3.29.** *The connectivity of  $G(V, E)$  increases to  $k+1$ , if exactly one parallel edge is added on each of the  $S^k - 1$  superedges.*

*Proof.* Any minimum edge-cut outside the supernodes has  $k + 1$  edges. There are exactly  $S^k - 1$ ,  $k + 1$  edge-cuts outside the supernodes. Based on Theorem 3.25, the internal connectivity of each supernode is at least  $k + 1$ . Thus the minimum edge-cut of the augmented supergraph is  $k + 1$ .

Figure 3.22 illustrates the lemma. □

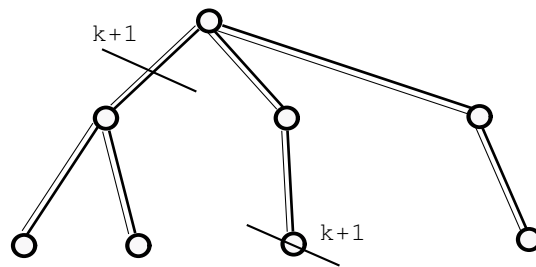


FIGURE 3.22. One edge is added in parallel to each superedge in  $SG^k$ .

Using Lemma 3.29 we can virtually add these edges just for the purpose of finding the internal connectivity of the supernodes. Please note that a virtual edge can be added between any two vertices belonging to the corresponding neighboring supernodes.

Thus, recursively, we can apply the canonical cactus representation to find the internal structure of the supernodes. Consider the following sequence:

- Step1: Input the supergraph  $SG^k$  with  $S^k$  supernodes, and  $S^k - 1$  superedges of weight  $k$ .
- Step2: Augment all superedges with one virtual edge each.
- Step3: Apply the canonical cactus representation on the augmented supergraph.
- Step4: Apply the ring-to-tree transformation to obtain a supergraph  $SG$ .
- Step5: Remove the augmenting virtual edges.

Consider the superedges in  $SG^k$  that were obtained by applying the ring-to-tree transformation presented in Property 3.20. It is important to mention that augmenting the superedges in  $SG^k$  translates into augmenting each circular partition edge in  $\mathcal{H}(G)$  with a virtual edge of weight 0.5. Figure 3.23 shows an example of an augmentation of  $SG^k$  and its equivalent augmentation

in  $\mathcal{H}(G)$ . However, based on Lemma 3.29, these non-integer weights will not affect the resulting connectivity of  $G(V, E)$  which is  $k + 1$ . Recall that the purpose of this augmentation is to find the edge-cuts of weight  $k + 1$ . Furthermore, any new edge-cut of weight  $k + 1$  will be found inside the supernodes. After removing all virtual edges (in Step 5) there will be no non-integer weights left.

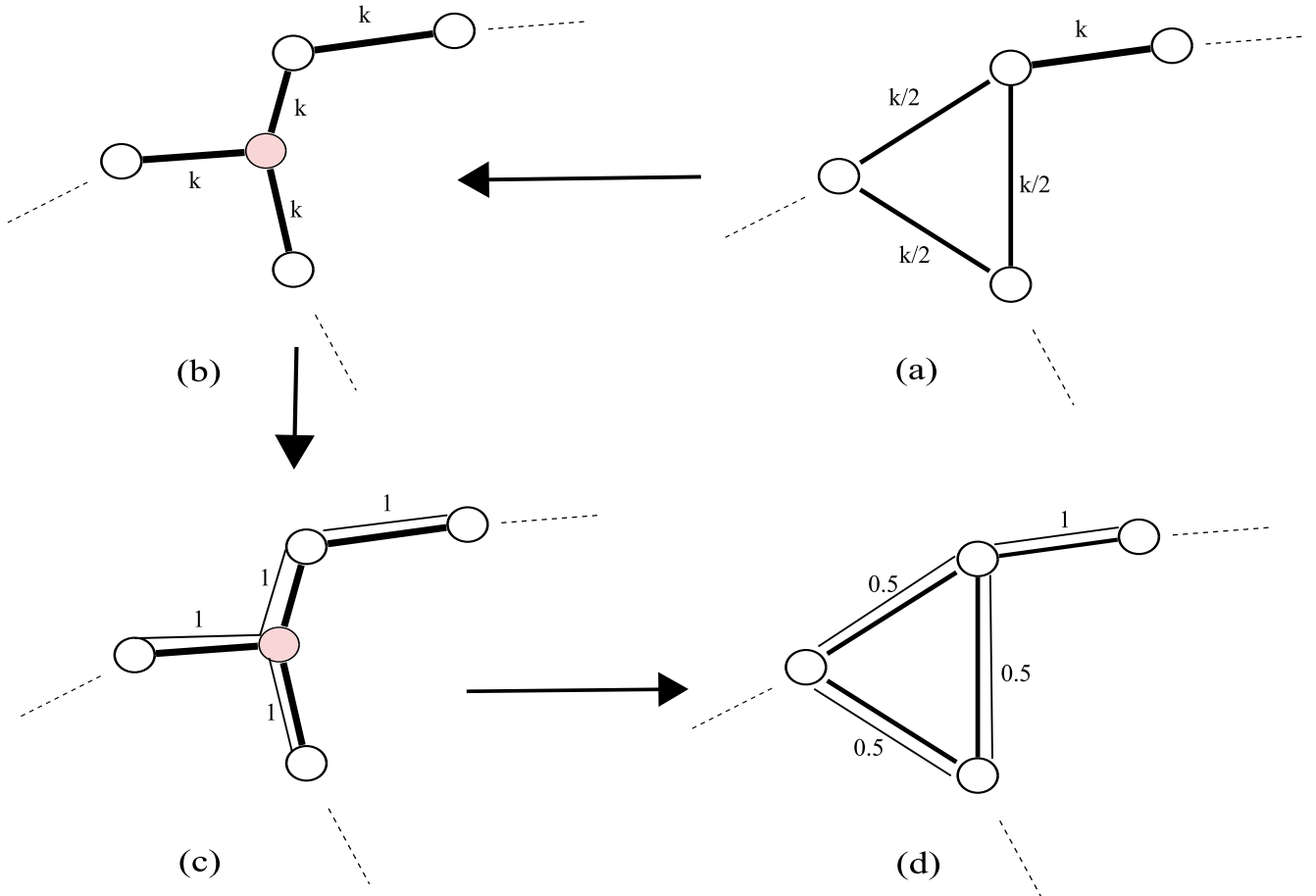


FIGURE 3.23. Example of augmentation when the original  $\mathcal{H}(G)$  contains circular partitions. (a) represents a part of  $\mathcal{H}(G)$ ; (b) represents the equivalent  $SG^k$ ; (c) represents the augmentation of  $SG^k$  with edges of weight 1; (d) represents the resulting augmentation of  $\mathcal{H}(G)$ ; the ring edges are augmented with edges of weight 0.5.

The output of the above sequence will be a tree supergraph  $SG^{k+1}$  with  $S^{k+1}$  supernodes. For this case  $S^{k+1} \geq S^k$ , exactly  $S^k - 1$  superedges of weight  $k$ , and  $S^{k+1} - S^k$  superedges of weight  $k + 1$ .

We now give the following result that establishes that the resulting structure is a tree.

**Theorem 3.30.** *If  $SG^{k+1}$  is obtained by applying the canonical cactus decomposition on the augmented  $SG^k$ , then  $SG^{k+1}$  is a tree.*

*Proof.* We want to show that applying the canonical cactus decomposition on the augmented graph  $SG^k$  will not create any cycles; i.e. each augmented superedge will connect at most two new supernodes in  $SG^{k+1}$ , say  $SN_x^{k+1}$  and  $SN_y^{k+1}$ . In other words we try to prove that any augmented superedge  $e$  of  $k + 1$  edges will not split into  $e_1$  and  $e_2$  edges,  $e_1 + e_2 = k + 1$  to two supernodes in the new augmented cactus decomposition. Please follow the proof on Figure 3.24.

We prove this result by contradiction. Based on Property 3.20, the structure inside a supernode  $SN_x^k$  will be a tree. Assume some augmented superedge  $e$  can be split into  $e_1$  and  $e_2$  edges. Thus, a circular partition is formed, and  $e_1 = e_2 = (k + 1)/2$ .

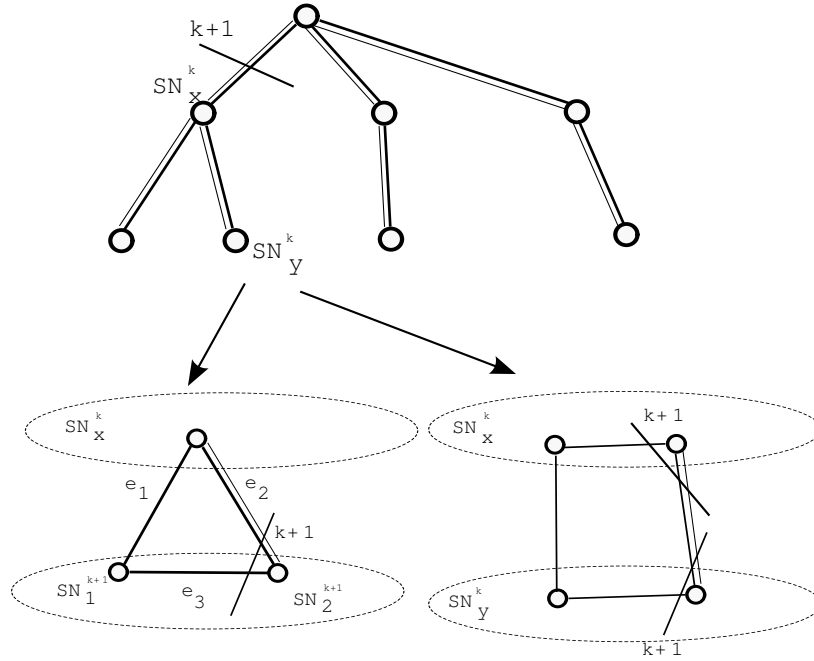


FIGURE 3.24. Illegal circular partitions when computing  $SG^{k+1}$  from  $SG^k$ .

If  $k$  is even then  $k + 1$  is odd. Based on Lemma 3.21, there are no circular partitions. If  $k$  is odd then  $k + 1$  is even. Thus there is a circular partition of at least 3 vertex sets,  $SN_x^k$  and two sets in the “ $k + 1$ ” decomposition, say  $SN_1^{k+1}$  and  $SN_2^{k+1}$ , with  $\omega(SN_x^k, SN_1^{k+1}) = e_1$ ,  $\omega(SN_x^k, SN_2^{k+1}) = e_2$ , and  $\omega(SN_1^{k+1}, SN_2^{k+1}) = e_3$ .  $\omega(node1, node2)$  denotes the number of edges between node1 and node2. Assume, without loss of generality, that the virtual augmenting edge belongs to  $e_2$ . It follows that  $e_2 + e_3 = k + 1$ . Removing the virtual edge we get  $e_2 + e_3 = k$ . Thus  $e_2 + e_3$  is a  $k$  edge-cut. But  $e_3$  did not belong to the edge set of the  $k$  cactus decomposition. This is a contradiction. Thus, the augmented superedge cannot split into a circular partition.  $\square$

Thus,  $SG^{k+1}$  will be a tree with superedges of weight  $k$  and  $k + 1$ . An example of  $SG^{k+1}$  which is obtained from  $SG^k$  is illustrated in Figure 3.25.

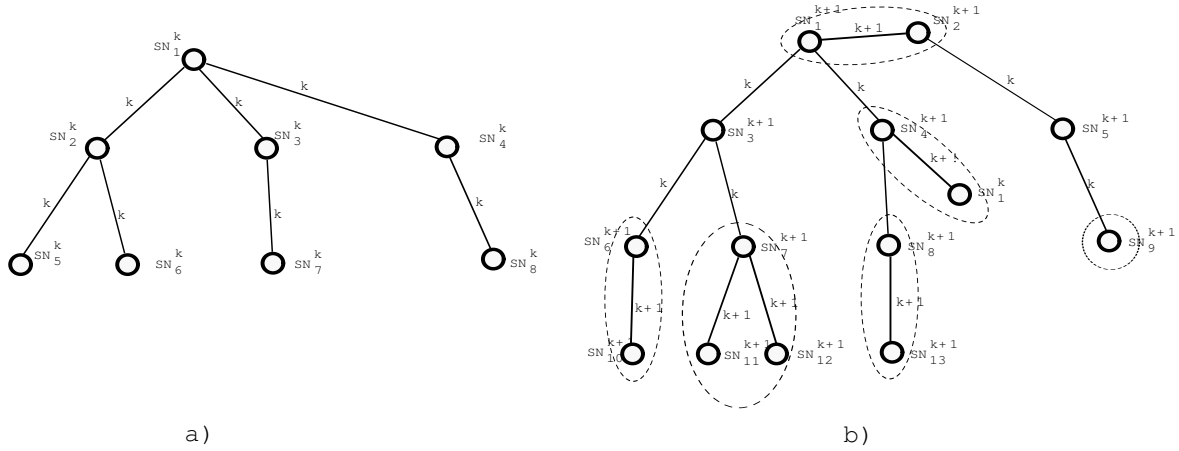


FIGURE 3.25. Example showing obtaining  $SG^{k+1}$  from  $SG^k$ . In (b), the supernodes  $SN_i^k$  are circled.

Thus, intuitively, we give the following general method for finding  $SG^{k+t+1}$  from a given supergraph  $SG^{k+t}$ :

- Add  $i$  virtual parallel edges to the superedges in  $SG^{k+t}$  with weight  $k + t + 1 - i$ ,  $1 \leq i \leq t + 1$ .
- Perform a canonical catus decomposition for the resulting connectivity of  $k + t + 1$ .
- Remove all virtual parallel edges to obtain  $SG^{k+t+1}$ .

Using a similar reasoning to the one presented in the proof of theorem 3.30, we find that  $SG^{k+t+1}$  is also a tree.

From the proof of Theorem 3.28, we know that a virtual node  $S$  connected to one vertex within each  $CN^k$  critical supernode and to any other  $k + 1 - C^k$  vertices will have a local connectivity of  $\kappa(S, G_d) = k + 1$ . See Figure 3.26. We define the “virtual  $k + 1$  dominance number” of node  $S$ , denoted by  $VD^{k+1}(S)$ , as the minimum number of vertices to select from the critical supernodes such that a total of  $k+1$  vertices can be selected to broadcast using only one wavelength. The remaining  $k + 1 - VD^{k+1}(S)$  vertices can be selected from any other supernodes. Figure 3.26 shows an example for  $k$  and  $k + 1$ .

We try to minimize the number of vertices selected from the critical supernodes, so long as the rest of the vertices can be selected from any other supernode. Thus, so far we can say that  $VD^{k+1}(S) = C^k$ . Generalizing, we use  $VD^{k+t}(S)$  for the virtual  $k + t$  dominance number of  $S$ , the

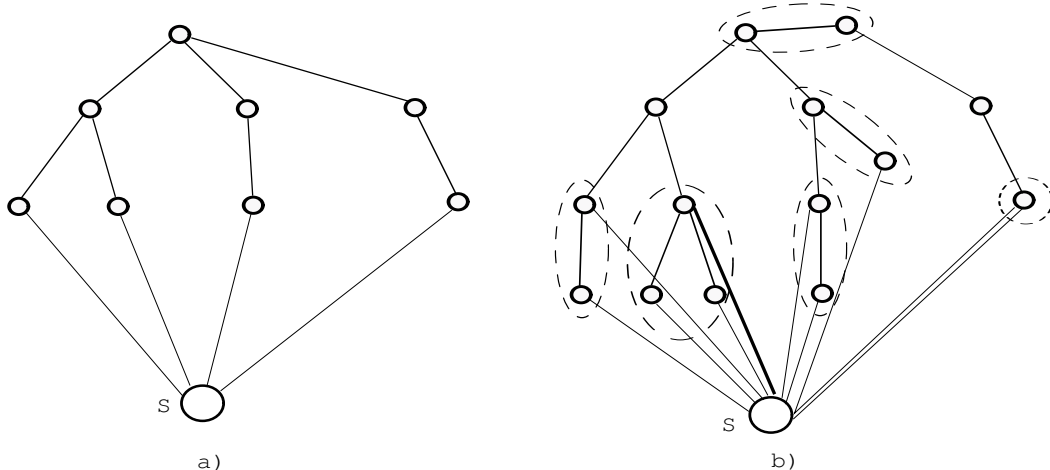


FIGURE 3.26. (a) Virtual node  $S$  connected to the critical nodes in  $SG^k$ . (b) Virtual node  $S$  connected to all critical nodes in  $SG^k$  and  $SG^{k+1}$ . The bold edge is unnecessary for  $VD^{k+1}(S)$

number of nodes selected from the critical nodes, such that a total of  $k + t$  nodes can be selected. In Figure 3.26.(b) we see that connecting  $S$  to  $C^k + C^{k+1}$  vertices, one in each critical node in  $SG^k$  and  $SG^{k+1}$ , we will be able to select a total of  $k + 2$  nodes. But  $VD^{k+t}(S) \leq C^k + C^{k+1}$  (usually less). As we observe in Figure 3.26 (b) the “bold” virtual edge is unnecessary. The question then becomes how to find  $VD^{k+t}(S)$ , in general, for an arbitrary  $t$ . In the following we give an algorithm to find  $VD^{k+t}$ .

We define  $w_v(< u, v >)$ , the virtual weight of superedge  $< u, v >$  in  $SG^{k+t}$  to be  $w_v(< u, v >) = w < u, v > + \min \{SV(u), SV(v)\}$ .  $SV(u)$  represents the number of virtual edges starting at virtual node  $S$  that reach supernode  $u$  without crossing superedge  $< u, v >$ . Consider the following algorithm sequence.

Algorithm (Find  $VD^{k+t}(S)$ ):

- Connect  $S$  to a vertex in each  $CN^{k+t}$ ;
- For all edges in  $SG^{k+t}$ ;
  - If  $w_v(< u, v >) \leq k + t$ ;
    - Get the node  $u$  which had the minimum  $SV(u)$ ;
    - Starting with  $CN^{k+t-1}$  and down, connect  $S$  to as few vertices as possible such that  $SV(u) + w < u, v > = k + t$ ;
- Endif;

-Endfor;

Thus the result in Lemma 3.27 (which is a special case for  $t = 1$ ) can be generalized into the following corollary.

**Corollary 3.31.** *If  $VD^{k+t}(S) \geq k + t + 1$ , then  $SG^{k+t-1}$  is the last supergraph that needs to be analyzed (i.e., it is not possible to find  $k+t$  nodes in the same group to broadcast with one wavelength).*

Corollary 3.31 gives a condition to terminate the recursive representation process of the supergraph  $SG$ . This condition is reached when the maximum number of vertices to select in one group to broadcast with one wavelength has been found. Thus, the maximum number of vertices,  $M$ , defined previously in Equation 3.12, is given by the last virtual dominance number  $VD^{k+t}(S)$  satisfying Corollary 3.31,  $M = k + t$ .

A group of vertices broadcasting with one wavelength will be called a wavelength group. The objective is to minimize the number of wavelength groups, once the final supergraph  $SG$  has been found. The number of wavelength groups will obviously translate into the number of wavelengths needed to perform conflict-free all-to-all broadcast. Therefore we will use  $\lambda$  to denote the number of wavelength groups.

To reduce the number of wavelength groups one may try a greedy approach. Please note that the groups may contain any number of vertices between  $k$  and  $k + t$  as it would make no sense to take groups of nodes with less than  $k$  vertices. It is obvious that increasing the number of groups of higher cardinality will decrease the total number of groups. Thus, a greedy approach would first select the largest number of groups of  $k + t$  vertices. Assume that  $X_{k+t}$  represents the number of groups of  $k + t$  vertices. If  $(k + t) * X_{k+t} < N$ , we will have to group the remaining vertices in as few wavelength groups as possible. Thus, to further reduce the number of groups, the algorithm will check the new maximum number of vertices that can be selected in a group, say  $k + p$ , where  $1 \leq p < t$ . The algorithm will now select the largest number of groups of  $k + p$  vertices and repeat the process until all  $N$  vertices are selected in wavelength groups. A simple greedy algorithm pseudocode can be stated as follows.

Input:  $SG^{k+t}$ ,  $M = k + t$

BEGIN

$new_M = M$  (maximum number of nodes selected in a wavelength group)

WHILE  $\{new_M \geq k\}$

- Find  $X_{new_M}$

- Remove all nodes selected in the  $X_{new_M}$  groups from the supergraph  $SG^{k+t}$

- Recompute  $new_M$

ENDWHILE

END

Such an algorithm is trivial. However, it is not clear if a greedy algorithm would always give the minimum number of wavelength groups. Assume, for example, that using the above greedy algorithm we have found  $X_{k+t}$  and  $X_{k+t-1}$ . Furthermore, assume that in the first algorithm step we only choose  $X'_{k+t} < X_{k+t}$ . In other words we do not choose to use the maximum number of groups of  $k + t$  vertices. Next the algorithm finds the maximum number of groups containing  $k + t - 1$  vertices, say  $X'_{k+t-1}$ . Notice that this number is greater or equal than  $X_{k+t-1}$  previously computed.

It may be possible for specific input graphs  $G(V, E)$ , to obtain:  $X_{k+t} + X_{k+t-1} < X'_{k+t} + X'_{k+t-1}$ . In such a case the greedy algorithm would not provide an optimal solution. This is mainly because, in general, a group selection of  $k+p$  vertices would affect the maximum number of groups of vertices  $X_{k+p-1}$  left to choose. Thus we can conclude that there will be a strong correlation between  $X_{k+t}$  and  $X_{k+t-1}$ , and all the subsequent groups of vertices, until  $X_{k+1}$ . Moreover, such a correlation may be different for different input graphs  $G_1(V_1, E_1) \neq G_2(V_2, E_2)$ .

Thus, we suggest for this step an exhaustive search. Analyzing the time complexity for such an exhaustive search we get the following interesting result. The time complexity of an exhaustive search algorithm for selecting the minimum number of wavelength groups is polynomial in  $N$ . Without loss of generality, assume that there is no correlation between the number of groups of different number of nodes. We can make this assumption because it upper-bounds the time complexity. Let's assume the maximum number of groups of  $k + t$  nodes is:

$$\left\lceil \frac{N}{k+t} \right\rceil.$$

Thus, an upper-bound on the number of total possible combinations of groups of different number of nodes is:

$$\left\lceil \frac{N}{k+t} \right\rceil * \left\lceil \frac{N}{k+t-1} \right\rceil * \dots * \left\lceil \frac{N}{k+1} \right\rceil = \dots$$

Ignoring the ceiling function, for simplicity, we get

$$\frac{N}{k+t} * \frac{N}{k+t-1} * \dots * \frac{N}{k+1} = \dots$$

$$\dots = \frac{N^t * k!}{(k+t)!} < N^t.$$

Thus, the time complexity of such an exhaustive search to find the minimum number of wavelength groups on  $SG^{k+t}$  is  $O(N^t)$  which is polynomial in  $N$ , the number of vertices in  $G(V, E)$ . Please recall that the power  $t$  represents the index of the recursion on the supergraph, and is always less than  $k$ . Now we can give the second condition for terminating the supergraph recursion process. This condition will reduce  $t$  further. We use  $\lambda^{k+x}$  to denote the number of wavelength groups found in supergraph  $SG^{k+x}$ .

We also use  $X_{k+j}$  to denote the maximum number of groups of  $k+j$  vertices we can select in  $SG^{k+p}$ ,  $1 \leq j \leq p$ .

**Lemma 3.32.** *For a given  $SG^{k+t}$ ,*

$$X_{k+j+1} \leq \frac{X_{k+j}}{2}, \forall j, 0 \leq j \leq t-1.$$

*Proof.* In the best case scenario, to maximize  $X_{k+j+1}$ , the number of critical nodes in  $SG^{k+j}$  and  $SG^{k+j+1}$  has to be the same. Thus,  $CN_i^{k+j+1} = CN_i^{k+j}$ . In such a case,  $VD^{k+j} = 2 * VD^{k+j+1}$ . Moreover, no other critical nodes are formed from the set of non-critical nodes  $NCN^{k+j}$ . It follows that  $X_{k+j+1} = \frac{X_{k+j}}{2}$ . This is the only case where  $X_{k+j+1}$  is maximized. For all other cases  $X_{k+j+1} < \frac{X_{k+j}}{2}$ . □

**Remark 3.33.**  $X_{k+j}$  for  $SG^{k+p}$  is less or equal to  $X_{k+j}$  for  $SG^{k+p+1}$ .

Obviously, the maximum number of wavelength groups of  $k+j$  vertices cannot increase from supergraph  $SG^{k+p}$  to supergraph  $SG^{k+p+1}$ .

The following theorem provides a second condition for stopping the recursion on the supergraphs.

**Theorem 3.34.** *If  $\lambda^{k+p} < \lambda^{k+p+1}$ , then  $\lambda^{k+p} < \lambda^{k+p+2}$ .*

*Proof.* We use  $x_{k+j}^{k+p}$  to denote the number of groups of  $k+j$  vertices selected in  $\lambda^{k+p}$ ,  $1 \leq j \leq p$ .

It follows that:

$$\begin{aligned} \lambda^{k+p} &= x_{k+p}^{k+p} + x_{k+p-1}^{k+p} + \dots + x_{k+1}^{k+p} + \dots \\ \dots + \frac{N - \left[ x_{k+p}^{k+p} * (k+p) + x_{k+p-1}^{k+p} * (k+p-1) + \dots + x_{k+1}^{k+p} * (k+1) \right]}{k} &= \dots \\ \dots &= \frac{N - \left[ p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p} \right]}{k}. \end{aligned}$$

Thus,  $\lambda^{k+p} < \lambda^{k+p+1}$  translates into:

$$\begin{aligned} \frac{N - \left[ p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p} \right]}{k} &< \dots \\ \dots &< \frac{N - \left[ (p+1) * x_{k+p+1}^{k+p+1} * + (p) * x_{k+p}^{k+p+1} + \dots + (k+1) * x_{k+1}^{k+p+1} \right]}{k} \\ (p+1) * x_{k+p+1}^{k+p+1} * + (p) * x_{k+p}^{k+p+1} + \dots + (k+1) * x_{k+1}^{k+p+1} &< \dots \\ &< p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p} \end{aligned}$$

Thus we can restate the theorem as follows:

If

$$\begin{aligned} (p+1) * x_{k+p+1}^{k+p+1} * + (p) * x_{k+p}^{k+p+1} + \dots + (k+1) * x_{k+1}^{k+p+1} &< \dots \\ &< p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p}, \end{aligned} \quad (3.14)$$

then

$$\begin{aligned} (p+2) * x_{k+p+2}^{k+p+2} * + (p+1) * x_{k+p+1}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2} &< \dots \\ &< p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p} \end{aligned} \quad (3.15)$$

From the hypothesis we know that groups of  $k+p+1$  vertices would not improve on the overall number of wavelength groups. Thus these groups should be discarded from  $\lambda^{k+p+2}$ . Thus the left hand side above becomes

$$(p+2) * x_{k+p+2}^{k+p+2} * + (p) * x_{k+p}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2}$$

From Lemma 3.32 we know that:

$$x_{k+p+2}^{k+p+2} < \frac{x_{k+p+1}^{k+p+2}}{2}$$

Thus,

$$\begin{aligned} (p+2) * x_{k+p+2}^{k+p+2} * + (p) * x_{k+p}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2} &\leq \dots \\ &\leq \frac{p+2}{2} * x_{k+p+1}^{k+p+2} + (p) * x_{k+p}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2} \end{aligned}$$

Based on Remark 3.33 it follows:

$$\begin{aligned} \frac{p+2}{2} * x_{k+p+1}^{k+p+2} + (p) * x_{k+p}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2} &< \dots \\ \dots &< (p+1) * x_{k+p+1}^{k+p+1} * + (p) * x_{k+p}^{k+p+1} + \dots + (k+1) * x_{k+1}^{k+p+1} \end{aligned}$$

Thus it follows that:

$$\begin{aligned} (p+2) * x_{k+p+2}^{k+p+2} * + (p+1) * x_{k+p+1}^{k+p+2} + \dots + (k+1) * x_{k+1}^{k+p+2} &< \dots \\ &< p * x_{k+p}^{k+p} * + (p-1) * x_{k+p-1}^{k+p} + \dots + (k+1) * x_{k+1}^{k+p} \end{aligned}$$

Which is exactly what we wanted to prove. □

We are in the position now to provide systematic steps for a conflict-free all-to-all broadcast method in order to reduce the number of wavelengths:

Given  $G(V, E)$ , with certain values for  $\delta$  and  $k$ .

1. Set  $i = 0$ , /\*  $i$  will be used in the recursive step \*/
2. Set  $\lambda^{k+i} = \lceil \frac{N}{k} \rceil$ .
3. If  $\delta = k$  then set  $\lambda = \lceil \frac{N}{k} \rceil$  and go to step 10.
4. Find  $SG^{k+i}$ .
5. If  $VD^{k+i}(S) \geq k+i+1$  go to step 10. Else continue.
6. Find  $\lambda^{k+i+1}$ . Use a greedy algorithm or an exhaustive search algorithm.
7. If a greedy algorithm was used in step 6 skip next step.

8. If  $\lambda^{k+i} < \lambda^{k+i+1}$  go to step 10. /\* second stopping condition \*/
9. Increment  $i$ . Go to step 4.
10. Assign  $\lambda = \lambda^{k+i}$

The routing and wavelength assignment algorithm (RWA) will be very simple in this case. Assign a wavelength for each group of vertices selected. Use the construction of Theorem 3.16 along with the algorithm in [75] to find the routes.

# Chapter 4

## Multi-Hop Routing

### 4.1 Introduction

This chapter addresses the issue of multi-hop routing in wavelength routed optical networks for non-blocking all-to-all broadcast. In a multi-hop routed network, the message makes multiple optical hops until reaching the destination. As explained in Chapter 3, an optical hop is defined as a continuous lightpath with no converters involved. The end-nodes of a lightpath/lighttree (except the source and the destination nodes), convert the signal from the optical domain to the electronic domain, process the message, and then convert the message back to the optical domain, to be transmitted in the next optical hop. This process is called the optical/electronic/optical conversion (OEO).

As mentioned in Chapter 2, multi-hop routing in WDM networks emerged as a necessity to cope with a number of optical physical limitations. Among the reasons for multihop are the need for signal regeneration on a lightpath/lighttree, the limitation on the number of wavelengths multiplexed on a fiber, and the number of optical receivers/transmitters needed at a node. As pointed out in Chapter 1, the 3-R operation on the optical signal (re-shaping, re-timing and re-amplification) needs to be performed whenever the quality of the signal drops below a specific threshold. Thus, very long one hop connections may need to be broken into multiple optical hops, in order to re-generate the signal. Furthermore, in one-hop routed networks (using no wavelength conversion), the number of wavelengths required to perform a specific set of routing requests may exceed the total number of wavelengths available on a fiber. Using multi-hop routing, the nodes that perform OEO conversions, besides regenerating the signal, have the ability of encoding the message on a different wavelength from the one it was received on. Thus, a multi-hop approach could considerably reduce the number of wavelengths needed on a fiber. Finally, for the specific routing case of non-blocking all-to-all broadcast studied in this thesis, the number of optical receivers needed per node for the one-hop case may be extremely large when the total number of

nodes in the network is large. Again, a multi-hop approach could tremendously reduce the number of receivers needed per node, as we shortly show.

Several studies brought into attention the merits of multi-hop WDM networks and the associated optical issues to be solved. Interested readers are referred to the work in [49], [37], [27] for more details on optical multi-hop routing.

In this chapter we attempt to reduce the optical resources for a given maximum number of hops, or alternately to use a minimum number of hops, constrained by the given resources. The optical resources we try to optimize are the number of wavelengths on a fiber and the number of receivers at a node. We propose and analyze simple but attractive multi-hop routing solutions. We aim to find a multi-hop RWA model that can achieve good results on the number of wavelengths and optical receivers and that can also be systematically achieved in polynomial time. The general lower bounds on the number of wavelengths and the number of optical receivers are strongly influenced by the RWA model used and the assumptions made. We find two elegant and realistic multi-hop routing models that are achievable in polynomial time. Furthermore, we find the necessary conditions to achieve optimal solutions for our proposed RWA models. As we will see the regular topologies of the ring and hypercube achieve near-optimal results. For the case of networks with arbitrary topologies, we provide a heuristic algorithm that gives very good results. All our solutions are very effective in terms of reducing the number of wavelengths and the number of optical receivers. We will shortly prove that both the number of wavelengths needed to perform non-blocking all-to-all broadcast as well as the number of optical receivers needed per node are on the order  $O(N^{\frac{1}{h}})$ , where  $N$  is the number of nodes in the network and  $h$  is the number of hops.

The rest of the chapter is structured as follows. In Section 4.1.1 we describe our multi-hop models for all-to-all broadcast and introduce some useful results. In Section 4.2 we study special cases of networks with regular topologies, namely the ring and the binary hypercube. In Section 4.3 we propose a heuristic algorithm for the general case of networks with arbitrary topologies and analyze its results.

### 4.1.1 Multi-Hop Routing Models

First we present the multi-hop routing models used throughout this chapter. There are several different ways to select the routes in each hop such that a complete information exchange is achieved [49], [27].

The authors in [27] defined the RWA in  $h$  hops for a request  $(u, v)$ , from source node  $u$  to destination node  $v$ , as follows. One needs to find  $h - 1$  intermediate nodes, such that  $(u_{j-1}, u_j)$  lightpaths are realized, for  $1 \leq j \leq h$ , and  $u_0 = u$ ,  $u_h = v$ . In the same reference [27], two RWA models for multihop are proposed specifically for the all-to-all broadcast communication pattern. In one model, called the “merge model”, each intermediate node will merge all received messages into one single message, such that in the next hop, only one wavelength will be needed per intermediate node to transmit the merged message.

We will make use of this merging feature of [27] as it comes natural for the specific case of all-to-all broadcast. However, please notice that the merging operation at each intermediate node introduces additional overhead that should not be overlooked for networks with large number of nodes ( $N$ ) or for cases using a large number of hops ( $h$ ).

In the following we present two novel multi-hop RWA models. As we will see, while reducing the number of wavelengths, the proposed multi-hop RWA models achieve a number of receivers very close to the lower bound.

Next we present two multihop routing models. Assume the multihop is done in  $h$  hops.

- **MHGM** (multi-hop gossip-multicast) routing. In the first hop, nodes within the same sub-network perform gossiping. In the remaining  $h - 1$  hops, selected nodes perform multicast to selected disjoint destination node subsets.
- **MHCB** (multi-hop collect-broadcast) routing. In the first hop selected “collector” nodes collect the information from nodes in disjoint node subsets. In the next  $h - 2$  hops, collector nodes collect the information from other selected collector nodes. In the last ( $h^{th}$ ) hop, selected collector nodes perform broadcast to all other nodes in the network.

We model the network as an undirected graph  $G(V, E)$ , in a fashion similar to Chapter 3. For both models we consider a hierarchical view of the network as  $h$ -level subnetworks (domains) organization, where  $h$  is the number of hops. Thus, we recursively partition the set of nodes  $V$ ,  $h - 1$  times, such that each node subset is connected (each node can be reached from any other node). Figure 4.1 shows an example of a network partitioned in 3 hierarchical levels. Each subnetwork (at each level) is considered to be connected. Notice that level 1 corresponds to the original (unpartitioned) network.

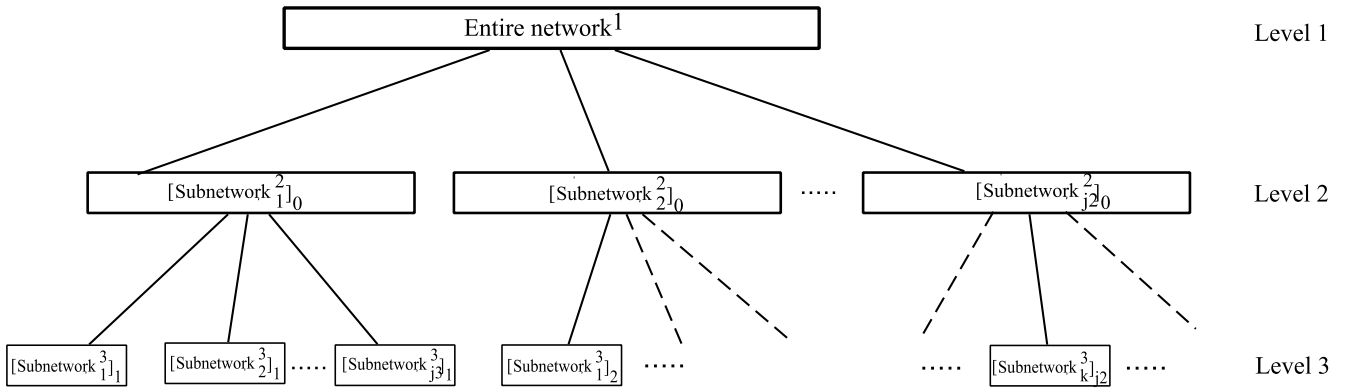


FIGURE 4.1. A network partitioned 3 times. Each level corresponds to a hop in the multihop routing.

We use the following notations:  $[V_i^j]_{k_1 k_2 \dots k_{j-1}}$  denotes node subset  $i$  at depth level  $j$ , with the parent node subset  $k_1$  at level  $j - 1$ ,  $k_2$  at level  $j - 2$ , and  $k_{j-1}$  at level 1. Thus,  $[V_i^j]_{k_1 k_2 \dots k_{j-1}} \subset [V_{k_1}^{j-1}]_{k_2 k_3 \dots k_{j-1}} \subset [V_{k_2}^{j-2}]_{k_3 k_4 \dots k_{j-1}} \subset [V_{k_{j-1}}^1]$ .

For simplicity we will use only  $[V_i^j]_{k_1}$  to denote a node subset included in the next hierarchical node subset  $[V_{k_1}^{j-1}]_{k_2}$ . We assume that each node subset at level  $j$  is partitioned into exactly  $s_j$  disjoint node subsets at level  $j + 1$ . The rationale of this will be given shortly, in Subsection 4.1.3. Thus,

$$\bigcup_{i=1}^{s_{j-1}} [V_i^j]_{k_1} = [V_{k_1}^{j-1}]_{k_2} \quad \text{and} \quad [V_{i_1}^j]_{k_1} \cap [V_{i_2}^j]_{k_2} = \emptyset, \forall k_1, k_2; \text{ if } k_1 = k_2, \text{ then } i_1 \neq i_2$$

In other words, the union of all node subsets with the same parent node subset in the hierarchy represents that specific parent node subset. Any two node subsets at the same partition level are disjoint. Please notice that it is possible to have  $i_1 = i_2$  if the two nodes have different parent node subsets.

Also, the union of all subsets in any level is the node set  $V$ ,

$$\bigcup_{\forall i,k} [V_i^j]_k = V. \quad (4.1)$$

We define  $(B_{i_1-i_2}^j)_{k_1}$  as a “border” node in  $[V_{i_1}^j]_{k_1}$  with a neighbor in  $[V_{i_2}^j]_{k_1}$ . Thus, there is an edge  $\langle (B_{i_1-i_2}^j)_{k_1}, (B_{i_2-i_1}^j)_{k_1} \rangle \in E$ . Please note that a border node in a subset  $[V_{i_1}^j]_{k_1}$  may have neighbors in more than one subset. Thus, for the purpose of the RWA method, we will use different notations for the same border node with neighbors in multiple subsets. For example we use  $(B_{i_1-i_2}^j)_{k_1} \equiv (B_{i_1-i_3}^j)_{k_1}$  when a border node in  $[V_{i_1}^j]_{k_1}$  neighbors a node in  $[V_{i_2}^j]_{k_1}$  and a node in  $[V_{i_3}^j]_{k_1}$ .

In addition, we use the following notations:

- $\mathcal{NS}([V_i^j]_k) = \{[V_x^j]_k \mid \exists \langle (B_{i-x}^j)_k, (B_{x-i}^j)_k \rangle \in E\}$ , the set of all node-subsets neighboring node-subset  $[V_i^j]_k$ ;
- $\mathcal{M}([V_{i_1}^j]_{k_1})$ , the collective message of subset  $[V_{i_1}^j]_{k_1}$ ;
- $\mathcal{D}((B_{i-x}^j)_k)$  the destination node subset for the multicast with the source being the border node  $(B_{i-x}^j)_k$ ;
- $(C_i^j)_k$  the collector node selected for subset  $[V_i^j]_k$

Thus, the two routing models described earlier can be formulated as follows.

### MHGM:

- Partition the set of nodes  $V$  recursively  $h - 1$  times, as explained above.
- In the first hop perform all-to-all broadcast among nodes within all subsets at level  $h - 1$ ,  $[V_i^{h-1}]_k$ . At each node, merge all received messages into collective messages  $\mathcal{M}([V_{i_1}^j]_{k_1})$ .
- For all subsequent hops, in decreasing order of  $j$ ,  $j = h - 2 : 1$  perform the following: let all border nodes  $(B_{i-x}^j)_k$  in level  $j$  multicast their message  $\mathcal{M}([V_i^j]_k)$  such that,

$$\mathcal{D}((B_{i-x}^j)_k) \cap \mathcal{D}((B_{i-y}^j)_k) = \emptyset, \forall x \neq y \quad (4.2)$$

$$\bigcup_x \mathcal{D}((B_{i-x}^j)_k) = [V_k^{j-1}]_m \setminus [V_i^j]_k, \forall x, [V_x^j]_k \in \mathcal{NS}([V_i^j]_k) \quad (4.3)$$

At each node merge all received messages into one collective message.

Relation 4.2 prevents nodes from receiving multiple messages from the same source. Relation 4.3 guarantees that all-to-all broadcast is accomplished.

**MHCB:**

- Partition the set of nodes  $V$ ,  $h - 1$  times, as explained above.
- In the first hop perform the following for all node subsets at level  $h - 1$ : Collect at node  $(C_i^{h-1})_k$  the messages from all the nodes within subset  $[V_i^{h-1}]_k$ . Merge all received messages into collective messages.
- For the next  $h - 2$  hops, in decreasing order of level  $j$ ,  $j = h - 1 : 2$ , perform the following. Let all collector nodes in level  $j$ ,  $(C_i^j)_k$  send their messages to collector node  $(C_k^{j-1})_m$ . Merge all received messages into collective messages.
- In the last hop ( $h$ ) perform the following: let all collector nodes at level one  $(C_i^1)$  perform broadcast to all other nodes in the network.

The two multi-hop routing models described above have several similarities. They both use a recursive  $h - 1$  level partitioning of the network into connected subnetworks. Moreover, both models operate at the same partitioning level for a given hop, as explained above. Thus, for hop  $s$  for example,  $1 \leq s \leq h$ , both models will operate at network partition level  $h + 1 - s$ . The main differences between the models are the following. The MHGM model yields a very robust routing scheme, where nodes are prevented from receiving multiple message instances from the same source. Also, all hops are identical, if the subsets (or domains) are regarded as supernodes in a supergraph, making the routing protocol very easy to implement in a recursive fashion. On the other hand, the MHCB model yields less overall demand on wavelengths than MHGM model, but has the disadvantage of delivering multiple instances of the same message to the receiving nodes.

The major challenge for implementing either of these two models is to find a way to partition the network such that each subnetwork is connected, and the number of partitions is predetermined. We will later see that for specific cases of regular topologies partitioning is not a major issue. Furthermore, for a special instance of the binary hypercube the multi-hop routing models proposed above can achieve optimal results. However, for the case of arbitrary topologies, network

partitioning becomes the main problem to solve as the rest of the multi-hop routing steps are fairly easy to implement. For the case of arbitrary topologies we propose an interesting and novel way for network partitioning that will satisfy nearly all of the above conditions.

We observe a very interesting property regarding the number of optical receivers needed per node for non-blocking all-to-all broadcast. The next subsection introduces this result on the number of receivers.

#### 4.1.2 Bound on the Number of Optical Receivers

As we mentioned above, our approach will try to reduce both the number of wavelengths as well as the number of receivers. We consider that each node is equipped with multiple transmitters and multiple receivers.

In common practice the number of transmitters does not have to equal the number of receivers. In one-hop routing networks with arbitrary topologies, as discussed in the Chapter 3, the number of receivers per node should be  $N - 1$ , where  $N$  is the number of nodes in the network. Obviously, for large  $N$ , one-hop routing becomes physically not feasible. At the same time, by using tap-and-continue, the number of transmitters needed at node  $N_i$  to broadcast in one hop is  $d(N_i)$ , the outdegree of that node.

Thus, in the following we will concentrate on the number of optical receivers, as the number of transmitters is usually small. Next we provide an interesting theoretical result on the number of optical receivers for a certain broad model of multihop networks.

Consider a generic multi-hop routing model that satisfies the following:

- The model is based on a hierarchic network partitioning.
- Each hop corresponds to a network partition level.

We will refer to a generic model satisfying the above two conditions as a Multi-Hop on Hierarchic Partitioning (MHHP) model. In the following we develop the lower bound for an MHHP model and prove its correctness regardless of the RWA strategy employed. We will compute this bound assuming an unlimited number of wavelengths per fiber.

Consider a WDM network with arbitrary topology. We use the concept of “receiver reuse” in a similar manner analogous to “wavelength reuse”. We assume a node can use the same receiver

in different hops to receive different messages. The overall number of receivers will be therefore given by the maximum number of receivers over all hops. The attempt to minimize this number translates into finding a solution for a general *min – max* problem.

**Theorem 4.1.** *The lower bound on the number of receivers needed per node for all-to-all broadcast in an  $N$ -node network with arbitrary topology in  $h$  hops using a MHHP model is  $N^{\frac{1}{h}}$ .*

*Proof.* We show that it is impossible to use less than  $N^{\frac{1}{h}}$  receivers. Consider a destination node receiving  $N - 1$  messages in  $h$  hops. Obviously this reduces to a fan-in tree of depth  $h$ , where the root is the destination and all the other nodes are the sources. Each level corresponds to a hop (see Figure 4.2). Also the number of receivers at a node is denoted by the indegree of that node. The min-max problem reduces to finding the minimum over all maximum number of receivers at all nodes in all levels. Minimizing the number of receivers means to minimizing the maximum node in-degree at all levels.

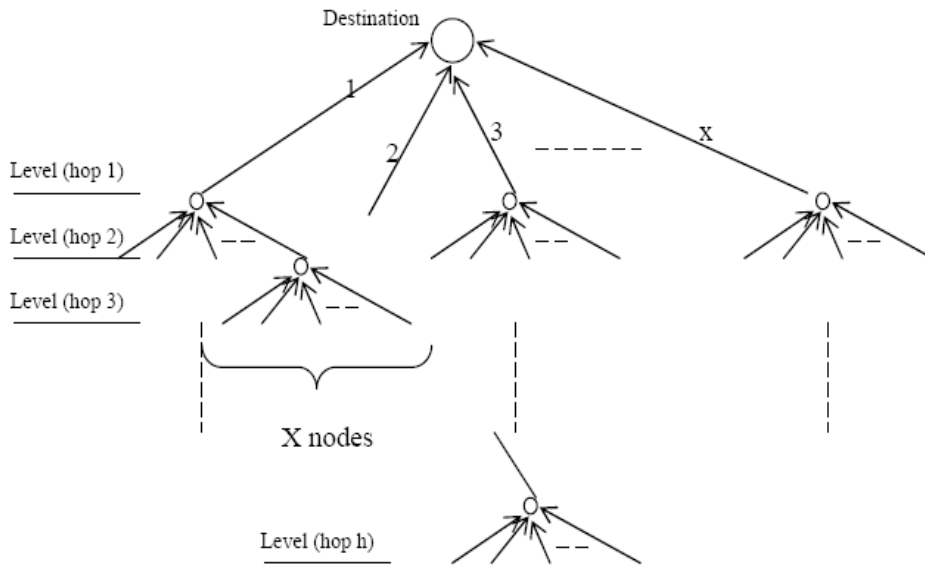


FIGURE 4.2. A fan-in tree. Nodes in each level represent the merging nodes in each hop

Thus the lower bound on the number of receivers, denoted as  $r_{min}$ , is given by:

$$r_{min} = \text{Min}(\text{Max}(\text{In}(N_i))) \text{ for all } N_i \in N \quad (4.4)$$

where the  $\text{In}(N_i)$  function represents the indegree of node  $N_i$ .

Because  $N$  is fixed, reducing the in-degree of a node will result in increasing the in-degree of another node. Thus we can conclude that to achieve the objective function 4.4, all in-degrees should be equal. We start with a balanced  $k$ -ary tree and prove by contradiction that it is the best choice for minimizing the number of receivers. Assume the in-degree is represented by  $x = r_{min}$ . To find the indegree  $x$  we have to satisfy and solve the following relation:

$$x^0 + x^1 + x^2 + \dots + x^h \geq N \quad (4.5)$$

This reduces to:

$$\frac{x^{h+1} - 1}{x - 1} \geq N$$

For  $x = 1$ , from (4.5) we get  $h + 1 \geq N$ , which obviously is not the case we are interested in. In the following we prove that for  $x = N^{\frac{1}{h}} - 1$  the inequality does not hold. To simplify the notations we use  $y = N^{\frac{1}{h}}$ . Thus, we have to prove that:

$$\frac{(y - 1)^{h+1} - 1}{y - 2} < N.$$

$$\frac{(y - 1)^{h+1} - 1}{y - 2} < \frac{(y - 1)^{h+1}}{y - 2} < N$$

From  $y = N^{\frac{1}{h}}$ , we use  $N = y^h$ ,

$$\frac{(y - 1)^{h+1}}{y - 2} < y^h.$$

Thus,

$$\frac{(y - 1)^{h+1}}{y^h} < y - 2 \Rightarrow \left( \frac{(y - 1)}{y} \right)^h < \frac{y - 2}{y - 1} \text{ and}$$

$$\left( \frac{(y - 1)}{y} \right)^h < \left( \frac{(y - 1)}{y} \right)^2 < \frac{y - 2}{y - 1}, \text{ for any integer } y > 2. \text{ Thus,}$$

$$(y - 1)^2 * (y - 1) < y^2 * (y - 2) \Rightarrow y^3 - 3 * y^2 + 3 * y - 1 < y^3 - 2 * y^2 \Rightarrow y^2 - 3 * y + 1 > 0$$

which is always true for any integer  $y > 2$ . We defined  $x = y - 1$ , therefore it is true for any  $x > 1$ . □

### 4.1.3 Necessary Conditions for Optimality

We consider the possibility of wavelength reuse to perform the required operations in different subsets in the same hop (level). Recall that the subnetworks in the same partition level are assumed to have the following properties:

- The nodes within each subnetwork are connected.
- Any two subnetworks in the same level are edge-disjoint.

In order to reduce the overall number of wavelengths  $\lambda$ , for a specific hop (partition level)  $j$ , each node subset should use the same number of wavelengths  $\lambda_j$  in that specific hop  $j$ .

Furthermore, we consider the routing to be synchronized. Synchronized multihop routing means that 2 consecutive hops do not overlap in time. Thus we can also re-use the same set of wavelengths in different hops. A good explanation of synchronous multihop routing is given in [27]. Thus, the resulting number of wavelengths needed for all-to-all broadcast in  $h$  hops should be taken as the maximum  $\lambda_j$ ,  $1 \leq j \leq h$ , taken over all  $h$  hops. Our goal is to minimize this number. Therefore, for the rest of this chapter we use the following “min-max” objective function to find the overall number of wavelengths:

$$\lambda = \min \{ \max \{ \lambda_1, \lambda_2, \dots, \lambda_j, \dots, \lambda_h \} \}$$

Consider 2-hop routing. Partition node set  $V$  into say  $s$  subsets  $V_i$ ,  $1 \leq i \leq s$ . We present an interesting and simple result for 2-hop routing that will be useful in deriving the overall number of wavelengths. We then generalize for multihop routing in more than 2 hops. We define  $\lambda_1$  to be the number of wavelengths used in the first hop and  $\lambda_2$  to be the number of wavelengths used in the second hop.

**Lemma 4.2.** *Using the 2-hop all to all broadcast models described above, the number of wavelengths*

$$\lambda = \min_s \{ \max \{ \lambda_1, \lambda_2 \} \}$$

*is minimum when  $\lambda_1 = \lambda_2$ .*

*Proof.*  $\lambda_1$  is dependent on the number of subnetworks,  $s$ . Thus we can say that  $\lambda_1$  is a non-increasing function, for increasing  $s$ . Please see Figure 4.3. Furthermore,  $\lambda_2$  is a non-decreasing

function in  $s$ . Please notice that the above is true for both routing models presented. For the MHGM case, when  $s = 1$ ,  $\lambda_1$  is actually the number of wavelengths needed to perform all-to-all broadcast in one hop, while  $\lambda_2 = 0$ . The opposite is achieved when no partition is done, i.e.  $s = N$ . Similarly, for the MHCB case, when  $s = 1$ ,  $\lambda_1$  is given by the number of wavelengths needed to collect the messages from  $N - 1$  nodes (recall that  $N = |V|$ ), while  $\lambda_2 = 1$ . When  $s = N$ ,  $\lambda_1 = 0$  and  $\lambda_2$  is the number of wavelengths needed to perform all-to-all broadcast in one hop. For both routing models, the minimum  $\lambda$  is obtained when these two functions meet. Thus

$$\lambda = \min_s \{ \max \{ \lambda_1, \lambda_2 \} \}$$

is minimum when  $\lambda_1 = \lambda_2$ . □

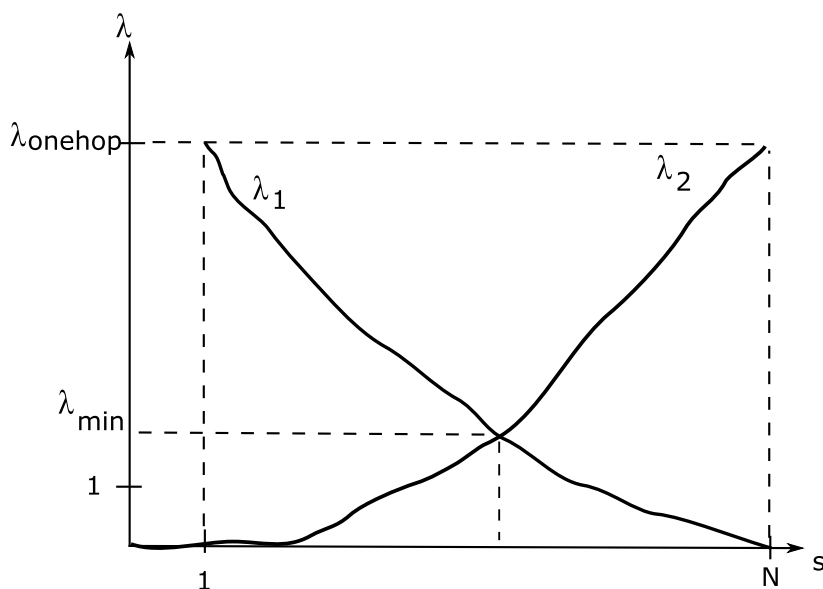


FIGURE 4.3. Example of number of wavelengths needed in 2 hops for the MHGM case, as a function of the number of subsets  $s$ .

The general result for  $h$ -hop routing is presented in the next corollary.

**Corollary 4.3.** *For both the MHGM and MHCB models, the minimum number of wavelengths*

$$\lambda = \min_s \left\{ \max_j \{ \lambda_j \} \right\}$$

*is achieved when*

$$\lambda_1 = \lambda_2 = \dots = \lambda_h. \tag{4.6}$$

*Proof.* Based on the mathematical model formulation presented earlier, decreasing the number of wavelengths in a particular hop (partition level) results in decreasing the node subsets-size of that respective level. Since the union of nodes in all subsets in any level should be the same ( $N$ ), based on relation 4.1 the size of the node subsets in some other hop (partition level) must increase, thus increasing the minimum number of wavelengths  $\lambda$ .

For both routing models based on Lemma 4.2,  $\lambda_j = \lambda_{j+1}, \forall j, 1 \leq j \leq h - 1$ . Thus, relation 4.6 is satisfied.  $\square$

## 4.2 Regular Topologies

We first introduce some simple results on the linear array which will prove useful in analyzing the special cases of regular topologies discussed next.

**Remark 4.4.** *The lower bound on the number of wavelengths to perform one-hop all-to-all broadcast in an  $N$ -node linear array is  $\lambda = N - 1$ .*

Indeed, using a simple construction we can see that the two opposing nodes, of degree one, can use two lighttrees using the same wavelength. The rest of the nodes need one wavelength per broadcast tree. Thus the total number of wavelengths is  $\lambda = N - 1$ .

For the rest of this chapter the “ceiling function” ( $\lceil \cdot \rceil$ ) will sometimes be omitted in the proofs for a better logical flow. However, because both the number of wavelengths and the number of receivers are integer numbers, the “ceiling” should always be considered.

**Lemma 4.5.** *All-to-all broadcast on the linear array in 2 hops can be realized using  $\lambda_{MHGM} = \lceil \sqrt{N} \rceil$  wavelengths for the MHGM model, or  $\lambda_{MHCB} = \lceil \sqrt{\frac{N}{2}} \rceil$  for the MHCB model, respectively.*

*Proof.* Partition the linear array into  $s$  subnetworks, each comprising of  $\lceil \frac{N}{s} \rceil$  nodes. Thus based on Lemma 4.2, and Remark 4.4, we obtain:

For MHGM:

$$\lambda_1 = \frac{N}{s} - 1 = s - 1 = \lambda_2 \tag{4.7}$$

$$N - s = s^2 - s$$

So we get an optimal partition for  $s = \lceil \sqrt{N} \rceil$ . From relation (4.7) we get,

$$\lambda_{MHGM} = s - 1 = \lceil \sqrt{N} \rceil - 1 = \lfloor \sqrt{N} \rfloor.$$

For MHCb:

$$\begin{aligned} \lambda_1 &= \frac{\frac{N}{s} - 1}{2} = s - 1 = \lambda_2 \\ \frac{N}{2s} &\approx s, \end{aligned} \tag{4.8}$$

So we get an optimal partition for  $s = \lceil \sqrt{\frac{N}{2}} \rceil$ . Again,  $\lambda = s - 1$ , thus we get

$$\lambda_{MHCb} = s - 1 = \lceil \sqrt{\frac{N}{2}} \rceil - 1 = \lfloor \sqrt{\frac{N}{2}} \rfloor.$$

Hence the proof. □

Generalizing for arbitrary number of hops  $h$  we obtain the following result.

**Theorem 4.6.** *All-to-all broadcast in  $h$  hops on the  $N$ -node linear array can be realized using  $\lambda_{MHGM} = \lfloor N^{\frac{1}{h}} \rfloor$  wavelengths for the MHGM model, and  $\lambda_{MHCb} = \lfloor \frac{1}{2^{\frac{h-1}{h}}} * N^{\frac{1}{h}} \rfloor$  for the MHCb model, respectively.*

*Proof.* Following an approach similar to the proof to Lemma 4.5, assume that partition level 2, up to partition level  $h - 1$ , have exactly  $s$  node subsets each. Based on Corollary 4.3 we get:

$$\lambda_1 = \dots \lambda_{h-1} = s - 1, \text{ for MHGM, and } \lambda_1 = \dots \lambda_{h-1} = \frac{s - 1}{2}, \text{ for MHCb.} \tag{4.9}$$

Please note that the number of partitions  $s_1$  at level 1 is given by

$$\frac{N}{s_2 * s_3 * \dots * s_{h-1}} = \frac{N}{s^{h-1}}.$$

Thus,  $\lambda_h$  for both models is given by

$$\lambda_h = \frac{N}{s^{h-1}} - 1$$

$$\lambda_1 = \lambda_h, s - 1 = \frac{N}{s^{h-1}} - 1, \text{ implies that } s = \lceil N^{\frac{1}{h}} \rceil, \text{ for the MHGM model;}$$

$$\lambda_1 = \lambda_h, \frac{s - 1}{2} = \frac{N}{s^{h-1}} - 1, \text{ implies that } s \approx \lceil (2 * N)^{\frac{1}{h}} \rceil, \text{ for the MHCb model.}$$

Thus, based on relation ( 4.9) we get:

$$\lambda_{MHGM} = s - 1 = \left\lceil N^{\frac{1}{h}} \right\rceil - 1 = \left\lfloor N^{\frac{1}{h}} \right\rfloor, \text{ and}$$

$$\lambda_{MHCB} = \frac{s - 1}{2} = \frac{\left\lfloor (2 * N)^{\frac{1}{h}} \right\rfloor}{2} = \frac{1}{2^{\frac{h-1}{h}}} * \lambda_{MHGM}$$

respectively. Hence the proof. □

Figure 4.4 shows an example of 3-hop all-to-all broadcast using MHGM on a 25-node linear array using 2 wavelengths. Notice that each hop operates on a network partition level. The wavelengths used are colored with blue and red. Please also notice that we assumed wavelength reuse in each hop.

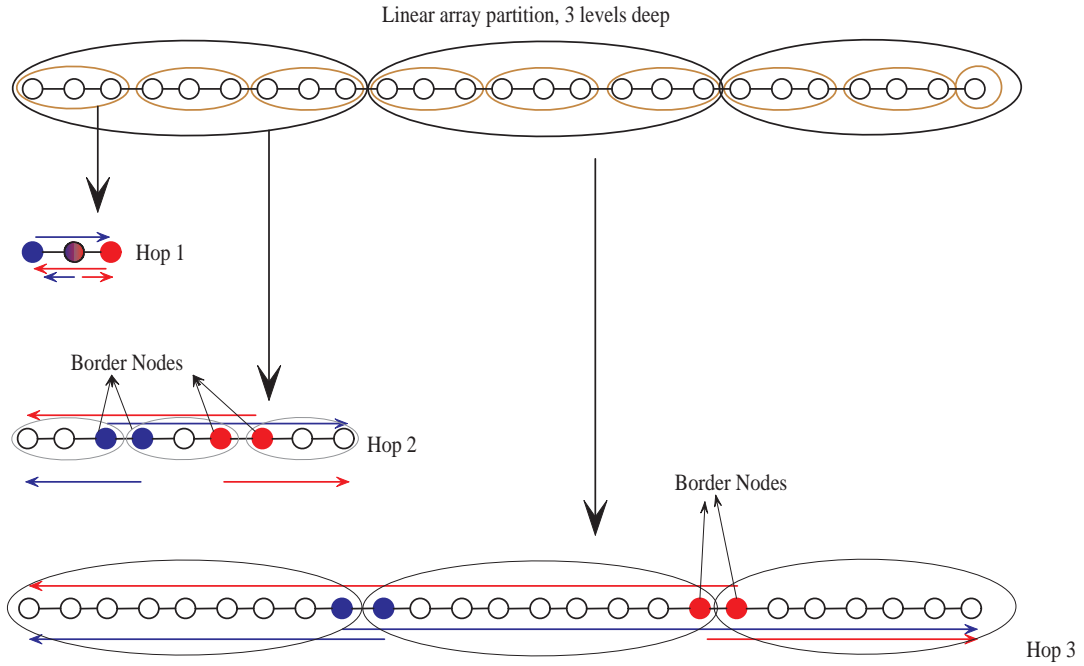


FIGURE 4.4. Example of multi-hop routing in a linear array.

Please note that the number of receivers needed is:

$$r_{MHGM} = N^{\frac{1}{h}};$$

$$r_{MHCB} = (2 * N)^{\frac{1}{h}};$$

while the number of transmitters is 2 for either model. Thus, for the linear array the MHGM model exhibits an optimal number of optical receivers.

### 4.2.1 Ring

The ring can be partitioned into subnetworks of equal size. Each subnetwork corresponds to a linear array. Thus, we will make use of the results on the linear array previously obtained. In the following we give the result on the number of wavelengths for  $h$ -hop routing in the  $N$ -node ring.

**Theorem 4.7.** *All-to-all broadcast in  $h$ -hops in an  $N$ -node ring can be realized using  $\lambda_{MHGM} = \left\lceil \left(\frac{N}{2}\right)^{\frac{1}{h}} \right\rceil$  for the MHGM model, or  $\lambda_{MHCB} = \left\lceil \frac{1}{2^{\frac{h-1}{h}}} * \left(\frac{N}{2}\right)^{\frac{1}{h}} \right\rceil$  for the MHCB model, respectively.*

*Proof.* We partition the ring as follows. The first level is comprised of  $x$  linear arrays of equal size. Then each linear array is recursively partitioned  $h - 1$  levels deep, each level consisting of  $s$  subsets. Thus the number of subsets  $x$  in the first partition level is  $x = \frac{N}{s^{h-1}}$ .

Based on Remark 4.4 we get:

$$\lambda_1 = \dots = \lambda_{h-1} = s - 1 \quad , \text{ for the MHGM model, and}$$

$$\lambda_1 = \dots = \lambda_{h-1} = \frac{s-1}{2} \quad , \text{ for MHCB model.}$$

$\lambda_h$  is the same for both models. Based on Lemma 3.1

$$\lambda_h = \left\lceil \frac{x}{2} \right\rceil = \left\lceil \frac{N}{2 * s^{h-1}} \right\rceil$$

and using Corollary 4.3 we get

$$\lambda_1 = \dots = \lambda_{h-1} = \lambda_h$$

$$s - 1 = \left\lceil \frac{N}{2 * s^{h-1}} \right\rceil, s \approx \left(\frac{N}{2}\right)^{\frac{1}{h}}, \text{ for the MHGM model, and}$$

$$\frac{s-1}{2} = \left\lceil \frac{N}{2 * s^{h-1}} \right\rceil, s \approx (N)^{\frac{1}{h}}, \text{ for the MHCB model.}$$

Thus, we get:

$$\lambda_{MHGM} = s - 1 = \left\lceil \left(\frac{N}{2}\right)^{\frac{1}{h}} \right\rceil - 1 = \left\lfloor \left(\frac{N}{2}\right)^{\frac{1}{h}} \right\rfloor$$

and

$$\lambda_{MHCB} = \frac{s-1}{2} = \left\lfloor \frac{N^{\frac{1}{h}}}{2} \right\rfloor = \left\lfloor \frac{1}{2^{\frac{h-1}{h}}} * \left(\frac{N}{2}\right)^{\frac{1}{h}} \right\rfloor.$$

Hence the proof. □

Please see Figure 4.5 for an example of 3-hop routing using the MHGM model on a ring of 31 nodes, using 2 wavelengths. Again, we use the colors blue and red for the two wavelengths employed. Also, nodes of a given color (e.g. red) will transmit using a particular wavelength (of the same node color). Thus, in the first hop, the level 3 subnetworks correspond to 3-node linear arrays. After all-to-all broadcasts among the nodes in level 3 subnetworks, each node will merge the received information into one message, ready to be transmitted in the next hop. In hop 2, the colored nodes correspond to boundary nodes  $B_i^j$  that will perform multicast to the indicated node subsets. Please notice that the nodes are prevented from receiving multiple instances of the same message. Finally, the last hop (hop 3) takes place in the ring. Here, the colored nodes represent boundary nodes  $B_i^j$  in a ring topology such that 4 boundary nodes can be selected to multicast using the same wavelength.

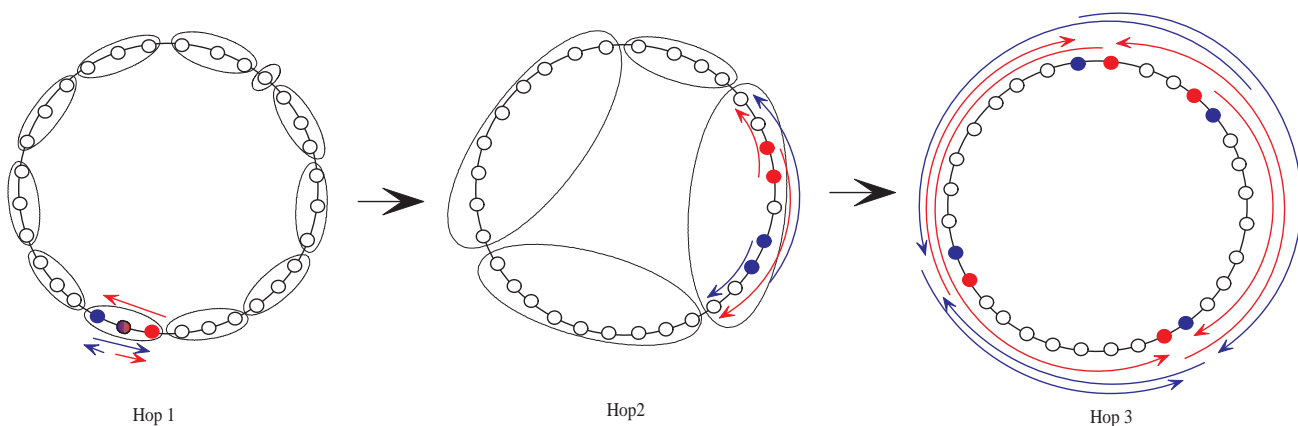


FIGURE 4.5. Example of multi-hop routing in a 31-node ring.

Next we compute the number of receivers needed per node, for multihop conflict-free all-to-all broadcast in the ring.

Consider the MHGM routing model. The first  $h - 1$  hops represent linear arrays of  $s$  subsets. Thus  $s - 1$  receivers are needed per node in the first  $h - 1$  hops. Recall that for MHGM  $s - 1$  is given by:

$$s \approx \left( \frac{N}{2} \right)^{\frac{1}{h}}.$$

In the last hop  $h$ , the ring is divided into  $x$  subsets. Thus,  $x - 1$  receivers are needed per node.

Recall that  $x$  is given by:

$$x = \frac{N}{s^{h-1}} = \frac{N}{\left(\frac{N}{2}\right)^{\frac{h-1}{h}}} = N^{\frac{1}{h}} * 2^{\frac{h-1}{h}}.$$

Thus, the number of receivers required by the MHGM model is:

$$r_{MHGM} = \text{Max} \{(s - 1), (x - 1)\} = N^{\frac{1}{h}} * 2^{\frac{h-1}{h}} - 1.$$

Observe that for a large value of  $h$ ,  $r_{MHGM}$  is close to being optimal.

Now, consider the MHCB routing model. In this case,  $s - 1$  is given by:

$$s - 1 \approx (N)^{\frac{1}{h}} - 1$$

and  $x - 1$  is given by:

$$x - 1 \approx \frac{N}{s^{h-1}} - 1 \approx N^{\frac{1}{h}}.$$

The number of receivers required by the MHCB model is:

$$r_{MHCB} = \text{Max} \{(s - 1), (x - 1)\} \approx N^{\frac{1}{h}}.$$

Notice that for multihop all-to-all broadcast in the ring, the MHCB model yields an optimal number of receivers per node.

## 4.2.2 Hypercube

Recall that an  $n$ -dimensional binary hypercube topology is defined as a connected topology with  $N = 2^n$  nodes, where each node is labeled with an  $n$ -bit binary label. Two nodes are directly connected if and only if their labels differ in exactly one bit. If two neighboring nodes differ in bit position  $k$ ,  $0 \leq k \leq n - 1$ , we will call the edge connecting these two nodes as an edge in dimension  $k$ .

The hypercube topology is very appropriate for the case of multihop routing, primarily because of its attractive partitioning properties. Next we provide a result on the number of wavelengths for routing in  $h$  hops in the hypercube.

**Lemma 4.8.** *Conflict-free all-to-all broadcast in  $h$  hops in the hypercube can be accomplished using no more than*

$$\lambda_{MHBC} = \lambda_{MHGM} = \left\lceil \frac{2^{\lceil \frac{n}{h} \rceil}}{\lceil \frac{n}{h} \rceil} \right\rceil \quad (4.10)$$

wavelengths.

*Proof.* Consider a hypercube of dimension  $n$ . Partition the  $n$  sets of dimension edges, 0 to  $n - 1$ , into the following edge subsets:

- edge subset  $S_1$ : dimensions 0 through  $\lceil \frac{n}{h} \rceil - 1$ ,
- edge subset  $S_2$ : dimensions  $\lceil \frac{n}{h} \rceil$  through  $2 * \lceil \frac{n}{h} \rceil - 1$ ,
- ...
- edge subset  $S_{h-1}$ : dimensions  $(h - 2) * \lceil \frac{n}{h} \rceil$  through  $(h - 1) * \lceil \frac{n}{h} \rceil - 1$ , and
- edge subset  $S_h$ : dimensions  $(h - 1) * \lceil \frac{n}{h} \rceil$  through  $n - 1$ .

Notice that the edge subsets defined above are edge disjoint. Furthermore, the nodes at the end of the edges in each edge subset,  $S_1$  to  $S_{h-1}$ , are connected into a hypercube of dimension  $\lceil \frac{n}{h} \rceil$ , with the last edge subset having the nodes connected into a hypercube of dimension  $n - (h - 1) * \lceil \frac{n}{h} \rceil$ .

Thus we have partitioned the hypercube of dimension  $n$  into  $h$  edge-disjoint smaller dimensional hypercubes. If one hop routing is performed at each hypercube partition, then by Theorem 3.11, only  $\lambda = \lceil \frac{2^x}{x} \rceil$  wavelengths are needed to perform complete exchange of information within each hypercube partition where  $x$  is the dimensionality of the hypercube partition. Notice that in the case of the MHCB routing model the number of wavelengths needed to collect the information at a node in an  $x$ -dimensional hypercube from all other nodes is still  $\lambda = \lceil \frac{2^x}{x} \rceil$ . Since the maximum hypercube partition in our case is  $x = \lceil \frac{n}{h} \rceil$ , the theorem follows.  $\square$

Figure 4.6 shows an example where 2-hop routing is performed in a 4 dimensional hypercube. Observe that dimensions 0 and 1 represent 2-dimensional hypercubes (colored with red), and dimensions 2 and 3 represent other disjoint 2-dimensional hypercubes (colored with blue). Using either the MHCB or MHGM model will yield 2 wavelengths.

Notice that the hypercube exhibits a very interesting property for our routing models.

All nodes of a  $\lceil \frac{2^x}{x} \rceil$  dimensional hypercube will be used as border nodes ( $B_i^j$ ) for multicasting in the next hop. Furthermore, all network partitions are edge disjoint, within the same hop (level)

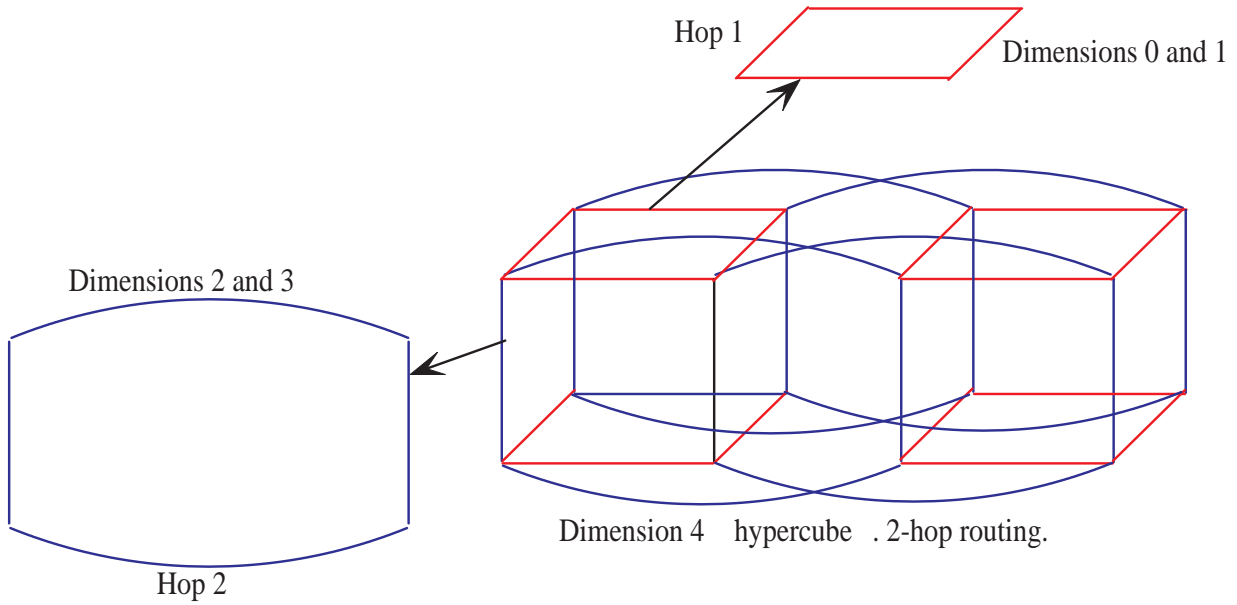


FIGURE 4.6. Example of 2-hop routing in 4-dimensional hypercube.

and also among different hops. This unique feature make our routing models very efficient on the hypercube.

Notice that the number of optical receivers needed per node for conflict-free all-to-all broadcast in the hypercube is given by the number of nodes in a hypercube partition, less one.

$$r_{MHCB} = r_{MHGM} = 2^{\lceil \frac{n}{h} \rceil} - 1$$

In the following we present the special case where multi-hop routing in the hypercube is optimal under the MHHP model. Consider the cases where  $h$  divides  $n$ . In this case all partitions will have the exact same number of subsets, which is

$$x = \left\lceil \frac{n}{h} \right\rceil = \frac{n}{h}.$$

We provide the next very interesting result for non-blocking all-to-all multi-hop broadcast in the hypercube.

**Theorem 4.9.** *Non-blocking all-to-all broadcast in the hypercube in  $h$  hops, using an MHHP model, in the case when  $h$  divides  $n$ , is optimal.*

*Proof.* We prove this result first for the number of wavelengths and then for the number of receivers.

Based on Lemma 4.8 the number of wavelengths used in each hop is:

$$\lambda = \frac{2^{\frac{n}{h}}}{\frac{n}{h}}.$$

Thus, the number of wavelengths used in each hop is the same for all hops. Based on Corollary 4.3, the number of wavelengths is optimal.

Recall that for the hypercube  $N = 2^n$ . The number of receivers in each hop is given by:

$$2^{\frac{n}{h}} = N^{\frac{1}{h}}$$

which is also optimal based on Theorem 4.1. □

### 4.3 Arbitrary Topologies

As we expected, the most difficult problem we have to solve in order to implement one of the two routing models described in Section 4.1.1 is to find a hierarchical network partitioning into a predetermined number of subnetworks, such that each subnetwork is connected and such that each subnetwork contains a specific number of nodes.

The problem of graph partitioning was studied in the literature mainly in two contexts. In the context of VLSI, where the graph is partitioned in two (or multiple) subgraphs, such that the total edge weight for the edges connecting the partitions is minimized. The other context was “clustering” in the area of parallel processing and multiprocessor systems. For this approach the topologies are considered regular and the nodes within a cluster are not necessarily connected. These models are not suitable for our context since the conditions to be satisfied for our network partitioning case are completely different.

As mentioned earlier network partitioning for conflict-free optical multi-hop all-to-all broadcast should ideally satisfy the following conditions:

- (1) The number of subnetworks (subgraphs) is predetermined and given;
- (2) The nodes in each subnetwork are connected; and
- (3) The number of nodes inside each subnetwork is constrained to a specific value.

In this subsection we consider arbitrary topologies. Thus we cannot make any assumption on the subnetworks’ edge connectivity. We only require the subnetworks be 1-connected. If a 1-connected subnetwork is also maximally edge connected then the number of wavelengths needed for performing a hop in MHGM must match the number of nodes in that subnetwork. For the

MHCB model the number of wavelengths needed for collecting the information will be upper-bounded by half the number of nodes in that specific subnetwork.

In the following we will see that our heuristic algorithm satisfies conditions (1) and (2) above. However, condition (3) becomes: All subnetworks will “almost” have the same node-size.

Next subsection introduces the concept of “virtual perfect matching”, and presents the grounds for the graph partitioning needed for conflict-free all-to-all broadcast.

### 4.3.1 “Virtual Perfect Matching” and Network Partitioning

The general definition of a perfect matching is taken from reference [26] and is restated below for convenience.

**Definition 4.10.** *A matching  $M$  in a graph  $G$  is a subset of edges no two of which have a common vertex. A matching is perfect if every vertex in  $G$  is incident to some edge in the matching.*

However, a perfect matching does not always exist for an arbitrary graph. Tutte presented the necessary condition for a graph to have a perfect matching in his famous “1-Factor Theorem”. This result is presented in detail in references [84] and [26]. Furthermore, some specific results can be found on some regular graphs such as Petersen’s theorem on the 3-regular 2-edge-connected graph and Anderson’s result on graphs of even order [26].

We attempt to partition the network based on a perfect matching. A perfect matching will partition the network into subnetworks each consisting of exactly 2 nodes that are connected by a matching edge. Thus conditions (2) and (3) for MHHP partitioning are met. We define the following concept of virtual perfect matching. Notice that this will apply only for our case of network partitioning.

**Definition 4.11.** *A virtual perfect matching represents  $\lfloor \frac{N}{2} \rfloor$  edge disjoint paths (virtual edges), connecting  $N$  (or  $N - 1$ , for the  $N = \text{odd}$  case) distinct nodes.*

Based on this definition we present the following simple but interesting result on the tree which represents the worst maximally edge connected topology with edge connectivity = 1.

**Lemma 4.12.** *Any tree has a virtual perfect matching.*

*Proof.* We prove that we can always find a number of edge-disjoint paths (virtual edges) in the tree such that removal of these paths will not disconnect the tree. Thus, we can recursively find edge-disjoint paths to cover all nodes in the tree.

Take any leaf node in the tree. Denote its predecessor as  $X$ . There are 2 cases:

- **Case I.**  $X$  has an odd number of successors; say  $2 * y + 1$ . Couple  $2 * y$  successors in  $y$  pairs using one path per pair. Couple the last predecessor with  $X$  using another path. The removal of these  $y + 1$  edge-disjoint paths will leave a connected tree.
- **Case II.**  $X$  has an even number of successors; say  $2 * y$ . Couple  $2 * y$  successors in  $y$  pairs using one path per pair. The removal of these  $y$  edge-disjoint paths will leave a connected tree. □

Figure 4.7 shows a virtual perfect matching example on a tree topology. Two nodes of the same color are considered to form a virtual edge in the virtual matching. Figure 4.8 shows the resulting topology by considering each virtual edge a supernode.

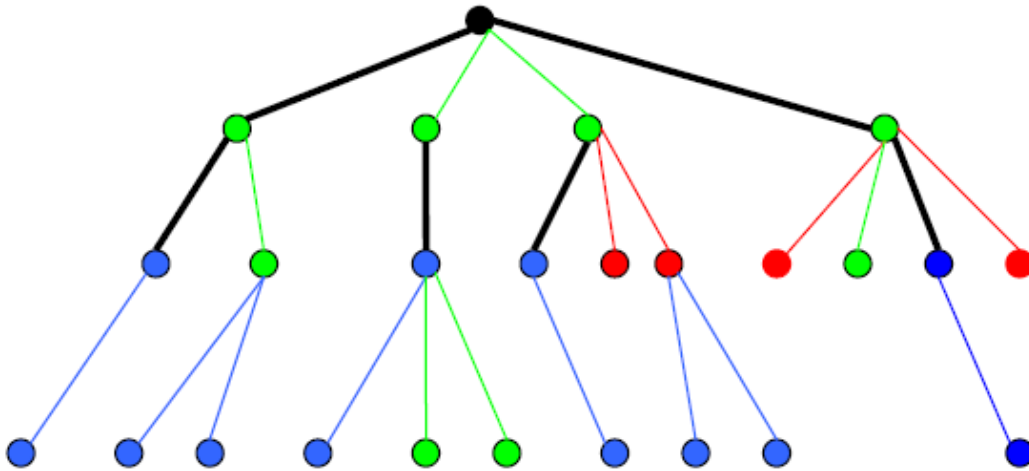


FIGURE 4.7. Example a virtual perfect matching in the tree.

If each node pair is considered a supernode then the resulting structure is a connected graph of supernodes. Thus, each supernode consists of a matching pair in the resulting graph. One important observation should be made on the resulting graph. Please notice the dotted bold lines in Figure 4.8. This represents a specific case where one physical link in the original tree connects more than two supernodes in the resulting graph. For our purpose of applying the virtual perfect matching algorithm recursively, only one dotted bold line should be considered in the final graph

representation. Both were shown in Figure 4.8 to emphasize that any one can be chosen for the final graph representation.

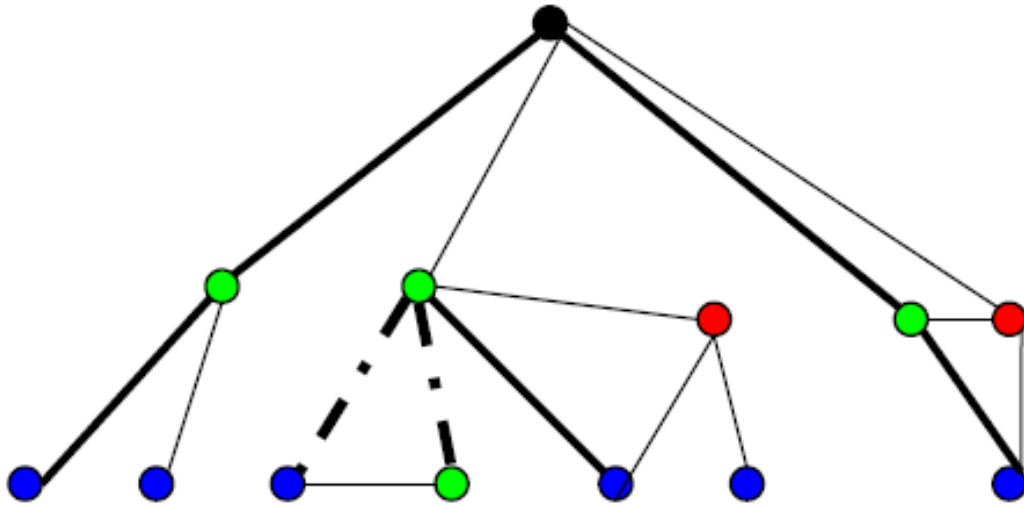


FIGURE 4.8. The connected resulting graph. Each colored supernode has 2 nodes. The dotted lines represent the following: there is only one physical link connecting one node to multiple nodes, thus only one dotted line has to be considered for the final graph representation.

Finally, the steps of an algorithm producing a virtual perfect matching in a tree can be simply described as follows:

**VPM Algorithm:**

1. While (there are uncolored nodes)
2. For any uncolored leaf
3. Color  $y$  pairs according to Case I or Case II;
4. Remove the  $y$  pairs from the tree;
5. Endfor;
6. Endwhile;

Thus, recursively, all nodes in the tree (except perhaps one) will be paired together resulting in the virtual perfect matching. Next we propose a simple but very efficient RWA heuristic for the case of non-blocking all-to-all multi-hop broadcast in arbitrary topologies.

### 4.3.2 Routing Heuristic

We start this subsection by presenting the following simple result.

**Theorem 4.13.** *Any connected graph has a virtual perfect matching.*

*Proof.* The proof follows directly from Lemma 4.12. Any connected graph has a spanning tree. Any tree has a perfect matching. Hence the proof.  $\square$

The virtual perfect matching operation can recursively be applied a maximum of  $\log N$  times. After  $\log N$  virtual perfect matchings the resulting supergraph consists of only one supernode comprising all  $N$  nodes in the network.

After  $p$  recursive such virtual perfect matchings, a supernode will have  $2^p$  nodes. Notice that whenever the graph has an odd number of nodes (or supernodes) one node/supernode will not be considered. This will result in some supernodes to have less than  $2^p$  nodes. A simple addition to the VPM algorithm above would be to uniformly distribute the nodes within the virtual perfect matching supernodes. However, this topic is outside the scope of our interest here. We consider the upper bound on the number of wavelengths. This is achieved when the supernodes consist of maximum number of nodes ( $2^p$  nodes in this case).

We can conclude that the number of supernodes will be then given by  $\frac{N}{2^p}$ .

The next result establishes the number of wavelengths needed for  $h$ -hop all-to-all broadcast in networks with arbitrary topologies under the special conditions summarized by the MHHP model.

**Theorem 4.14.** *All-to-all non-blocking broadcast using the MHGM routing model in  $h$ -hops can be realized in an  $N$ -node network with arbitrary topology and connectivity  $k$  using  $\lambda = 2^p$  wavelengths, where  $p$  is given by:*

$$\left\lceil \frac{1}{h} * \log \left( \frac{N}{k} \right) \right\rceil.$$

*Proof.* The first partition has connectivity  $k$ . For the rest of the levels the connectivity is 1. As mentioned above, the number of wavelengths needed in an MHGM hop when each subnetwork has an edge-connectivity of 1 equals the number of nodes/supernodes in that subnetwork. We have

shown above that this number is  $2^p$ . Thus we have to satisfy:

$$\frac{N}{k * 2^{p*(h-1)}} = 2^p, \text{ or}$$

$$2^p = \left(\frac{N}{k}\right)^{\frac{1}{h}},$$

Thus,

$$p = \log \left[ \left(\frac{N}{k}\right)^{\frac{1}{h}} \right] = \left\lceil \frac{1}{h} * \log \left(\frac{N}{k}\right) \right\rceil,$$

where  $p$  represents the number of virtual perfect matchings to perform until a new connected domain is achieved. Thus, the number of wavelengths required for  $h$  hop routing is  $\lambda = 2^p$ .  $\square$

Please notice that  $p$  is taken above as the ceiling. Thus:

$$\lambda = 2^p \neq \left(\frac{N}{k}\right)^{\frac{1}{h}}$$

The case of MHCB is very similar with only one exception. The number of wavelengths needed for the collection operation will equal half the number of nodes/supernodes in the subnetwork. Thus we will consider  $\frac{2^p}{2} = 2^{p-1}$  wavelengths needed for each of the  $h - 1$  hops.

**Theorem 4.15.** *All-to-all non-blocking broadcast using the MHCB routing model in  $h$ -hops can be realized in an  $N$ -node network with arbitrary topology and connectivity  $k$  using  $\lambda = 2^{p-1}$  wavelengths, where  $p$  is given by:*

$$p = \left\lceil \frac{1}{h} * \log \left(\frac{2 * N}{k}\right) \right\rceil \tag{4.11}$$

*Proof.* Similar to the proof to Theorem 4.14, the first partition has connectivity  $k$ . For the rest of the levels the connectivity is 1. Thus we have to satisfy:

$$\frac{N}{k * 2^{p*(h-1)}} = 2^{p-1},$$

$$2^p = \left(\frac{2 * N}{k}\right)^{\frac{1}{h}}.$$

Thus,

$$p = \log \left[ \left(\frac{2 * N}{k}\right)^{\frac{1}{h}} \right] = \left\lceil \frac{1}{h} * \log \left(\frac{2 * N}{k}\right) \right\rceil.$$

Hence the proof.  $\square$

Based on the previous two theorems a simple RWA heuristic can be designed for multi-hop non-blocking all-to-all broadcast in arbitrary topologies. In the following we show the main steps of such a heuristic algorithm.

Find  $p$  based on relation ( 4.11)

For  $i=1:h$

For  $j=1:p$

Find a spanning tree;

Apply VPM algorithm;

Redraw the resulting graph;

End for;

Define each connected supernode as a hop domain;

Save each domain;

End for;

In each hierarchical perform the specific MHGM or MHCB steps.

Next we compute the number of receivers needed per node for multihop conflict-free all-to-all broadcast in arbitrary topologies.

MHGM case:

In each of the first  $h - 1$  hops  $2^p - 1$  receivers are needed. In the last hop we have  $\frac{N}{k * 2^{p*(h-1)}}$  subnetworks. Thus the number of receivers needed is given by:

$$r_{MHGM} = \max \left\{ 2^p - 1, \frac{N}{k * 2^{p*(h-1)}} - 1 \right\}.$$

MHCB case:

In each of the first  $h - 1$  hops  $2^{p-1} - 1$  receivers are needed. In the last hop we have  $\frac{N}{k * 2^{p*(h-1)}}$  subnetworks. Thus the number of receivers needed is given by:

$$r_{MHCB} = \max \left\{ 2^{p-1} - 1, \frac{N}{k * 2^{p*(h-1)}} - 1 \right\}.$$

**Example:**

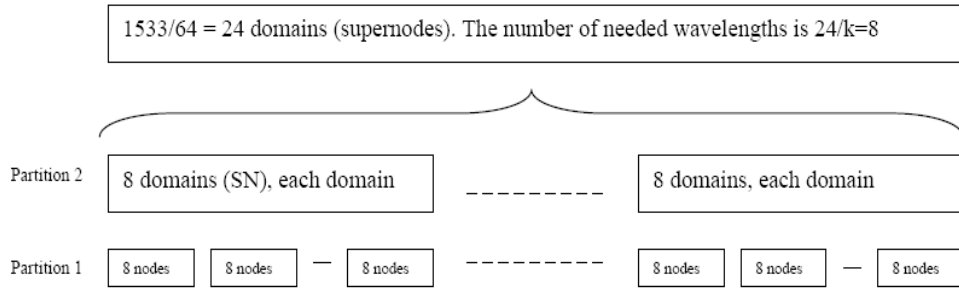


FIGURE 4.9. Example of 3-hop all to-all broadcast using the heuristic for the arbitrary topologies.

Consider any arbitrary topology with the following properties:  $N = 1533$ ,  $k = 3$ ,  $h = 3$ . We will try to find the upper-bound on the number of wavelengths and provide a network partitioning to apply the MHGM model in 3 hops. Based on the above theorem,  $p = 3$ . Thus we would need 8 wavelengths. Figure 4.9 shows the network partitioning. It is easy to follow the MGHM steps using this figure. In each hop exactly 8 wavelengths are to be used. Thus 8 is an upper-bound on the number of wavelengths.

Notice that the RWA heuristic presented above produces extremely good results, as the number of wavelengths, as well as the number of receivers are still of the order  $O(N^{\frac{1}{h}})$ .

# Chapter 5

## Conclusions

This thesis presented a number of interesting theoretical results on reducing the optical resources needed for conflict-free all-to-all broadcast in WDM optical networks. The all-to-all broadcast communication pattern has been at the core of our efforts since it may be essential in implementing link state information dissemination mechanisms, needed by any routing algorithm and its traffic engineering protocols.

We studied the problem of minimizing the number of wavelengths needed for non-blocking all-to-all broadcast in one hop in WDM networks in the absence of wavelength converters. The RWA (routing and wavelength assignment) problem for global exchange of information was shown in [6] to be an NP-complete problem formulating it as a path coloring problem. We have theoretically shown that when considered along with tap-and-continue and split capable nodes, the RWA problem may be solvable in polynomial time. In our case the problem of path coloring becomes a disjoint tree coloring problem such that the number of colors (wavelengths) is minimized. To the best of our knowledge our work is a first attempt at solving such a problem for optical networks. We offered optimal solutions on the number of wavelengths for specific regular topological cases such as the ring, torus and the binary hypercube. We also presented a very interesting optimal solution for the maximally edge-connected general networks with arbitrary topologies based on a result by Edmonds [24]. Furthermore, we extended our research for the more involved case of non-maximally edge-connected topologies. We have presented a near-optimal solution along with an RWA algorithm.

We have also addressed the more practical case of conflict-free all-to-all broadcast in multiple hops. We have presented an interesting result on the number of receivers per node needed to perform all-to-all broadcast. We have shown that this bound is applicable to a large class of multi-hop RWA models. We have presented two attractive models for multihop all-to-all broadcast along with the algorithms for route selection and wavelength assignment. We have demonstrated that

the number of wavelengths needed to perform non-blocking multihop all-to-all broadcast decreases considerably, compared to the one hop case, when using our routing models. Furthermore, we have shown that the number of receivers is drastically reduced as well, and that it is close to the lower bound theoretically derived. We applied the two routing models to the ring and the binary hypercube. The results obtained are optimal for the ring and some special instances of the hypercube. Finally, we have presented a heuristic for applying these two models to networks with arbitrary topologies. A “virtual perfect matching” scheme has been proposed for the first time for the purpose of recursive network partitioning in an efficient way from a multihop routing viewpoint.

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# Vita

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