APPENDIX A

HEC RAS 3.0 GOVERNING EQUATIONS

The HEC RAS 3.0 unsteady solver is adapted from Barkau's UNET model (Brunner, 2001). UNET utilizes a four point implicit scheme to solve the unsteady flow equations (Barkau, 1997). The continuity and momentum equations are:

$$\frac{\partial A}{\partial t} + \frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} - q_l = 0 \qquad \text{Continuity}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (VQ)}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f\right) = 0 \qquad \text{Momentum}$$

where:

x	=	distance along the channel
t	=	time
Q	=	flow
A	=	cross-sectional area
S	=	storage
q_l	=	lateral inflow per unit distance

The finite difference approximations of the terms in the continuity equation are:

$$\Delta Q = (Q_{j+1} - Q_j) + \theta(\Delta Q_{j+1} - \Delta Q_j)$$

$$\frac{\partial A_c}{\partial t}\Delta x_c \qquad 0.5\Delta x_{cj}\frac{\left(\frac{dA_c}{dz}\right)_j \Delta z_j + \left(\frac{dA_c}{dz}\right)_{j+1} \Delta z_{j+1}}{\Delta t}$$

$$\frac{\partial A_f}{\partial t}\Delta x_f \qquad 0.5\Delta x_{fj}\frac{\left(\frac{dA_f}{dz}\right)_j \Delta z_j + \left(\frac{dA_f}{dz}\right)_{j+1} \Delta z_{j+1}}{\Delta t}$$

$$\frac{\partial S}{\partial t}\Delta x_f \qquad 0.5\Delta x_{fj}\frac{\left(\frac{dS}{dz}\right)_j \Delta z_j + \left(\frac{dS}{dz}\right)_{j+1} \Delta z_{j+1}}{\Delta t}$$

The finite difference approximations of the terms in the momentum equation are:

$\dfrac{\partial(Q_c\Delta x_c + Q_f\Delta x_f)}{\partial t\Delta x_e}$	$\dfrac{0.5}{\Delta x_e\partial t}\left(\partial Q_{cj}\Delta x_{cj} + \partial Q_{fj}\Delta x_{fj} + \partial Q_{cj+1}\Delta x_{cj} + \partial Q_{fj+1}\Delta x_{fj}\right)$
$\dfrac{\Delta\beta VQ}{\Delta x_{ej}}$	$\dfrac{1}{\Delta x_{ej}}\left[(\beta VQ)_{j+1} - (\beta VQ)_j\right] + \dfrac{\theta}{\Delta x_{ej}}\left[(\beta VQ)_{j+1} - (\beta VQ)_j\right]$
$g\bar{A}\dfrac{\Delta z}{\Delta x_e}$	$g\bar{A}\left[\dfrac{z_{j+1} - z_j}{\Delta x_{ej}} + \dfrac{\theta}{\Delta x_{ej}}(z_{j+1} - z_j)\right] + \theta g\Delta\bar{A}\dfrac{(z_{j+1} - z_j)}{\Delta x_{ej}}$
$g\bar{A}\left(\bar{S}_f + \bar{S}_h\right)$	$g\bar{A}\left(\bar{S}_f + \bar{S}_h\right) + 0.5g\bar{A}\left[(\Delta S_{fj+1} + \Delta S_{fj}) + (\Delta S_{hj+1} + \Delta S_{hj})\right]$ $+ 0.5\theta g\left(\bar{S}_f + \bar{S}_h\right)(\Delta A_j + \Delta A_{j+1})$
$\bar{A}$	$0.5(A_j + A_{j+1})$
$\bar{S}_f$	$0.5(S_{fj+1} + S_{fj})$
∂A_j	$\left(\dfrac{dA}{dZ}\right)_j\Delta z_j$
∂S_{fj}	$\left(\dfrac{-2S_f}{K}\dfrac{dK}{dz}\right)_j\Delta z_j + \left(\dfrac{2S_f}{Q}\right)_j\Delta Q_j$
$\partial\bar{A}$	$0.5(\Delta A_j + \Delta A_{j+1})$

Subscript *c* indicates channel, *f* indicates overland flow, and *j* indicates the space domain.

APPENDIX B

QUAL2E GOVERNING EQUATIONS

QUAL2E is a water quality model that can simulate up to 15 water quality constituents. For this investigation, only nitrogen is modeled. The model is applicable to dendritic streams that are well mixed. Each stream reach is divided into a number of computational elements wherein a hydrologic balance for stream flow and material balance in terms of concentration are written. Both advective and dispersive mechanisms are considered in the material balance. Mass is gained or lost from the computational element by transport, injections or withdrawals, as well as internal process such as release from benthic sources or biological transformations (Brown, 1987).

Hydraulically, the program computes a series of steady state water surface profiles. The calculated stream flow rate serves as a basis for determining the mass fluxes into and out of each element due to flow. For constituent evolution and transport, QUAL2E solves for the concentration using the implicit backward finite difference method.

For nitrogen specifically, the nitrogen cycle is divided into organic nitrogen, ammonia nitrogen, nitrite nitrogen, and nitrate nitrogen compartments. The model takes into account the stepwise transformation of organic nitrogen to ammonia, to nitrite, and finally to nitrate.

Assumptions of the model include that each computational element is completely mixed and that the hydraulic regime is steady state, and that the major transport mechanisms and advection and dispersion are significant only along the main direction of flow.

For mass transport, the governing equations are:

$$\frac{\partial C}{\partial t} = \frac{\partial\left(A_x D_x \frac{\partial C}{\partial x}\right)}{A_x \partial x} - \frac{\partial\left(A_x \bar{u} C\right)}{A_x \partial x} \frac{dC}{dt} + \frac{s}{V}$$

where:

x	=	distance
C	=	concentration
t	=	time
D_x	=	dispersion coefficient
A_x	=	cross-sectional area
s	=	external sources or sinks
$\bar{u}$	=	mean velocity

The right hand terms respectively represent dispersion, advection, constituent changes due to growth and decay and external sources and sinks.

The equations governing the transformation of nitrogen from one form to another are:

$$\frac{dN_4}{dt} = \alpha_1 \rho A - \beta_3 N_4 - \sigma_4 N_4 \qquad \text{Organic nitrogen}$$

$$\frac{dN_1}{dt} = \beta_3 N_4 - \beta_1 N_1 + \frac{\sigma_3}{d} - F_1 \alpha_1 \mu A \qquad \text{Ammonia nitrogen}$$

$$\frac{dN_2}{dt} = \beta_1 N_1 - \beta_2 N_2 \qquad \text{Nitrite nitrogen}$$

$$\frac{dN_3}{dt} = \beta_2 N_2 - (1 - F)\alpha_1 \mu A \qquad \text{Nitrate nitrogen}$$

where:

N_1 = concentration of ammonia nitrogen

N_2 = concentration of nitrate nitrogen
N_3 = concentration of nitrite nitrogen
N_4 = concentration of organic nitrogen
α_1 = fraction of algal biomass that is nitrogen
β_1 = rate constant for the biological oxidation of ammonia nitrogen, temperature dependent
β_2 = rate constant for the biological oxidation of nitrate nitrogen, temperature dependent
β_3 = rate constant for hydrolysis of organic nitrogen to ammonia nitrogen, temperature dependent
ρ = algal respiration rate
A = algal biomass concentration
σ_3 = benthos source rate for ammonia nitrogen
σ_4 = rate coefficient for nitrogen settling, temperature dependent
d = mean depth of flow
F_1 = fraction of algal nitrogen uptake from ammonia pool
μ = local specific growth rate of algae
P_N = preference factor for ammonia nitrogen

APPENDIX C

RMA2 GOVERNING EQUATIONS

RMA2 solves the depth integrated equations of mass and momentum in the two horizontal directions (Donnel, 1997). The solved equations are:

$$h\frac{\partial u}{\partial t}+hu\frac{\partial u}{\partial x}+hv\frac{\partial u}{\partial y}-\frac{h}{p}\left(E_{xx}\frac{\partial^2 u}{\partial x^2}+E_{xy}\frac{\partial^2 u}{\partial y^2}\right)+gh\left(\frac{\partial a}{\partial x}+\frac{\partial h}{\partial x}\right)$$
$$+\frac{gun^2}{(1.486h^{1/6})^2}+\left(u^2+v^2\right)^{1/2}-\zeta V_a^2\cos\psi-2h\omega v\sin\phi=0 \qquad (1)$$

$$h\frac{\partial v}{\partial t}+hu\frac{\partial v}{\partial x}+hv\frac{\partial v}{\partial y}-\frac{h}{p}\left(E_{yx}\frac{\partial^2 v}{\partial x^2}+E_{yy}\frac{\partial^2 v}{\partial y^2}\right)+gh\left(\frac{\partial a}{\partial y}+\frac{\partial h}{\partial y}\right)$$
$$+\frac{gvn^2}{(1.486h^{1/6})^2}+\left(u^2+v^2\right)^{1/2}-\zeta V_a^2\sin\psi-2h\omega v\sin\phi=0 \qquad (2)$$

$$\frac{\partial h}{\partial t}+h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=0 \qquad (3)$$

where:

h	=	depth
u,v	=	velocities in the Cartesian directions
x,y,t	=	Cartesian coordinates and time
ρ	=	fluid density
E	=	Eddy viscosity coefficient, for *xx* = normal direction on *x* axis surface; for *yy* = normal direction on *y* axis surface; for *xy* and *yx* = shear direction on each surface
g	=	acceleration due to gravity
a	=	bottom elevation
n	=	Manning's roughness coefficient
ζ	=	empirical wind shear coefficient
V_a	=	wind speed
ψ	=	wind direction

ω	=	rate of earth's angular rotation
φ	=	local latitude

Equations 1,2, and 3 are solved by the finite element method using the Galerkin Method of weighted residuals. The shape functions are quadratic for velocity and linear for depth. Integration in space is performed by Gaussian integration. Derivatives in time are replaced by a nonlinear finite difference approximation. Variables are assumed to vary over each time interval in the form

$$f(t) = f(0) + at + bt^x \qquad t_0 \le t < t_o + \Delta t$$

which is differentiated with respect to time, and cast in finite difference form. The solution is fully implicitly and the set of simultaneous equations is solved by Newton-Raphson non-linear iteration. The computer code executes the solution by means of a front type solver, which assembles a portion of the matrix and solves it before assembling the next portion of the matrix.

APPENDIX D

RMA4 GOVERNING EQUATIONS

RMA4 solves the depth integrated equations of the transport and mixing process (Donnel, 2001). The form of the depth averaged equations is:

$$h\left(\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}-\frac{\partial}{\partial x}D_x\frac{\partial c}{\partial x}-\frac{\partial}{\partial y}D_y\frac{\partial c}{\partial y}-\sigma+kc+\frac{R(c)}{h}\right)=0$$

where

h	=	depth
u,v	=	velocities in the Cartesian directions
x,y,t	=	Cartesian coordinates and time
c	=	concentration of pollutant for a given constituent
D_x, D_y	=	turbulent mixing (dispersion) coefficient
k	=	first order decay of pollutant
σ	=	source/sink of constituent
R(c)	=	rainfall/evaporation rate

This equation is solved by the finite element using Galerkin weighted residuals. Spatial integration of the equations is performed by Gaussian techniques and the temporal variations are handled by nonlinear finite differences similar to the method described for RMA2.

VITA

Stephan Alexander Capps was born on April 21, 1977, in Stuttgart, Federal Republic of Germany. After graduating high school in Fayetteville, North Carolina, he enlisted in the United States Army as an Infantryman and served at Fort Campbell, Kentucky. In 1985, he attended the United States Military Academy at West Point, New York, and graduated with a bachelor of science in 1989.

Upon receiving his commission as Second Lieutenant in the U. S. Army Corps of Engineers, he was stationed in Germany with the 7th Engineer Brigade in Karlsruhe. During his tour there, he served in three engineer battalions, including the 249th Engineer Battalion with which he deployed to Operations Desert Shield and Desert Storm in Southwest Asia.

After his tour of duty in Germany, he was assigned to Fort Lewis, Washington, and served as a battalion operations officer, United Nations Military Observer in the Western Sahara, brigade training officer, and Company Commander of Headquarters Support Company, 864th Engineer Battalion, and A Company, 249th Engineer Battalion.

In 1999, he was assigned as adviser to the 1088th Engineer Battalion, Louisiana National Guard. He started pursuing a master's degree at Louisiana State University at this time and graduated in May 2003.

Stephan Capps is currently a Major in the United States Army and is serving as a Project Engineer with the Far East District, U. S. Army Corps of Engineers, at Camp Casey, Republic of Korea.