EXPERIMENTAL AND COMPUTATIONAL MODELING
OF GRANULAR MATERIALS

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ABSTRACT

4-inch cubical sand specimens were been tested under drained conditions using a wide range of stress paths. The testing was performed by means of a true (cubical) triaxial device that has been conditioned for this study. This testing set-up consisted of a servo-controlled, single-boundary type apparatus, with six flexible membranes that create a center cavity surrounding the soil specimens. This thesis illustrates the functioning of the triaxial device, including detailed descriptions of the device components, sample preparation, and a practical application related to the mechanical characterization of granular materials using critical state soil mechanics. An experimental program, based on the above-described testing device, was created and implemented in order to calibrate a simple constitutive model (Cam Clay Model), and to compare the predictions of this model with the observed mechanical behavior of the selected material under triaxial state of stress. The results of the testing program for this study are typically presented in the Deviatoric Stress vs. Shear Strain Plane, and in the Triaxial Space. These results indicated that the testing program implemented for the calibration of the Cam Clay Model was successful in reproducing the general mechanical behavior of the granular material tested herein. General conclusions and recommendations for further research are included in at the end of this paper.
CHAPTER 1

INTRODUCTION

1.1 General

The understanding of the mechanical behavior of granular material under different states of stress conditions is of great importance for the analysis of possible failure within a soil mass that supports structures such as embankments, piles, spread footings, earth dams, etc. In diverse situations, these structures may induce a wide range of loading conditions to the soil mass beneath them, causing deformations that, in extreme cases, may lead to a total collapse. This mechanical behavior is directly associated with the strength and deformation characteristics of soils.

In order to study the strength and deformation characteristics of a granular material, an experimental program, based on a three-dimensional setting, is proposed. This three-dimensional setting will mimic different stress states that a soil may undergo due to external loading and the testing device will measure the consequent deformation. Furthermore, stresses should be applied independently in three principal directions and deformations be measured along the same three orthogonal directions.

In the case of a cubical (“true”) triaxial apparatus, the implementation of the experimental program will bring an extensive understanding of the soil stress-strain-strength behavior under multiaxial states of stress. With this information, predictions of the soils response to loading may be made by means of constitutive models. The selection, development and calibration of a constitutive model will depend on the analysis of a specific boundary assessment problem. The experimental program should examine the material under several sets of loading conditions and confining pressures. These tests will in assessing
strength and deformation characteristics of the material, and, in doing so, allowing for modeling of the soil behavior by means of “calibrated” constitutive models.

1.2 Scope of Work

The understanding of the mechanical response of a granular material upon being subjected to multiaxial loading conditions is the focus of this research work. Furthermore, this knowledge will help to clarify the elaboration of analytical studies commonly found in geotechnical engineering involving granular materials.

The results of a group of true triaxial tests will be used to acquire the necessary data for the analysis and numerical modeling of the observed soil behavior. The tests are drained, stress-controlled true triaxial test using 4.00” cubical specimens of poor-graded sand (SP). The data is also used to assess the nature of the major, intermediate, and minor principal stress-strain responses of soil specimens under selected stress paths commonly used in advanced soil modeling. Specifically, Triaxial Compression (TC), Triaxial Extension (TE) and Simple Shear (SS) stress paths will be used. Some of these stress paths cannot be attained in a conventional triaxial device.

Also, from the testing results, the stress-strain-strength characteristics of the material are analyzed in the octahedral plane \((\sigma_1 : \sigma_2 : \sigma_3)\), and the influence of the void ratio on the shape of the yield surface on this plane is studied.

Prior to the presentation of the experimental plan, a review of the previous work on the use of multiaxial apparatuses is carried out. In the same way, the review of articles on critical state constitutive models serves as a platform for the computational analysis and modeling of granular materials with the modified Cam-Clay model (MCCM), proposed by Roscoe and Burland (1968).
1.3 Thesis Content

A brief description of the organization of the chapters that form this thesis follows:

Chapter 2 provides a summary of previous work reviewed for this investigation in the area of multiaxial soil testing, constitutive numerical modeling of soil’s response, and the behavior of granular materials.

Chapter 3 presents a review of the cubical triaxial device, the experimental apparatus used in this study, and a step-by-step description of the specimen preparation procedures for a cohesionless material.

Chapter 4 presents a detailed description of the Critical State Soil Mechanics (CSSM) based plasticity model. This includes details of CSSM, elastic-plastic model of soil’s behavior, and the implementation of a numerical driver to model this behavior.

Chapter 5 summarizes the experimental program that produced the data for the experimental and numerical analyses of the granular material.

Chapter 6 includes the experimental analysis of the behavior of the material in an octahedral plane. In addition, it presents the results of the comparison between the experimental and computational analyses.

Finally, chapter 7 presents a summary, conclusions, and a list of recommendations for future research.
CHAPTER 2
LITERATURE REVIEW

Sture and Desai (1979) developed an innovative multiaxial cubical test apparatus that uses fluid cushions to apply a three-dimensional stress state to a 4.00-inch cubical sample. In this device the deformations are measured with groups of linear variable displacement transducers placed in each face of the cubical cell. In their paper, the writers emphasized the simplicity of the apparatus assembly, and the uncomplicated procedure for the sample preparation. An agreement was found while comparing the results from this multiaxial apparatus with those produced by conventional triaxial devices in specific instances where such comparisons were possible. Aspects found in earlier multiaxial apparatuses (such as boundary interference problems) do not appear to be significant in this investigation. In general, this multiaxial device provides a very convenient and accurate platform for modeling the behavior of a particulate media under three-dimensional state of stress.

Desai et al (1982) created a new model for the true triaxial apparatus for testing materials of relatively high strength. Several improvements were made to the original concept to make it capable of testing a broad range of materials, such as wood, ballast, aggregates, rock and concrete. In a previous work (1979), Sture and Desai presented a multiaxial device with a capacity of 250 psi when $\sigma_1 = \sigma_2 = \sigma_3$. In contrast, Desai et al (1982) presented a device with a high capacity of 20,000 psi under isotropic consolidation. The performance of this triaxial apparatus was found to be satisfactory in providing consistent behavior for several geologic materials with higher strength and stiffness.

Macari and Arduino (1995) presented a detailed overview of soil modeling in engineering practice. The authors focused on the implementation of the Modified Cam Clay model
(MCC). In the past, a big disadvantage for constitutive models in engineering practice has been their complexity. Because of this, its simplicity and practicality, the MCC has become widely used in recent years. In their work Macari and Arduino described a generalized formulation for the MCC, including a third stress invariant (e.g. $\sigma_2 \neq \sigma_3$). The numerical simulation of the model was done by means of a constitutive driver code. A brief description of the implementation of this constitutive driver code into a finite element formulation, and an example of the calibration of the parameters of the MCC were included in this work. From this investigation it was found that MCC model could not correctly account for the essential characteristics of highly overconsolidated clays.

**Hoyos and Macari (2001)** modified a servo-controlled multiaxial (cubical) testing device to test 4.00” cubical specimens of unsaturated soil under suction-controlled conditions for a broad range of stress paths. Specifically, an important factor they studied was the influence of matric suction on the mechanical behavior of unsaturated soils. The authors described the development of the apparatus, step-by-step assembly procedure, and a validation of its suitability for testing unsaturated soils (axis translation technique). In this investigation it was established that the modifications implemented by Hoyos and Macari did not significantly influence the stress-strain behavior reaction of the specimens tested. Substantiation of the axis-translation technique confirmed the suitability of the apparatus to test soils under suction-controlled conditions and for multiaxial stress paths that cannot be attained in a conventional triaxial apparatus.

**Macari and Hoyos (2001)** used a stress/suction-controlled true triaxial apparatus to study the stress-strain-strength behavior of an unsaturated soil under multiaxial stress states and suction controlled conditions. Their experimental program incorporated the axis-translation technique
to induce matric suction states to a series of 4.00” silty-sand cubical specimens. In this research, a typical stress path imposed to a sample applied hydrostatic compression followed by conventional triaxial compression, triaxial compression, or simple shear in the first octant of the octahedral stress plane. Their results established that matric suction was a factor of significant influence on the stress-strain-strength behavior of the soil studied. As for hydrostatic cases, it was noted that influence of matric suction in the degree of anisotropy in recompacted specimens. Furthermore, for triaxial instances, matric suction was found to affect in several aspects the potential failure envelopes in the octahedral stress plane.

Chang et al (1999) considered the evaluation of critical state strength parameters by extending the modified Cam Clay model (MCCM) to account for anisotropy of the soil mass. Typically, the MCCM has been used to obtain critical state strength parameters from conventional triaxial compression tests on isotropically consolidated soils. In their new approach, Chang et al implemented an extension to the original MCCM to evaluate undrained triaxial and plane strain tests on anisotropically consolidated clays. This extension was developed with the objective of establishing the relation between the critical state friction angles from several triaxial and plain strain tests based on the implementation of the Lade’s failure criterion and the plastic potential from the MCCM. The selection of the Lade’s failure criterion for this research is, in part, due to the previous verification of this criterion with extensive experimental results, and the occurrence of a variation of critical state parameters with stress conditions (adequate for anisotropy). The authors also considered the prediction of undrained shear strength establishing a relation between the isotropic overconsolidation ratio (R) and the conventional overconsolidation ratio (OCR). The results of their research indicate that this extension for the MCCM can be successfully implemented on critical state
parameters from diverse compression tests on normally consolidated and lightly to moderately overconsolidated soils.

**Wang et al (2002)** proposed the implementation of a new approach to study the application of critical state models to triaxial testing of unsaturated soil samples under controlled suction. Typically, critical state models have been based on experimental data of compacted soils samples. In their paper, Wang et al stated the importance of the application of a different sample preparation method to better identify the mechanical behavior of unsaturated soils; specifically, their shear strength and critical state characteristics. The soil samples for their research were prepared gradually consolidating a silt slurry by applying one-dimensional pressure increments from 2 to 100 kPa, encased in a cylindrical mold. The samples then were tested on a strain-controlled triaxial apparatus modified for suction application. The tests for their research included triaxial compression tests of saturated and unsaturated specimens, at different confining pressures and suction conditions. The results of the testing program indicate that the previously referenced mechanical characteristics of unsaturated soil specimens are similar to those for saturated specimens. The critical state lines for the unsaturated simple soil fabric specimens (gradually consolidated) for this study corresponding to different soil suctions were found to be parallel lines to those for the saturated soil specimens. The authors recommended the implementation of additional testing on simple soil fabric samples in order to better identify the effects of soil fabric and stress history on soil behavior.

**Matsuoka et al (2002)** modified an existing true triaxial device for granular materials to test unsaturated soil specimens. The modifications performed to the original device included the attachment of a mechanism to apply matric suction to the undrained silty soil specimens.
considered for this study using the negative pore-water pressure method 
\( s = -u_w > 0; u_a = 0 \). The apparatus used for their study was originally 
developed for true triaxial testing (three different principal stresses) on 
granular materials. The consistency of the modified system was 
confirmed by comparing the data using the true triaxial device with 
the results of the conventional triaxial test under similar conditions.

The new additions to the original triaxial apparatus implemented by 
Matsuoka et al, included a module for the application of matric suction to a 
specimen during testing. This allowed them to test unsaturated soil 
specimens under three different principal stresses at a constant suction. 
A numerical constitutive model, based on the extended spatially 
mobilized plane, was used to predict the mechanical behavior of the 
silty soil samples for their study. Static compaction was used during the 
specimen preparation phase.

The results of their study indicated that the stress-strain relationship of 
the tested silty soils can be particularly assembled extended spatially 
mobilized plane. The measured shear strengths were observed to generally 
concur with the extended spatially mobilized plane failure criterion.
CHAPTER 3

REVIEW OF THE CUBICAL TRIAXIAL DEVICE

The cubical triaxial device used in this study consists of three major components: the Cubical Cell (frame and walls), the Pressure Control System, and the Data Acquisition System. The following sections discuss significant details about these three principal components, plus a description of the specimen preparation procedure.

3.1 The Cubical Cell

The structure of the Cubical Cell system is mainly composed by a frame that is made of a high strength alloy, aluminum 6061 T6-T651, which provides the necessary stiffness to resists the operational pressures of the system (up to 250 psi). The external measurements of the aluminum frame are 8.9 x 8.9 x 8.9 in. (22.6 x 22.6 x 22.6 cm). The frame has a cubical opening of 4.2 in. (10.67 cm) to allow the placement of the test specimen inside the cell. A general view of the Triaxial Frame Assembly is shown in Figure 3.1.

Figure 3.1: Triaxial Frame Assembly.
There is an arrangement of 6 bolt connections on each wall of the cubical cell to securely fasten the wall assemblies. Two small 0.13-inch-diameter cylindrical cavities extend diagonally into the frame. The cavity located on the upper area of the cubic cell can be used to connect a backpressure line to the top of the specimen. On the bottom and opposing side, a vacuum line can be installed to the bottom of the sample through the lower cylindrical cavity. The frame stand, sits firmly underneath allowing the frame to rotate 90°. This feature makes the wall assemblies accessible from the top and bottom.

There are six wall assemblies encasing all sides of the specimen. Figure 3.2 presents a lateral view of a wall assembly disconnected from the cubical cell frame. The wall assemblies, like the frame, are made of aluminum 6061 T6-T651. Six openings machined uniformly in all of the wall assemblies are arranged in patterns that are the same in opposite faces. Each wall is attached to the frame with washers and nuts.
There is a flexible fluid cushion (silicone rubber membrane, Figure 3.2) connected to the inner-face of each wall assembly. This fluid cushion creates a pressure pocket that stands against the soil sample and homogeneously transfers the pressure supplied by a water reservoir located in the back panel of the pressure control system.

Inlet and outlet cavities are located in the wall assembly, with female quick-connectors placed on the outer side of the wall. The inlet is used to fill-up the face with water, and to supply pressurized water during specimen testing. The outlet allows the air present in the face to go out while filling-up the face. During the testing of a sample, the water pocket in the wall assembly must be filled with water, thus the outlet must be disconnected. A graphic representation of the wall assembly is shown in Figure 3.3.

Linear Variable Differential Transformers (LVDT’s) are used for displacement measurements (see Figure 3.3). Three LVDT’s are placed on each wall, for a total of eighteen.
LVDT’s are composed of a fixed outer coil, and an inner core that moves as the specimen deforms. A spring is located inside a non-magnetic steel tube that is attached to the wall. This spring ensures that a good contact between the LVDT and the flexible membrane that is in contact with the soil specimen exists. As the sample deforms, the core of the LVDT moves and generates an analog signal to be processed by the data acquisition system. To measure the surface displacement on each side, the average of each group of 3 LVDT’s is taken as the total displacement for that face.

The eighteen LDVT’s were calibrated before performing the tests. The calibration was done using a LabVIEW program, aluminum plates (0.1” thick), and a wooden stand. One wall at the time was placed on the stand with an opening in the center to let the extension of the cores facing down and in contact with the aluminum plates. The LabVIEW program called “calibration.ltv” allows the operator to insert one steel plate at the time, then records the corresponding voltage reading generated by the insertion of that steel plate. A calibration file is created with the following information: LVDT number, displacement, and voltage. With this information, the operator may create a plot of Displacement vs. Voltage; which is a linear function. From this plot, the slope of the line is taken in inches per Volt (calibration factor), and inserted in another LabVIEW module used to run the tests.

3.2 Data Acquisition System

In addition to the LVDT’s, the triaxial cubical cell system also networks with other transducers such as pore pressure transducers, confining pressure transducers, and pressure regulators (details of these devices are explained later in this chapter). The mechanism that allows the interaction, via the computer, between the sensors and the operator is known as The Data Acquisition System (DAQ). In other words, the data acquisition system collects and
translates the signals from the different transducers connected to the cubical cell, bringing these signals to the computer ready for processing, examination, storage, and other data processing. In addition, the Data Acquisition may generate a command signal for drivers, such as pressure regulators and electronics valves.

Figure 3.4: The Data Acquisition System.

The way the DAQ system works is very straightforward. An schematic of the DAQ system operation is shown in Figure 3.4. An input file containing command values is read by DAQ software. For example, in this investigation the command values where the desired pressures (in psi) to be applied by the electronic drivers included in the cubical cell system. The software translates the command values and sends a digital output signal to the DAQ board. Then, the DAQ board generates an analog output signal that is conditioned by a signal-
conditioning module and finally is delivered to the various drivers in the system to produce changes on the physical conditions (pressure, temperature, dimensions, etc.) of the object that is being tested. In return, each transducer produces an analog input signal that passes through the signal-conditioning module and is then delivered to the DAQ board. The board converts this analog signal to a digital signal that is stored in the computer by the DAQ software in an output file. A picture of the main components of the DAQ system is presented in Figure 3.5.

The data acquisition system is controlled by a DAQ software developed by National Instruments (LabVIEW 6.1). The specific codes utilized in this investigation where written by the group of researchers from the University of Colorado (at Boulder) that participated in the construction of the Cubic Cell System (Dewoolkar et al, 1997). With this tool, the user is capable of easily creating interfaces to interactively control the system by using its graphical development environment based on the G programming language. LabVIEW communicates directly with the DAQ board, also manufactured by National Instruments: the AT-MIO-16E-10 board. This device is entirely plug and play compatible with Windows 95/98/2000. The AT-MIO-16E-10 does not
have DIP switches, potentiometers, or jumpers, in other words, this board can be fully calibrated, and configured utilizing LabVIEW 6i. The AT-MIO-16E-10 has a resolution of 12-bit, and is capable of handling either 16 single-ended channels or 8 double-ended analog input channels at a maximum sampling rate of 100 kHz. This board also features two 12-bit and two 24-bit analog outputs, with a 20 MHz counter timer. Gains may be selected from 0.5 to 100 to be applied to the input signals. It is worth to mention that during the course of this investigation, the DAQ system had to be configured several times due malfunctioning of the original computer provided with the cubical cell system by The University of Colorado at Boulder. Specifically, incompatibilities were noted between the National Instrument hardware and newer computer using faster processors.

The next element in the DAQ system is the National Instruments SCXI-1000 chassis. The SCXI is a four-slotted chassis (Figure 3.6), designed to encase up to four National Instruments modules. Figure 3.6: Chassis SCXI-1000 (top), Connection Box (center) and Power Supply (bottom).
Instrument SCXI modules, providing a low-noise environment for signal conditioning, power and control circuitry.

Three SXCI modules are connected to the chassis. Two SCXI-1122 modules in slots 1 and 2, and one SCXI-1124 in the fourth slot. Slot number three is not in use at this time, however, it may be used for future expansion of the system. A picture with a front view of the SXCI modules connected to the chassis is presented in Figure 3.7 on the next page.

The SCXI-1122 is a module for signal conditioning of strain gages, RTDs, thermocouples, volt, and millivolt sources, and 4 to 20 mA process-current sources. The SCXI-1122 module has 16 isolated multiplexed channels with gains ranging from 0.01 to 2000, and two isolated excitation channels with voltage and current excitation. There are two modes of operation for this module, either two-wire scan mode with all 16 input channels used for input, or the four-wire scan mode with the eight upper channels configured as sense lead for connecting inputs and the lower eight channels configured as current output channels. For this investigation, the module inputs are multiplexed to a single output, which drives a single data acquisition board channel. To provide better connectivity between the input signals and the SCXI-1122 module, a National Instruments SCXI-1322 terminal block is used. This terminal block is very convenient because it has screw-in terminals to attach the input. A picture of the chassis system is presented in Figure 3.7.

The 16 channels in the SCXI-1122 module, housed in the first slot of the SCXI chassis, are in use by the first 16 LVDTs, utilizing channels 0 to 15. Channels 0 to 5 in the module encased in the second slot, are used by LVDTs 17 and 18, a pore pressure transducer (PPT), and the output signals from three electronic regulators. The remaining 10 channels, channels 6 through 15, are available for other purposes.
When data from a specific transducer is to be monitored, the channel to which it is connected has to be addressed in a specific manner in LabVIEW. These addresses are based on how the system is configured. According to this configuration, the output analog signals that command the electronic regulators are sent through an SCXI-1124 module.

Six isolated digital-to-analog converters may be prearranged in the SCXI-1124 module. Depending on the operator needs, these converters can be configured to produce either voltage, or current outputs. This module is designed for DC output and not for waveform generation. From the six channels available in the module, three are used to operate the electronic air regulators in the control panel, and three are not in use at this time. In addition, a shielded terminal block (SCXI-1325), with screw terminals, is used to provide easy signal attachment.

3.3 The Pressure Control System

The pressure control system is the mechanism that provides the pressurized water that applies stresses to a sample inside the cubical cell. The pressure control system also serves as a vacuum supplier for the mold of the specimen, and for de-airing water. During sample
preparation (granular material), vacuum is required to hold the shape of the specimen while moving it from an acrylic mold into the cell frame.

Another important feature of this system is the visual display of the applied specimen pressure. This is very important because it provides the operator an additional view of the pressure settings inside the frame and shows any problem that the computer may not be able to readily detect.

3.3.1 The Control Panel

Aboard the control panel are all the components (connections, valves, sensors, water reservoirs, etc.) that are needed for the stress application to a soil sample. The front and back panels of this load application mechanism are shown in figures 3.8 and 3.9 respectively. The front panel houses five digital pressure gauges, three electronic regulators, four manual regulators, one vacuum valve, and a number of two-way and three-way valves, while the back
The control panel has six cast acrylic cylinders, a pore pressure transducer, and more two-way and three-way valves.

Water, vacuum and pressurized air are supplied to the control panel through an inlet port, located in the left side of the back panel, as shown in Figure 3.9. An outlet port located on the right side of the back panel provides compressed water and vacuum to the cubical cell.

Three cylinders in the back panel are used as water reservoirs for the three different axes (X, Y, and Z). Each one of the reservoirs is connected to a manual as well as an electronic regulator.

Depending on the operators needs, the control panel may be arranged to use either the electronic regulator or the manual regulator per axis. According to this, the load application for any given test may be configured to fully automated, semi-automated, or completely manual. An additional set of valves and tubing are located between the front panel and the water reservoirs to provide more flexibility to the entire control panel.

Figure 3.9: Back Panel of The Pressure Control System.
3.3.2 The X, Y, And Z Regulators

Each axis has a water reservoir that is connected to a digital gage and a three-way valve, both located in the front panel. The gage shows the actual pressure applied to the reservoir with a precision of plus or minus 0.5% of the full scale (500psi). The operator may choose the pressure source by turning the three-way valve to electronic and manual position. In the manual position, a manual regulator series 70 (manufactured by Bellofram) supplies the pressure to the reservoir (Figure 3.10). This regulator has an output range from 1.5 to 150 psi, with a maximum inlet pressure of 250 psi.

As for the load application, electronic regulators were used for all the tests presented in this study. Three Proportion-Air QB1TFEE250 servo control valves are placed in the front panel (one per axis), as shown in Figure 3.11. The QB1TFEE250 is capable of providing a maximum pressure of 250 psi with an input pressure greater or equal to 250 psi. This regulator requires a 15 VDC @ 300 mA. Three AIR PS-300 units are placed inside the connection box to power the QB1TFEE250 regulators.
Beneath its aluminum case, the QB1TFEE250 is a single loop model consisting of valves, manifold, internal pressure transducer, and electronic controls. The output pressure is proportional to an electrical signal input. Once the signal is received, two solenoid valves control the pressure. One of the valves functions as inlet control, the other as pressure relieve. An internal pressure transducer that provides a feedback signal to the electronic controls measures the pressure output. The feedback signal is compared against the command control. A difference between the two signals causes one of the solenoid valves to open, allowing flow in or out of the system. Precise pressure is maintained by driving these two solenoid valves in agreement with the prescribed testing stress path.

3.4 Specimen Preparation

The first step towards repeatability and consistency on the testing results with this multiaxial device is to assure quality control on the preparation of the testing specimens. As mentioned earlier, this multiaxial apparatus holds 4.00” cubical specimens. These specimens can be made of cohesive or cohesionless material. When dealing with cohesive materials (e.g., clays, sandy clays, silty clays, etc.), the preparation is very straightforward. A cohesive specimen
can be easily trimmed to a desired cubical shape and size, or compacted in a mold outside the cubical frame. However, the focus of this investigation is to study the behavior of granular materials. The following is a description of the preparation process of cubical sand specimens. Specific details about this process were obtained from the Cubical Cell User's Manual (Dewoolkar et al, 1997) provided by the University of Colorado, at Boulder.

To give a cubical shape to the specimen, a cast acrylic mold is used (Figure 3.12). This mold is composed of 4 cast acrylic plates, arranged in a way that they will be in touch with four sides of the specimen. A natural latex membrane is to be placed covering the inner lateral faces of the mold, overlapping the top and bottom edges. Orings or rubber bands are located around the overlaps to secure the membrane to the mold. This mold has 4 vacuum ports located at the center of each side. These ports intent to keep the latex membrane aligned to the inner face of the mold, maintaining its cubical shape. Even though vacuum provides good

Figure 3.12: Cubical acrylic mold with latex membrane before air pluviation.

Even though vacuum provides good
adhesion at the center of each side, the corners of the membrane do not follow a right angle shape. To solve this detail, a coat of vacuum grease was applied along all the edges of the mold. It was found that the vacuum grease granted a more effective way of placing the membrane around the inner faces of the mold, preventing wrinkling of the sides latex membrane, and reducing the time spent on connecting additional vacuum lines.

Once the lateral sides of the latex membrane are in place, a square membrane supported by an acrylic plate is glued with rubber cement to the top of the mold to confine the topside of the sample.

![Diagram of the sand-raining device](image)

**Figure 3.13: Diagram of the sand-raining device**

Now, the mold assembly is ready to accommodate the granular material to be tested by the triaxial apparatus. This material is to be placed inside the mold by means of an air pluviasion technique. A sand-raining device (Alshibli et al, 1996) was used to prepare the specimens. A schematic of the sand raining system is presented in Figure 3.13. This device consist of a funnel placed on the top, followed by 4 sieve #4 stacked together at an angle of 45 degrees between any two consecutive sieves, and finally a platform to hold the mold
assembly. Several sample densities can be attained by varying the size of the opening of the funnel and/or the height of the drop between the bottom of the sieves and the platform holding the mold assembly. For example, increasing the size of opening of the funnel and/or reducing the height of the drop would produce a higher void ratio (a less dense specimen).

To hold the shape of the specimen while extracting it from the mold, a small amount of vacuum is applied through a vacuum line (Figure 3.14). The opening of this line is covered with filter paper to prevent small soil particle from getting to the control panel and damaging the vacuum system. The vacuum line has to be carefully inserted inside the sample before placing the bottom latex membrane.

As previously mentioned, the bottom membrane is attached to the specimen using rubber cement as a bonding agent. The contact surface between the side membranes and the bottom membrane should be very clean and free any soil particle as possible. Soil particles trapped along the contact surface between the membranes may create voids that could cause loss of negative pressure inside the sample. Having the specimen under vacuum, the top and
bottom orings are released, and the sample is pushed from the bottom and cautiously placed inside the cubical frame as shown in Figure 3.15.

Figure 3.15: Placement of a cohesioless specimen into the cell frame.

The latex membranes (top, sides and bottom) will remain attached to the specimen during testing. In general, the latex membranes cannot be reused for further testing if a good bonding is achieved during specimen preparation.
CHAPTER 4

CRITICAL STATE SOIL MECHANICS

The following sections present a general discussion on Critical State Soil Mechanics (CSSM) based on concepts compiled by David Wood in his work *Soil Behaviour and Critical State Soil Mechanics*. This discussion was included in order to aid in the understanding of soil modeling founded on constitutive relations. These concepts are actually the basis of the experimental program presented later in Chapter 5.

4.1 Critical State Soil Mechanics

The Critical State of a material corresponds to a point in soil shearing where no more strength may be attained and no more volume change occurs (Figure 4.1). The critical state concept suggests that some shear tests on soil will produce final stress-strain-strength states which lie on the critical state line in the $q-p'$ and $e$ or $v$ space; where $q$ is the deviator stress $(\sigma_1 - \sigma_3)$, $p'$ represents the net mean effective stress $(\sigma_1 + \sigma_2 + \sigma_3/3)$, $e$ is the void ratio, and $v$ is the specific volume $(1+e)$. It is possible to characterize the complex behavior of soils by certain combinations of isotropic consolidation response characteristics and stress paths.

![Figure 4.1: Critical State in $(q: \varepsilon_q)$ and $(e: \varepsilon_q)$ planes.](image-url)
limited to the trial plane. It is asserted that there is a unique relationship between shear stress, $q$, and the mean normal effective stress, $p'$, and the void ratio, $e$, or the specific volume, $v$, at failure, residual or critical state.

The concept of critical state soil mechanics has a fundamental aspect called the **critical state line**. The critical state line has the property that failure of initially isotropically consolidated samples will occur once the stress state of the samples reaches the line, irrespective of the test path followed by the samples on their way to the critical state. This behavior is observed to be the same in conventional triaxial compression tests (both, drained and undrained). The projection of the critical state line (CSL) onto the $p'$-$q$ (Figure 4.2) space may be expressed as $q = Mp'$, where $M$ is the slope of the critical state line. It may also be represented in the $v$-$p'$ space (Figure 4.3) as $v = \Gamma - \ln p'$, where $\Gamma$ is the specific volume $(v)$ corresponding to $p' = 1kN / m^2$ on the critical state line.

Figure 4.2: Critical State Line in the $(q:p')$ Plane (modified from class exercise CE7300, Fall 2000).
4.2. The Roscoe Surface

Figure 4.4 presents the projection of undrained and drained stress paths in a two-dimensional space $q$-$p'$. The representation of these stress paths in a three-dimensional

Figure 4.3: The Critical State Line on the $v$-$LN(p')$ Space (modified from class assignment CE7000, Fall 2000)

Figure 4.4: Projection of Roscoe Surface in $p'$-$q$ Space. (Modified from class assignment CE7000, Fall 2000).
space \((p':q:v)\) will form a “3-D” Surface. Note that the drained and undrained stress paths shown in Figure 4.4 form contours of constant specific volume, and have the same critical state line.

It is clear that the contours from drained and undrained tests have similar trends and are consistent with each other. The assumption that all isotropically normally consolidated clay specimens will behave in a similar manner gives rise to what is known as the “Roscoe Surface”.

This behavioral trend can be standardized by normalizing both axes, \(p'\) and \(q\), with respect to the isotropic effective mean stress (confining pressure) \(p_o'\), from the start of the test.

\[
\frac{q}{p_o'} \quad \frac{p'}{p_o'}
\]

Figure 4.5: Paths of CD and CU test plotted in a non-dimensional space (modified from Wood, 1990).

4.3 Elastic-Plastic Model for Soils

Recognizing that yield surfaces exist for soils, the next element to consider is the behavior of soils within the yield surface. It follows that stress changes inside the yield surface will be elastic. Once the stress state reaches the current yield surface, a combination of elastic and plastic response will result. For this investigation it is important to determine the
magnitude and the direction of the plastic deformation in relation with the change in size of
the yield surface.

The discussion of the elastic-plastic model for soil in this chapter will be limited to the
cases of stress and strain in the conventional and true triaxial apparatuses. Furthermore, the
model will be described in terms of triaxial stress parameters $p'$, $q$ and strain parameters $v$, $\varepsilon$.

4.4 Elastic Volumetric Strains

“Yield surfaces mark the boundary between an elastically achievable state of stress
and a plastically admissible stress state. Stress changes inside the yield surface are
recoverable deformations” (Wood, 1990). In order to avoid complications one may assume
that the material is isotropic. The elastic stress-strain relationship may be expressed as:

$$\delta v = \frac{\delta p'}{K'} \quad (4-1)$$

$$\delta \varepsilon = \frac{\delta q}{3G'} \quad (4-2)$$

Where $K'$ and $G'$ are the bulk and shear modulus of the soil respectively, both in terms of
effective stress. It is worth noting that in expressions 4-1 and 2-2 recoverable changes in
volume are associated only with changes in the mean stress $p'$.

A yield locus of a soil in the $q$-$p'$ space is illustrated in figure 4.6. For a given state of
stress, the specific volume, $v$, may be determined by projecting the point into the $v-p'$ space.
From one point to another, inside the yield surface, changes will be elastic.

The loading history of the soil has a direct influence on the position, shape, and size of
the yield surface of the soil. Figure 4.6 shows the stress path for a soil that is loaded along
one-dimensional compression. The yield surface, associated with this stress state, is shown
and its related to the $v-p'$ curve, also shown in figure 4.6. It should be noted that the $v-p'$
represents the normally consolidated line and upon unloading it follows the elastic response curve, also known as the unload-reload line (url). Another feature of this diagram ($v-p'$) is that if one plots it in a logarithmic scale, instead of an arithmetic scale, one obtains a straight line response (very much alike to what is done to consolidation test results in conventional $e-
abla\sigma_v-processes).

\begin{equation}
\ln pv = v - \lambda \ln p' \quad (4-3)
\end{equation}

and the expression for the unloading-reloading (url in Figure 4.3) curve becomes:

\begin{equation}
q = P' - Cyl
\end{equation}

Figure 4.6: Yield Surface in the $q-p'$ Space (Modified from Wood, 1990).
\[ v = v_\lambda - \kappa \ln p' \]  \hspace{1cm} (4-4)

where \( \lambda \) is the slope of the normal compression line and \( \kappa \) is the slope of the unloading-reloading line. \( v_\lambda \) and \( v_\kappa \) are the intercepts on these lines at \( p' = 1 \) kPa. Note that the values of these last two parameters are dependent on the units selected to measure stress. Equation 4-4 is expressed in an incremental form as:

\[ \delta v = -\kappa \frac{\delta p'}{p'} \]  \hspace{1cm} (4-5)

where \( e \) specifies that they are elastic, recoverable, volume changes. An increase in volumetric strain is given by:

\[ \delta e_p = -\frac{\delta v}{v} \]  \hspace{1cm} (4-6)

The incremental form of this equation may be rewritten in terms of volumetric strain as:

\[ \delta e_p' = -\kappa \frac{\delta p'}{vp'} \]  \hspace{1cm} (4-7)

comparing this expression with equation (4-1) affirms that \( K' = vp'/\kappa \). Variations in

![Diagram](Figure 4.7: Normal compression line and unloading-reloading line in compression plane (modified from Wood, 1990)).
deviatoric stress inside the yield surface of an isotropic elastic soil, will produce no changes in volume but will cause elastic deviatoric, or triaxial shear strains: $\delta \varepsilon^e$. This value can be obtained from equation (4-2) with an appropriate value of shear modulus, $G'$.

4.5 Plastic Volumetric Strains and Plastic Hardening

A soil that yields as shown in figure 4.8 is under a change in stress state that is trying to penetrate the yield surface. With a stress path that goes from point K to point L (L is located on what is called yield surface No.2) one is able to know the shape of the new yield surface. To simplify the analysis, it is assumed that the shape of the new yield surface stays the same, irrespective of the stress path that created it. Also going from point K to L, there is a change in volume that shown in the bottom graph as $\Delta V$. Where $\Delta V = \Delta V^e + \Delta V^p$ and $\Delta V^p = \Delta V_{\kappa1} - \Delta V_{\kappa2}$.

By focusing on the region of the compression space around the points where the unloading-reloading meet the normal compression line (in figure 4.4) an alternative expression can be obtained for the volume change.

![Figure 4.8: Expansion of yield locus (modified from Wood 1990).](image)
Point A corresponds to mean stress $p'_{o_1} = p'_{o_2}$ and point B is the point on the normal compression line with $p'_{o_1} = p'_{o_2}$. The change in specific volume that can be recovered between the unloading and reloading curve $url_1$ and $url_2$ which matches point A and B respectively is given by:

$$
\delta v^p = -\lambda \ln \left( \frac{p'_{o_2}}{p'_{o_1}} \right) + \kappa \ln \left( \frac{p'_{o_2}}{p'_{o_1}} \right) = - (\lambda - \kappa) \ln \left( \frac{p'_{o_2}}{p'_{o_1}} \right) \tag{4-8}
$$

The first term of this expression refers to the total change in volume as the net mean stress goes from point A to Point B. The second term represents the recovered volume change when the net mean stress is reduced again. In the maximum value, the above equation becomes:

$$
\delta v^p = - (\lambda - \kappa) \frac{\delta p'_{o}}{p'_{o}} \tag{4-9}
$$

in terms of volumetric strain:

$$
\delta \varepsilon^p_v = (\lambda - \kappa) \frac{\delta p'_{o}}{vp'_{o}} \tag{4-10}
$$

The total volumetric strain increment and the total change in specific volume can be presented as the sum of their elastic and plastic components, respectively:
\[ \delta \varepsilon^p = \delta \varepsilon^c + \delta \varepsilon^p \]  
\[ (4-11a) \]

and

\[ \delta \nu = \delta \nu^c + \delta \nu^p \]  
\[ (4-11b) \]

### 4.6 Plastic Potentials

Beyond the simple model described earlier in this investigation, a new element can be introduced to the plasticity analysis. Any yielding taking place at a stress state \( Y \) (in the \( q-p' \) space, figure 4.10) will be linked with the occurrence of some plastic irrecoverable volumetric strain, \( \delta \nu^p \), and some plastic shear strain, \( \delta \varepsilon^p \). Point \( Y \) in Figure 4.10 represents the magnitudes of these two components with axes parallel to \( p' \) and \( q \). \( \delta \nu^p \) and \( \delta \varepsilon^p \) form a plastic strain vector \( \text{YS} \). This vector can be created by drawing an orthogonal line through point \( Y \) with a slope given by: \( \frac{\delta \nu^p}{\delta \varepsilon^p} = -\frac{\delta \nu^p}{\delta \varepsilon^p} \).

![Figure 4.10: Plastic strain increment vector normal to plastic potential curves (Wood, 1990).](image)

A number of different combinations of stresses may occur on a soil particle at any given instant during its history causing yielding.

Yielding may occur under several different permutations of stresses in the history of the soil. For each one of this combinations, a vector of plastic strains can be drawn as more and more
data becomes available these lines may be joined up to form a group of curves to which the plastic strain vectors are orthogonal. These curves are known as “the Plastic Potential Curves”.

A normality condition is considered to take place when an engineering material has the shape of its plastic potential almost identical to its yield surface. This can be seen in Figure 4.11 presented below. The normality trend is closely related to an associated flow rule. This relation is further discussed in the subsequent Section 4.7 Normality or Associated Flow.

![Figure 4.11: Families of plastic potential (dashed) and yield locus (solid) (from Wood 1990).](image)

4.7 Normality or Associated Flow

If the yield surfaces and the plastic potential surfaces for a specific material are identical, then the material is said to obey the postulate of normality: the plastic strain increment vector is in the direction outward normal to the yield surface. However, the material can be said to follow a law of associated flow: the nature of the plastic deformations, or flow, is associated with the yield surface of the material.
In the previous paragraphs, elastic-plastic models of soils were described in a general way. Yield loci and plastic potentials were sketched, without any further attempt to generate possible mathematical expressions for these curves. Apart from this, the following is a description of a mathematical model that collects all this information into more practical applications.

When the Cam clay model was originally described by Roscoe and Burland (1968), it was called modified Cam clay to distinguish it from an earlier model called Cam clay (Roscoe 1968).
and Schofield, 1963). Basically, the model is described in terms of the effective stress values $p'$ and $q$ which are important to the area of soil response in conventional triaxial test.

In an attempt to study the yielding behavior of NC clays, Roscoe and his coworkers conducted several tests on samples of saturated clays. They found that the effective stress paths for several test were geometrically similar, and their ultimate stress states were a straight line on a $q-p'$ space. Another important feature was that the ultimate states of stress were observed to lie on a curve, which was similar to the isotropic consolidation line on a compression space.

In the MCCM, the stress path passes through several yielding surfaces (also called hardening caps) causing plastic deformations. The yielding will continue to occur until the material reaches a critical void ratio ($e_{cr}$), after which the void ratio remains constant during subsequent deformations. In other words, this critical void ratio can be considered as the ultimate state of the material. It has been observed that a soil with a void ratio lower than the critical value will deform in such a manner as to increase its volume while at a void ratio higher than the critical value the deformations will decrease in volume.

Yield criterion, and post yield behavior are two important factors with the plastic behavior of the material. The yield criterion defines the limit of purely elastic behavior. When the state of stress comes in contact with the current yield surface, the material undergoes elastic-plastic deformations. During this process, the material hardens and the yield surface expands to a new position. In order to describe the elastic-plastic response of the soil it is essential that we develop explicit relations in an incremental (flow) fashion. The formulations will then be used to predict the response of a soil as the stresses or strains are incrementally increased or decreased, depending on the purpose of the investigation.
The MCCM is represented by an ellipse in the $p':q$ space. This ellipse is centered on the $p'$ axis, and can by plotted by using the following expression:

$$\frac{p'}{p_o'} = \frac{M^2}{M^2 + \eta^2}$$  \hspace{1cm} (4-12)

where $\eta$ is the relation $q/p'$. To incorporate this particular form of yield locus into the general considerations discussed in the elastic-plastic concepts for soil, the equation of the ellipse can be expressed as:  

$$f = q^2 - M^2[p'(p_o' - p')] = 0$$  \hspace{1cm} (4-13)

It is assumed that the soil obeys the normality conditions; So having an expression for the family of yield loci, we note that the same equation serves as a representation for the family of plastic potential curves in the $p':q$ plane:

$$g = f = q^2 - M^2[p'(p_o' - p')] = 0$$  \hspace{1cm} (4-14)

Then the vector of plastic strain increments $\delta e_p^p: \delta e_q^p$ is in the direction of the outward normal to the yield locus. This implies that

$$\frac{\delta e_p^p}{\delta e_q^p} = \frac{\partial g/\partial p'}{\partial g/\partial q} = \frac{M^2(2p' - p_o')}{2q} = \frac{M^2 - \eta^2}{2\eta}$$  \hspace{1cm} (4-15)

when plastic deformations are occurring. It is assumed that the yield loci expand at constant shape, the size is controlled by the tip stress $p_o'$, and that the expansion of the yield loci, the hardening of the soil, is linked to the normal compression of the soil. Now, following the idea of hardening rule, we assume a linear relationship between specific volume $v$ and the logarithm of mean effective stress $p_o'$ during isotropic normal compression of the soil:

$$v = N - \lambda \ln p_o'$$  \hspace{1cm} where $N$ is a soil constant specifying the position of the isotropic
compression line in the compression plane $p':v$. Then the magnitude of plastic volumetric strains is given by

$$\delta e^p_p = \left[ (\lambda - \kappa)/\nu \right] \frac{\delta p_o'}{p_o} \quad (4-16)$$

and the elements of the hardening relationship become

$$\frac{\partial p_o'}{\partial e^p_p} = \frac{v p_o'}{\lambda - \kappa} \quad (4-17)$$

$$\frac{\partial p_o'}{\partial e^q} = 0$$

Now, by combining $\delta e^p_p = \kappa \frac{\delta p'}{v p'}$ and $\delta e^q = \frac{\delta q}{3G}$, we can summarize the elastic stress-strain response in the matrix equation:

$$\begin{bmatrix} \delta e^p_p \\ \delta e^q \end{bmatrix} = \begin{bmatrix} \nu p' \lambda - \kappa & 0 \\ 0 & \nu G' \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix} \quad (4-18)$$

Then the plastic stress-strain response can be represented by the expression:

$$\begin{bmatrix} \delta e^p_p \\ \delta e^q \end{bmatrix} = \frac{\lambda - \kappa}{vp' (M^2 + \eta^2)} \begin{bmatrix} (M^2 - \eta^2) & 2\eta \\ 2\eta & 4\eta^2/(M^2 - \eta^2) \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix} \quad (4-19)$$

4.9 The Modified Cam Clay Model in Excel

Excel, a spreadsheet software produced by Microsoft, Inc., was found to be a very practical environment for the numerical implementation of the Modified Cam-Clay Model during this investigation.

Three spreadsheets were produced to generate the necessary data for modeling the behavior of granular materials. In the first spreadsheet, the operator is required to input the slope of the critical state line, confining pressure and increments on deviator stress and net
mean stress (for 40 plastic steps). From this information the spreadsheet will generate the
numerical values of the points forming an ellipse for each plastic increment in the $q:p'$ plane,
until failure is reached. The second spreadsheet deals with the changes on specific volume
related to the numerical modeling using The Modified Cam Clay Model. Here, three
parameters are required: $\lambda$, $\kappa$, and $N$. Finally, the third spreadsheet generates the plot for
Deviator Stress ($q$) vs. Shear Strain ($\varepsilon_q$).
CHAPTER 5

THE EXPERIMENTAL PROGRAM

In order to implement a valid constitutive model that represents as accurate as possible states of loading and deformation on soils, it is important to conduct a series of test along varying loading paths. This provides a wide-ranging understanding of soil behavior that may be used to develop, select and calibrate adequate numerical models. In other words, to reproduce the material’s behavior in a mathematical model. Because the behavior of soils is not uniform under different states of stress-deformation, the selection of stress paths will be of great importance to reproducing the loading conditions needed for each analysis of specific boundary value problems.

In the triaxial apparatus stress paths are set to mimic different loading conditions that a soil deposit may undergo in its natural environment and as a result of external loading conditions. Figure 5.1 shows a structure fixed in a nonlinear half-space above the ground.

Figure 5.1: (a) Structure embedded in a soil mass undergoing axial and lateral loading. (b) Symbolic stress paths in $p:q$ plane (a and b modified from Desai et al, 1987).
water table. This structure is subjected to axial and lateral loads, causing different loading paths among various elements (points A, B, C, D, and E in Figure 5.1a) contained by the soil mass. A symbolic representation of these stress paths in the ($p' : q$) stress plane is shown in Figure 5.1b. Considering the cubical cell, a set of specific tests may be achieved in a soil specimen. Figure 5.2 presents a schematic representation in the principal stress plane of the most common stress paths produced using this apparatus.

![Figure 5.2: Stress Paths in the principal stresses plane ($\sigma_1 : \sigma_2 : \sigma_3$).](image)

An experimental program was implemented to obtain the necessary parameters to calibrate the modified Cam-Clay model. A set of 20 tests was performed on a granular material classified as SP (poorly graded sand) according to the Unified Soil Classification System (USCS), test standard No. ASTM D 2487. A grain size distribution analysis was performed in order to determine specific parameters required for classification of the granular material.

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material tested in this investigation. Table 5.1 presents the gradation parameters obtained from the gradation test.

Table 5.1: Gradation parameters of tested sand.

<table>
<thead>
<tr>
<th>Material (USCS)</th>
<th>( D_{10} )</th>
<th>( D_{30} )</th>
<th>( D_{60} )</th>
<th>( Cu )</th>
<th>( Cc )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>0.6</td>
<td>0.85</td>
<td>0.95</td>
<td>1.58</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Shown in Table 5.1, \( D_{10} \), \( D_{30} \) and \( D_{60} \) are the typical particle sizes corresponding to the 10, 30 and 60 percentile finer by weight respectively as determined from the gradation test. Also presented in this table are the Coefficient of Curvature (Cc), and the Coefficient of Uniformity (Cu). These coefficients are defined as:

\[
Cc = \frac{D_{60}}{D_{10}}
\]

\[
Cc = \frac{D_{30}^2}{(D_{10})(D_{30})}
\]

Two different void ratios, \( e = 0.41 \) and \( e = 0.65 \), were selected for the preparation of the testing samples. Three stress paths with constant net mean stress, Triaxial Compression (TC), Triaxial Extension (TE) and Simple Shear (SS), were chosen starting at three different confining pressures, 100, 200 and 400 kPa. Also hydrostatic compression tests (HC) were selected for the two soil densities.

For the Hydrostatic Compression test (HC), the specimen is subjected to an initial isotropic state of stress, \( p_o = \sigma_o \). Then the pressure in each axis is increased the same amount (\( \Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 \)). This test provides information about the volumetric bulk behavior of a material. Note that in this test no shear stresses are introduced.
In the case of the Triaxial Compression (TC), after a confining pressure is applied, \( \sigma_1 \) is increased \( \Delta \sigma_1 \), while \( \sigma_2 \) and \( \sigma_3 \) are decreased \( \Delta \sigma_1/2 \). For the Triaxial Extension, \( \sigma_2 \) and \( \sigma_3 \) are increased and \( \sigma_1 \) is decreased to maintain the octahedral normal stress (or mean stress, \( p \)) constant.

In the Simple Shear Test (SS) one of the stresses is held constant while the other two are increased and decreased respectively by the same magnitude. For example: keeping constant \( \sigma_2 \), the changes of stresses would be \( \Delta \sigma_1 = -\Delta \sigma_3 \), and \( \Delta \sigma_2 = 0 \).

Excluding the Hydrostatic Compression test, all the stress paths described in this study may be characterized by a parameter known as: stress ratio “b”. The stress ratio, b, is defined as,

\[
b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}
\]

where \( \sigma_1 \) is the major principal stress, \( \sigma_2 \) is the intermediate principal stress, and \( \sigma_3 \) is the minor principal stress. Hence, for Triaxial Compression \( b=0.0 \), for Simple Shear \( b=0.5 \), and for Triaxial Extension \( b=1.0 \).
Before shearing a typical test specimen, the confining pressure is increased in each axis; so that \( \Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 \) until a desired value of mean stress is achieved. Note that until this point no shear stresses are introduced to the specimen. Once the confining pressure is reached, the shearing of the sample takes place with constant net octahedral stress, \( \sigma_{\text{oct}} \), which may be expressed as \( \sigma_{\text{oct}} = (\sigma_1 + \sigma_2 + \sigma_3)/3 \). Likewise, the octahedral shear stress, \( \tau_{\text{oct}} \), is defined by the following expression,

\[
\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}
\]

![Figure 5.4: Octahedral normal stress and octahedral shear stress in the octahedral stress space (modified from Macari and Hoyos, 2001).](image-url)
The following table, Table 5.2, presents a summary of the experiments performed in this investigation, along with their respective type of test, void ratio, and confining pressure.

Figure 5.5: Schematic representation in \((q:p':b)\) space of the TC, SS and TE stress paths implemented using the Multiaxial Cell during the development of this study.
Table 5.2: Group of tests performed for this investigation.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>e</th>
<th>Test Type</th>
<th>Confining Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC100-41</td>
<td>0.41</td>
<td>Triaxial Compression</td>
<td>100</td>
</tr>
<tr>
<td>TC200-41a,b</td>
<td>0.41</td>
<td>Triaxial Compression</td>
<td>200</td>
</tr>
<tr>
<td>TC400-41</td>
<td>0.41</td>
<td>Triaxial Compression</td>
<td>400</td>
</tr>
<tr>
<td>SS100-41</td>
<td>0.41</td>
<td>Simple Shear</td>
<td>100</td>
</tr>
<tr>
<td>SS200-41</td>
<td>0.41</td>
<td>Simple Shear</td>
<td>200</td>
</tr>
<tr>
<td>SS400-41</td>
<td>0.41</td>
<td>Simple Shear</td>
<td>400</td>
</tr>
<tr>
<td>TE100-41</td>
<td>0.41</td>
<td>Triaxial Extension</td>
<td>100</td>
</tr>
<tr>
<td>TE200-41</td>
<td>0.41</td>
<td>Triaxial Extension</td>
<td>200</td>
</tr>
<tr>
<td>TE400-41</td>
<td>0.41</td>
<td>Triaxial Extension</td>
<td>400</td>
</tr>
<tr>
<td>TC100-65</td>
<td>0.65</td>
<td>Triaxial Compression</td>
<td>100</td>
</tr>
<tr>
<td>TC200-65</td>
<td>0.65</td>
<td>Triaxial Compression</td>
<td>200</td>
</tr>
<tr>
<td>TC400-65</td>
<td>0.65</td>
<td>Triaxial Compression</td>
<td>400</td>
</tr>
<tr>
<td>SS100-65</td>
<td>0.65</td>
<td>Simple Shear</td>
<td>100</td>
</tr>
<tr>
<td>SS200-65</td>
<td>0.65</td>
<td>Simple Shear</td>
<td>200</td>
</tr>
<tr>
<td>SS400-65</td>
<td>0.65</td>
<td>Simple Shear</td>
<td>400</td>
</tr>
<tr>
<td>TE100-65</td>
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<td>Triaxial Extension</td>
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<tr>
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<td>0.65</td>
<td>Triaxial Extension</td>
<td>400</td>
</tr>
<tr>
<td>HC-41</td>
<td>0.41</td>
<td>Hydrostatic Compression</td>
<td>-</td>
</tr>
<tr>
<td>HC-65a,b</td>
<td>0.65</td>
<td>Hydrostatic Compression</td>
<td>-</td>
</tr>
</tbody>
</table>
CHAPTER 6
TEST RESULTS AND NUMERICAL MODELING

6.1 General

The results of the experimental program described in Chapter 5 are presented in this Chapter. A brief analysis of the results is presented along with the general response discussion. The results are summarized in terms of general stress-strain-strength behavior and are compared with simple numerical models based on Critical State Soil Mechanics concepts. Even though Critical State Soil Mechanics was originally developed for cohesive soils the model is used for this simulation because of the stress-control nature of the cubical device in which no post-peak softening response is detected.

6.2 Tests Results

As shown in table 5.1 the experimental program consisted of 18 shear tests and 2 isotropic compression tests. Tests TC200_41 and HC_65 were duplicated to show the repeatability of the testing apparatus as shown in figures 6.1a and 6.1b.

Appendix A presents all of the tests from this study. This section outlines general trends of behavior and presents a discussion of the response.

Figures 6.2 – 6.7 present the typical response of the soil under three different loading paths. Each figure shows a comparison side-by-side of the results in two different planes, Shear Octahedral Stress vs. Shear Strain plane \((\tau_{oct} : \varepsilon_{q})\), and Shear Octahedral Stress vs. Principal Strains plane \((\tau_{oct} : \varepsilon_{1,2,3})\). It should be noted that compression and extension, as used in this investigation, will are presented negative and positive respectively.
Figure 6.1: (a) Tests TC200_41a and TC200_41b on the deviator stress vs. total shear strain plane \((q: \varepsilon_q)\). (b) Hydrostatic Compression tests HC65a and HC65b on the Volumetric Strain vs. Log of Net Octahedral Stress plane \((\varepsilon_v : \log(\sigma_{oct}))\).
Figure 6.2: Triaxial Compression Tests performed on sand specimens with $e = 0.41$, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain, $(\tau_{oct} : \varepsilon_{q})$, plane. (b) Shear Octahedral Stress vs. Total Principal Strain, $(\tau_{oct} : \varepsilon_{i})$, plane.
Figure 6.3: Simple Shear tests performed on sand specimens with $e = 0.41$, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain, $(\tau_{oct} : \varepsilon_t)$, plane. (b) Shear Octahedral Stress vs. Total Principal Strain, $(\tau_{oct} : \varepsilon_i)$, plane.
Figure 6.4: Triaxial Extension tests performed on sand specimens with $e = 0.41$, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain, $\left( \tau_{oct} : \epsilon_q \right)$, plane. (b) Shear Octahedral Stress vs. Total Principal Strain, $\left( \tau_{oct} : \epsilon_j \right)$, plane.
Figure 6.5: Triaxial Compression tests performed on sand specimens with $e = 0.65$, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain plane ($\tau_{oct} : \varepsilon$). (b) Shear Octahedral Stress vs. Total Principal Strain plane ($\tau_{oct} : \varepsilon_p$).
Figure 6.6: Simple Shear tests performed on sand specimens with e = 0.65, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain plane ($\tau_{oct} : \varepsilon_t$). (b) Shear Octahedral Stress vs. Total Principal Strain plane ($\tau_{oct} : \varepsilon_i$).
Figure 6.7: Triaxial Extension tests performed on sand specimens with $e = 0.65$, at 100, 200 and 400kPa of confining pressure. (a) Shear Octahedral Stress vs. Total Shear Strain plane ($\tau_{oct} : \varepsilon_{q}$). (b) Shear Octahedral Stress vs. Total Principal Strain plane ($\tau_{oct} : \varepsilon_{l}$).
6.3 Analysis of Results

Peak values were obtained to assess the presence of a Critical State Surface for each of the two soil densities tested in this investigation. Figures 6.8 and 6.9 show the points representing the peak values for each of the densities as well as each of the loading path in the deviator stress vs. net mean stress plane ($q:p'$).

Figure 6.8: Points representing the critical state line for TC, SS and TE tests performed for this study on sand specimens with $e = 0.41$. 

\begin{align*}
M_{TC} & \rightarrow q = 1.7857 \ p' \\
M_{SS} & \rightarrow q = 1.5667 \ p' \\
M_{TE} & \rightarrow q = 1.1393 \ p'
\end{align*}
Figures 6.10 and 6.11 show the results of various tests; superimposed onto the $\pi$-plane of principal stress space. Connecting the peak values of the different loading path followed during the specimen testing forms critical state surfaces. Three loading paths (TC, SS, and TE in figure 6.10) provide three different points defining part of the critical state surface. The rest of the points are obtained by assuming the material to be homogeneous and
isotropic. This permits to plot all the possible permutations of the original loading paths onto the principal stress space (figure 6.11). All the yield surfaces for the two densities evaluated in this study are presented in Figures 6.12 through 6.15. It is worth noting that the lower density exhibits a rounder response.

![Figure 6.10: Example of the projection of three loading paths represented in the π - plane.](image1)

![Figure 6.11: Example of the loading paths forming a yield surface.](image2)
Figure 6.12: Failure surfaces on the octahedral \( (\sigma_1 : \sigma_2 : \sigma_3) \) stress plane for dense sand specimens \((e=0.41)\) at different values of net mean octahedral stress, 
\[ \sigma_{\text{oct}} = 100\, kPa, \quad \sigma_{\text{oct}} = 200\, kPa, \quad \text{and} \quad \sigma_{\text{oct}} = 400\, kPa. \]
Figure 6.13: Three-dimensional view of the yield surfaces for the dense sand in a principal stress space.
Figure 6.14: Failure surfaces on the octahedral \((\sigma_1 : \sigma_2 : \sigma_3)\) stress plane for loose sand specimens \((\varepsilon=0.65)\) at different values of net mean octahedral stress, 
\[
\sigma_{\text{oct}} = 100\text{kPa}, \quad \sigma_{\text{oct}} = 200\text{kPa}, \text{ and } \sigma_{\text{oct}} = 400\text{kPa}.
\]
Figure 6.15: Three-dimensional view of the yield surfaces for the loose sand in a principal stress space.
6.4 Numerical Simulation

This section presents the comparison between the experimental results and the numerical simulation of the testing methods. Even though the Cam-Clay models were developed for cohesive soils, they have been adapted for this study in order to show general trends of behavior. It is worth noting that for each of the test series an individual values of “M” (slope of the critical state line) were used as shown figures 6.1 and 6.2 (e.g. for e=0.41: \(M_{TC} = 1.7857\), \(M_{SS} = 1.5567\), and \(M_{RE} = 1.1393\)). More sophisticated models are capable of capturing these variations in critical state line slope (e.g. Macari and Arduino, 1998).

The following figures, Figures 6.16, 6.17, and 6.18, present a comparison between the experimental response of the material, and a consecutive model of this response. This comparison is presented in a side-by-side format of the different groups of loading paths applied to loose and dense sand specimens studied in this work. From these figures can be noted that the numerical model was able to capture generalized trends of the sand behavior under shear loading conditions.

Note that in the case of the denser material, the numerical model was more efficient mimicking its behavior under shearing.
Figure 6.16: Experimental (solid) and Cam Clay (dotted) results of triaxial compression tests ($b = 0$) varying the net mean octahedral stress, $\sigma_{oct} = 100 \text{kPa}$, $\sigma_{oct} = 200 \text{kPa}$, and $\sigma_{oct} = 400 \text{kPa}$; for: (a) void ratio = 0.41.
(b) void ratio = 0.65.
Figure 6.17: Experimental (solid) and Cam Clay (dotted) results of simple shear tests (b = 0.5) varying the net mean octahedral stress, $\sigma_{oct} = 100kPa$, $\sigma_{oct} = 200kPa$, and $\sigma_{oct} = 400kPa$; for: (a) void ratio = 0.41. (b) void ratio = 0.65.
Figure 6.17: Experimental (solid) and Cam Clay (dotted) results of triaxial compression tests (b = 1.0) varying the net mean octahedral stress, $\sigma_{oct} = 100kPa$, $\sigma_{oct} = 200kPa$, and $\sigma_{oct} = 400kPa$; for: (a) void ratio = 0.41. (b) void ratio = 0.65.
CHAPTER 7
SUMMARY AND CONCLUSIONS

7.1 Summary

This thesis has presented work on the implementation of a triaxial cubical device to study the experimental and computational response of loose and dense sand cubical specimens under general triaxial state of stress. An experimental program was implemented in order to illustrate the general constitutive behavior selected clean, poorly-graded sand, and to provide the necessary data for the numerical modeling of the soil behavior based on the calibration of a simple elastic-plastic model (the Modified Cam-Clay Model). Sets of shearing and hydrostatic triaxial tests (at constant net mean octahedral stress) were performed on remolded cubical (4.00” per side) poorly graded sand (SP) specimens by means of a stress-controlled Multiaxial Apparatus. This multiaxial device was found to meet efficiently most of the requirements for the execution of the testing program.

The data generated by the experimental program was arranged and presented in a side-by-side comparison between \((\tau_{oct} : \varepsilon^t_q)\) and \((\tau_{oct} : \varepsilon)\) plots. This information was also used to elaborate the failures envelopes in the octahedral stress plane for the experimental analysis between the testing specimens with \(e = 0.41\) and the testing specimens with \(e = 0.65\). With respect to the numerical modeling, the experimental results were shown against the numerical outcome from the MCCM in deviator stress vs. total shear strain \((q : \varepsilon^t_q)\) plots.

7.2 General Conclusions

The general response of loose and dense sand was found to be in agreement with traditional behavior. Specifically, it was observed a stiffer response for dense specimens, and a softer
response for loose specimens. Even though quality data was generated from the experimental program, no post-peak response could be obtained due to the loading characteristics of the multiaxial apparatus. At failure, the triaxial device continues to increase the stress on the testing specimen, making almost impossible to account for any accurate measurement of the post-peak behavior of the soil. Another factor affected by these characteristics was the determination of the critical state conditions. In most cases, the critical state conditions were determined to be between 1% to 3% of total shear strain.

The position and shape of the failure envelopes in the octahedral stress plane were very dependent on the density of the material. For the denser samples, the yield surfaces displayed a almost triangular-like shape (stiffer response). On the other hand, the failure surfaces for the less dense material exhibited a much rounder reaction (softer response).

It was demonstrated that the Modified Cam-Clay Model was able to reproduce certain general characteristics of the soil behavior under shearing. Others were not well reproduced. Overall, from the beginning of the tests to about halfway to failure, the numerical model was in fairly good agreement with the experimental soil behavior. After this, the numerical behavior was much stiffer than the actual experimental data. It is also worth to mention that the model was less effective mimicking the behavior of the loose sand.

7.3 Recommendations for Future Work

A major modification should be implemented in the LabView program that controls the testing process in the triaxial apparatus. The code should be revised and adjusted to change the system from stress-controlled to strain-controlled multiaxial device. This will allow the experimenter to better define the critical state condition on tested specimen.
A new set of drained tests should be completed to generate more data points to fully describe the shape of the failure surfaces on the octahedral stress plane. The determination of the stress paths should be dependent on the stress ratio, \( b \). For example, a loading path with \( b = 0.25 \) would provide an intermediate point between TC and SS; and a test with \( b = 0.75 \) would generate an intermediate point between SS and TE. All this variations in stress ratio are very feasible in the “true” triaxial device.

Improvements should be made on the actual specimen preparation process to make it more efficient. The fact that all the latex membrane sheets have to be glued together makes the process not very effective and time-consuming. A new membrane should be designed and fabricated with at least 5 sides already attached to each other. This would eliminate the valuable time spent marking, cutting, and gluing the latex membranes.

Digital imaging of the test specimen can be further accomplished by modifying the cell frame to allow the extraction of a solidified (with epoxy) failed sample. This modification would involve the splitting of the aluminum frame, and the installation of hinges and locks to freely open and close the triaxial cell.
REFERENCES


Macari, E.J., “Class Notes for Advanced Soil Mechanics I,” Louisiana State University, Department of Civil and Environmental Engineering, Course CE7000, Fall 2000.

APPENDIX

RESULTS OF EXPERIMENTAL PROGRAM
Figure A1: Results from a Triaxial Compression Test on a sand specimen with $e = 0.41$ at $s_{oct} = 100$ kPa.
Figure A2: Results from a Triaxial Compression Test on a sand specimen with $e = 0.41$ at $s_{oct} = 200$ kPa.
Figure A3: Results from a Triaxial Compression Test on a sand specimen with $e = 0.41$ at $s_{oct} = 400$ kPa.
Figure A4: Results from a Simple Shear Test on a sand specimen with $e = 0.41$ at $s_{oct} = 100$ kPa.
Figure A5: Results from a Simple Shear Test on a sand specimen with $e = 0.41$ at $s_{oct} = 200$ kPa.
Figure A6: Results from a Simple Shear Test on a sand specimen with $e = 0.41$ at $s_{oct} = 400$ kPa.
Figure A7: Results from a Triaxial Extension Test on a sand specimen with $e = 0.41$ at $s_{oct} = 100 \text{kPa}$.
Figure A8: Results from a Triaxial Extension Test on a sand specimen with $e = 0.41$ at $s_{oct} = 200$ kPa.
Figure A9: Results from a Triaxial Extension Test on a sand specimen with $e = 0.41$ at $s_{oct} = 400$ kPa.
Figure A10: Results from a Triaxial Compression Test on a sand specimen with $e = 0.65$ at $s_{oct} = 100$ kPa.
Figure A11: Results from a Triaxial Compression Test on a sand specimen with $e = 0.65$ at $s_{oct} = 200$ kPa.
Figure A12: Results from a Triaxial Compression Test on a sand specimen with $e = 0.65$ at $s_{oct} = 400$ kPa.
Figure A13: Results from a Simple Shear Test on a sand specimen with $e = 0.65$ at $s_{oct} = 100$ kPa.
Figure A14: Results from a Simple Shear Test on a sand specimen with $e = 0.65$ at $s_{oct} = 200$ kPa.
Figure A15: Results from a Simple Shear Test on a sand specimen with $e = 0.65$ at $s_{\text{oct}} = 400$ kPa.
Figure A16: Results from a Triaxial Extension Test on a sand specimen with $e = 0.65$ at $s_{oct} = 100$ kPa.
Figure A17: Results from a Triaxial Extension Test on a sand specimen with $e = 0.65$ at $s_{oct} = 200$ kPa.
Figure A18: Results from a Triaxial Extension Test on a sand specimen with $e = 0.65$ at $s_{oct} = 400$ kPa.
Figure A19: Grain Size Distribution Curve for the sand (SP) tested in this investigation.
VITA

Orlando Boscan was born in Maracaibo, Venezuela, on November 11, 1974, the son of Raquel Pirela de Boscan and Orlando Boscan Badell. After completing his work in “Santa Angela” High School, Maracaibo, Venezuela, in 1991, he entered to the Zulia State University at Maracaibo, Venezuela. After a period of two years, affected by several university strikes, he jointed University Rafael Urdaneta in Maracaibo, Venezuela. During the next four and a half years he completed his undergraduate work obtaining the degree of Bachelor of Science with a major in civil engineering in May 1998. He dedicated the next year and a half to study the English language at the English Language and Orientation Program (ELOP), institution associated to Louisiana State University (LSU) in Baton Rouge, Louisiana, and to take undergraduate courses in petroleum engineering at LSU. In January 2000, he entered the Graduate School of LSU at Baton Rouge, Louisiana. In August 2000, he married Tiffany Gilbreath of Birmingham, Alabama. Daughter Carolina Boscan was born in January 2001. After nearly completing in June 2002 his graduate work in Civil Engineering at LSU, he joint Fugro South, Inc. (formerly know as McClelland Engineers) of Houston Texas, a consulting engineering firm specialized in Geotechnical Engineering operating nationwide in the United States and overseas. Daughter Nadia and Laura Boscan were born in July 2001 and April 2004, respectively.