ROUTE PLANNING OF AUTOMATED GUIDED VEHICLES FOR CONTAINER LOGISTICS

A Thesis

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**By
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ABSTRACT

Automated guided vehicles (AGVs) are widely used in container terminals for the movement of material from shipping to the yard area and vice versa. Research in this area is directed toward the development of a path layout design and routing algorithms for container movement. The problem is to design a path layout and a routing algorithm that will route the AGVs along the bi-directional path so that the distance traveled will be minimized. This thesis presents a bi-directional path flow layout and a routing algorithm that guarantee conflict-free, shortest time routes for AGVs. Based on the path layout, a routing algorithm and sufficient, but necessary conditions, mathematical relationships are developed among certain key parameters of vehicle and path. A high degree of concurrency is achieved in the vehicle movement. The routing efficiency is analyzed in terms of the distance traveled and the time required for AGVs to complete all pickup and drop-off jobs. Numerical results are presented to compare performance of the proposed model. The research provides the foundation for a bi-directional path layout design and routing algorithms that will aid the designer to develop complicated path layouts.

Keywords: Automated guided vehicles, container logistics, conflict-free routing.

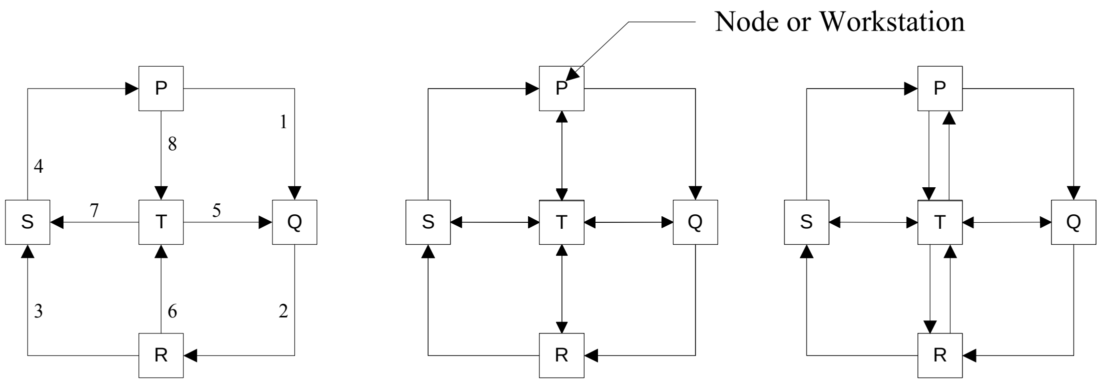
CHAPTER 1

INTRODUCTION

Automated guided vehicles (AGVs) are self-driven vehicles used to transport material from one location on the facility floor to another without any accompanying operator, and are widely used in material handling systems, flexible manufacturing systems, and container handling applications. With the advance of technology, more sophisticated machines are available, which considerably reduce machining and internal setup time. The aim of production planning has shifted from fast production to the efficient transportation of material between the workstations and in and out of storage. Flexible material handling systems are required to perform an efficient routing of material. The use of AGVs increases flexibility, since the flow path can easily be reconfigured to accommodate production changes. The design of material handling guide path has a significant implication on the overall system performance and reliability, since it has a direct impact on the travel time, the installation cost, and the complexity of the control system software.

1.1 Flow Path Networks

AGV flow path networks can be classified into three categories, namely, unidirectional, bi-directional, and mixed models. The operational control of a unidirectional model [Figure 1(a)] is very simple, since the controller need not require the functionality to accommodate bi-directional travel. However, the simplicity comes at the cost of reduced system throughput. A mixed model [Figure 1(c)] can be used to overcome deficiencies in the unidirectional model.



(a) Unidirectional Model (b) Bi-directional Model (c) Mixed Model

Figure 1. Flow Path Network Models

In Figure 1(a), for example, with the unidirectional model, the only way to go from node T to node P is to follow lanes 7 and 4, even though there is a shorter path (lane 8) with the bi-directional model. In this model, some aisles of the guide path system operate on a bi-directional mode, while the rest operate on a unidirectional mode. A bi-directional model [Figure 1(b)] with controller can be used to achieve the same objective. Bi-directional models achieve significant reduction in the total travel distance and the space requirements for a flow path network, and are economical with fewer vehicles.

As one of the enabling technologies, scheduling and routing of AGVs has attracted considerable attention, and many scheduling and routing algorithms for AGVs have been proposed. Routing algorithms are classified into two categories, namely, general path topology and specific path topology. While designing scheduling and routing algorithms, a number of interrelated decisions must be made; these include determining the guide path layout and characteristics, the number and type of vehicles, the location, type, and buffer capacities of pickup/deposit (P/D) stations, routing algorithms, the type of communications, the type and characteristics of the control system (e.g., centralized, decentralized, zone or distributed, etc.). In order to improve the system in terms of throughput and response time, good routing algorithms and path layouts are necessary. Much research is directed towards the development of routing algorithms and path layouts for a specific application. Current research in this area is directed toward the container handling application.

1.2 The Problem

Recent AGV research has focused on container handling logistics. In a container port, an AGV originates from a location near one of the container cranes at a container ship, with a specific destination within the yard area. Similarly, an AGV could reverse the direction of its travel.

Here, we consider the following routing problem. AGVs are assumed to originate from fixed locations in a bi-directional path layout, and they are directed to a different location on the path. The AGVs can move in both directions on the same lane without turning around, as shown in Figure 1(b). Every AGV has a distinct origin as well as a distinct destination. The objective is to efficiently route all AGVs, such that they reach their destinations without conflict or congestion and within the shortest possible time. Presently, the application is being studied at a Singapore container port. However, no satisfactory solution to the problem of scheduling and routing of AGVs has been found. Hence, there is a need to discover the problem solution both in theory and in a realistic application.

The proposed research focuses on the bi-directional path layout and routing algorithm for a container handling application. A bi-directional path layout consists of two parallel lanes, L_1 and L_2, and a bridge connecting the lanes at the workstations.

Vehicles are allowed to travel in both directions, and the functionality is accomplished by providing a bridge connecting two parallel lanes at the P/D station. All P/D jobs are divided into two disjoint subsets depending on the positions of the P/D jobs. Accordingly, AGVs are also classified into two disjoint subsets, which will run parallel along a bi-directional path layout in opposite directions.

In this system, the task and routes for each vehicle are determined in advance as part of the system design, not part of the controller planning function, and the system is controlled thorough a centralized control mechanism. Thus, the possible communication between an AGV and the central controller is kept to a minimum. Also, even if the loading and unloading time is not uniform, it does not affect the routing, as these operations are scheduled at the beginning and end of the P/D jobs. The proposed path layout and routing algorithm will route AGVs without conflict, and within the shortest possible time.

1.3 Research Objective and Scope

The problem that can form a basis for the above shortcomings has been addressed here specifically. It is followed by the objective, scope and application strategy.

1.3.1 Goal

The research aims to reduce the distance that an AGV must travel to move material or container from one workstation to another in a bi-directional path layout. Reduction in travel distance will increase the throughput of the system. Conflict-free AGV routing, travel time reduction, and increase in the system throughput are the indicators toward the improvement of the system performance, which motivated this study.

1.3.2 Objective

The research deals with developing a bi-directional path layout and an algorithm for routing AGVs on the path layout in a non-conflicting manner, as commonly found in container ports. Given a set of P/D jobs and the AGVs, the objective is to route the AGVs along the path, such that the distance traveled will be minimum. A single vehicle is allowed to carry only one job at a time. Hence, the primary objectives of the research are:

(i) To study the existing routing algorithms and path layout design for a specific path topology, and to use the analogous ideas for AGV routing model.

(ii) To design the bi-directional path layout in such a way that the AGV routing is achieved in a non-conflicting manner.

(iii) To develop an algorithm that will route AGVs to carry the P/D jobs within the shortest possible time.

(iv) To develop a criteria for conflict-free routing.

1.3.3 Scope

The research deals with routing AGVs along a bi-directional path layout. The container distribution schedule is assumed to be known on a certain time horizon, which generates a set of P/D tasks. The workstations are assumed to be equally spaced. Initially, the vehicles rest at the respective workstation in a park. When a P/D job is assigned to an AGV by the central controller, the route for the AGV is determined beforehand. Deadheading is the empty travel trip of the AGV from one workstation to another, which is the AGV trip to pick up material in this research.

1.4 Applications

AGVs have pervasive applications in flexible manufacturing systems (FMS). AGVs are used to move pallets, parts, and raw material among the workstations. AGV systems are used in docking terminal operations for the storage and retrieval of containers. AGVs used as a part of flexible manufacturing systems can be utilized in either of the two possible modes, namely, (a) carriers and (b) careers and workstations.

The vehicles, used as carriers, provide the transportation medium between the workstations. On arrival at the workstation, the load is delivered on the load stand. When the load processing is finished, the vehicle is called to transport the load to the next station.

The vehicles, used as carriers and mobile workstations, provide transportation service and also function as mobile workstations. The vehicle picks up a load and as it advances along the line, the operation is performed on the loaded parts. This application can be found in the automotive industry.

In recent years, AGVs have been used in seaports for container handling that greatly improves the overall operational efficiency. Container shipping has become a popular means to convey high-value products. Each container vessel entering the port is assigned to a gantry crane. All the containers assigned for transshipment are discharged from the vessel onto AGVs by gantry cranes; the AGVs then transport the containers to specific storage locations in the yard area. Outgoing containers are uploaded onto the ship after the majority of incoming ones have been unloaded from the vessel. The outgoing containers are carried by AGVs from the yard to the quay area, where they are loaded onto the ship by quay crane.

CHAPTER 2

RELEVANT LITERATURE REVIEW

Vehicle route planning involves selection of a route for the vehicle, in addition to scheduling the vehicle's journey through the route. The path layout design and a routing algorithm for conflict-free AGV routing have been addressed in several papers in past research. Past research on the AGV system can be classified into three categories: routing algorithms for general path topology, for specific path topology, and vehicle scheduling algorithms.

2.1 Routing Algorithms for General Path Topology

Broadbent *et al.* (1985) first introduced the concept of conflict-free routing. The routing procedure described is based on Dijkstra's shortest path algorithm. Potential conflicts among the vehicles are detected by comparing path occupation times, and thereby avoided in advance. Glover *et al.* (1985) developed a polynomially bounded shortest path algorithm, called the partitioning shortest path (PSP) algorithm, which finds the shortest distance from one node to another in a network.

Egbelu and Tanchoco (1986) showed that the throughput can be increased with a bi-directional path network. However, control of the bi-directional path AGV system can be complex because of the problem of the traffic control at the intersections. Daniels (1988) first developed a partitioning shortest path (PSP) algorithm based on branch-and-bound method, used to route vehicles in a bi-directional path. The method can detect and find a conflict-free shortest time route for a newly added AGV without changing the existing route of others. The correctness and feasibility (feasible conflict-free and the shortest time path for the new AGV from origin to the destination) of the algorithm was theoretically proven.

Huang *et al.* (1989) proposed a polynomial time labeling algorithm to find the shortest time path for routing a single vehicle in a bi-directional path network. This algorithm allows the path segments to be shared within their free time windows. The algorithm also finds the shortest path through the use of time windows on arcs or nodes in order to avoid collision. Kim and Tanchoco (1991) presented Dijkstra's shortest-path algorithm for conflict-free shortest-time routing of AGVs in a bi-directional path. In a time window graph, where the node set represents the free time windows, and the arc set represents the reachability among free time windows, the graph is used to determine whether the vehicle will reach from one time window to another. Then AGV routing is accomplished through the free time windows of the time window graph instead of the physical nodes of the path network.

Krishnamurthy *et al.* (1993) developed a column generation technique to minimize the makespan, while routing AGVs along the bi-directional path network in a non conflicting manner. Later, Narasimhan *et al.* (1993) extended the model to generate the conflict-free routes for AGVs with varying speed. Kim and Tanchoco (1993) proposed a model for the operational control of a bi-dirctional AGV system. The model describes a conservative myopic strategy, to coordinate the movement of vehicles in a bi-directional path layout. Under the conservative myopic strategy, one vehicle is considered at a time; all the previous decisions are strictly maintained, and a subsequent travel schedule is assigned only after the vehicle becomes idle.

Kim *et al.* (2002) presents a construction algorithm for designing a guide path of an AGV system. The total travel time is used as a decision criteria and the direction of the path segments on a unidirectional path layout is determined. A reinforcement learning (Q learning) technique is used to estimate the travel time of the vehicles on the path layout. The proposed algorithm reduces the vehicle travel distance, when compared with Kim and Tanchoco (1993).

Taghaboni and Tanchoco (1995) proposed an incremental route planning algorithm, which can route AGVs quickly and efficiently. The algorithm selects the next node to which the vehicle will travel, based on the status of neighboring nodes and the global network information. The vehicle is rerouted until it reaches the destination in a non- conflict manner. However, the incremental route planning algorithm can not achieve a high efficiency, when the number of tasks, the guide path layout complexity, and the number of vehicles increase.

Langevin *et al.* (1996) present a dynamic routing algorithm, which gives an optimal integrated solution for planning, dispatching, routing, and scheduling of AGVs in a flexible manufacturing system. The algorithm based on dynamic programming was proposed and solved on the rolling time horizon, which finds the transportation plan that minimizes the makespan.

Gaskins and Tanchoco (1987) first formulated the path layout problem as a zero-one integer programming model. The emphasis was placed on the optimization of the path layout rather than the routing algorithm. The objective is to find the unidirectional flow path, which will minimize the total transportation distance. Kaspi and Tanchoco (1990) proposed the branch-and-bound method with a depth-search first technique, which gives the best path design, provided that the P/D station locations and the facility layout are given. The model reduces the computational time at the cost of the path design compared to Gaskins and Tanchoco (1987).

Sinriech and Tanchoco (1991) proposed the Intersection Graph Method (IGM) for solving an AGV flow path optimization model developed by Kaspi and Tanchoco (1990). A procedure based on the technique of branch-and-bound and an algorithm, satisfying the reachability condition for the nodes in the AGV flow path network, is presented. Only intersection nodes are used to find an optimal solution. With this improved procedure, the number of branches of the main problem is almost half the number of branches described in the model developed by Kaspi and Tanchoco (1990). Goetz and Egbelu (1990) studied the same problem in a different approach. The problem of selecting the guide path, as well as the location of a pickup and drop-off stations, was addressed in the paper. A linear integer program was formulated to minimize the total distance traveled by AGVs.

Narsimhan and Batta (1999) proposed a rule based heuristic for routing of AGVs in the presence of unexpected interruptions. A route generated database is used to obtain the previously generated vehicle paths, and a flexible re-routing strategy is used, when the vehicle incurs interruption. Batta *et al.* (1999) presented a dynamic conflict-free routing of AGVs in a bi-directional path layout, with the AGVs themselves being unidirectional. In this model, a network representation of an AGV is presented, and the operational control factors are addressed. An effective route generation technique is developed that routes the AGVs, moving with varying speed.

Singh and Tiwari (2002) proposed an intelligent agent framework to determine an optimal conflict-free route for an AGV system. The model describes a multi-agent approach to the operational control of AGVs in dynamic environment. A rule based system and an efficient routing algorithm is presented for finding a conflict-free shortest-time path for AGVs in a bi-directional, as well as a unidirectional flow path network. The concept of loop formation in a flow path network is introduced, which deals with the parking of idle vehicles.

Wu and Zeng (2002) present a colored Petri net model for deadlock avoidance in an AGV system, whereby the model is developed and an effective control law is presented. The deadlock is completely avoided by observing the state of the system and checking the free spaces available in some of the circuits. The model was developed for an AGV system in a unidirectional path layout, which decreases the system performance. Revoltis (2000) proposes conflict resolution strategy in a bi-directional path layout, which ensures robust AGV conflict resolution. The operational flexibility is maintained by free vehicle travel on an arbitrarily structured guide path network. The zone control strategy is adopted, and AGVs are synthesized incrementally. An effective and efficient structure control policy is developed for AGVs resource allocation systems.

2.2 Routing Algorithms for Specific Path Topology

Based on the path layout, path topologies with the routing algorithms have been proposed. The path topology is a virtual arrangement of the elements of a network. The networks may differ in physical interconnections, distances among the nodes and signal type. AGV Path topologies are single-loop, multi-loops, and mesh, etc. Recent research on AGV routing is directed towards the specific path topology.

Tanchoco and Sinriech (1992) suggested an optimal closed loop guide path layout configuration for an AGV system. They developed an algorithm based on integer programming to find the optimal single loop. In the model, if all the vehicles run in the same direction with uniform speed, there will be no collision, because the optimal singe loop path has no intersections. Lin and Dgen (1994) provided an algorithm for routing control of a tandem AGV system. The system is composed of several non overlapping loops, and the stations within each loop are served by a single dedicated vehicle. If the destination station is not located within the same loop, a load needs more than one vehicle to carry out the task. A task-list time-window algorithm is employed to find the shortest route from a source workstation to the destination without disrupting the travel schedules of other vehicles.

Sinriech and Tanchoco (1994) proposed Segmented Flow Topology (SFT), which is similar to the multi-loops path layout in Lin and Dgen (1994). The SFT can be used in conjunction with connected, partitioned, and split-flow network. The general SFT is comprised of one or more zones, each of which is separated into non-overlapping segments with each segment serviced by a single vehicle. The vehicle can move bi-directionally in the segment. Therefore, the routing control for such a path topology is very simple.

2.3 Scheduling Algorithms

The AGV systems considered above usually have a relatively small number of vehicles and P/D jobs. In this case, the problem of AGV scheduling is trivial, when compared with the existing solutions of path network optimization and routing control. However, for an AGV system with a great number of P/D jobs and a large size of AGV fleet, such as container handling in a seaport, the scheduling of AGVs become nontrivial. The scheduling algorithm must be able to select a vehicle, and perform the route planning for the selected vehicle. It should be studied separately from the routing problem and a path layout design. However, recent research on AGV scheduling is relatively scarce and limited in scope.

Taghboni and Tanchoco (1988) presented a LISP based controller that will find a route and schedule an AGV along that route in a non conflicting manner. Ulusoy and Bilge (1993) addressed the problem of simultaneous scheduling of machines and AGVs in a flexible manufacturing system. The problem is decomposed into two sub-problems, machine scheduling and vehicle scheduling. A heuristic is developed that will generates new machine schedule at each iteration, and from operations time of the machine schedule, time windows are constructed for each material handling trip. The second sub-problem is solved as a sliding time window problem.

Based on mixed-integer programming algorithm, Akturk and Yilmaz (1996) proposed a micro-opportunistic approach (MOSA) to schedule vehicles and jobs in a decision-making hierarchy. The proposed algorithm combines job-based and vehicle-based approaches into a single algorithm, in which both the critical jobs and the travel time of unloaded vehicles are considered simultaneously.

Qiu and Hsu (2001) presented an algorithm to schedule and route AGVs on a bi-directional path layout. Based on assumptions and criteria conditions, a model is developed, achieving a high degree of concurrency in AGV movement. When AGVs are scheduled on the path layout as per the given algorithm and condition, there will be no conflict, congestion, or deadlock. Bish *et al.* (2001) developed a heuristic for the analysis of new vehicle scheduling and location problem. The problem is to assign each container to yard location and dispatch vehicles to the containers so as to minimize the time it takes to download all the containers from the ship. The effectiveness of the heuristic is analyzed from both the worst case and computational point of view.

2.4 Drawbacks of Previous Research

General path routing algorithms emphasize on finding the optimal route without considering the path layout design. Therefore, computational complexity increases with the size of network and the number of vehicles. On the other hand, routing algorithms for specific path topologies, focused on a path layout. Hence, the routing algorithms are simple, but the system throughput is low. The research is not sufficient to combine the path layout design with efficient routing algorithm, which will route AGVs along a bi-directional path layout in a conflict- free manner. From the survey of previous research, the following drawbacks are addressed.

(a) Broadbent *et al.* (1985), Daniels (1988), Huang et al. (1989), Kim and Tanchoco (1991,1993) developed a bi-directional model for an optimal AGV routing, but they did not give more emphasis on the path layout design. Since the path networks are not optimized, the computational complexity increases with the size of the path network.

(b) Gaskin and Tanchoco (1987), Kaspsi and Tanchoco (1990), Tanchoco and Sinriech (1991,1992), Goetz and Egbelu (1990), Hsu and Huang (1994) presented models based on path optimization and integer programming. However, due to a unidirectional nature of the model, the system throughput and the path utilization is low.

(c) Hsu and Huang (1994) idealized the assumption of the buffer capacity. Tanchoco and Sinriech (1994), Lin and Dgen (1994) models suffer from low system throughput, and indirect transportation may cause an increase in travel time. Qiu and Hsu (2001) developed routing algorithms for a specific path topology. However, the system throughput is low due to non optimal nature of the routing algorithm and path layout.

In this research, the path layout and routing algorithm is developed for a container terminal, which will route AGVs along a bi-directional path layout in a non conflicting manner. The research will overcome some of the drawbacks of the previous research. It is anticipated that the proposed research will provide better results compared to the previous model for a specific path layout.

CHAPTER 3

MODEL DEVELOPMENT

This section of research describes the formulation of an AGV system model, which consists of a bi-directional path layout and a routing algorithm. A bi-directional path layout is designed to formulate the model. In order to simplify solution to the model, we make some definitions and assumptions. The bi-directional path layout, assumptions, and definitions are as described below.

3.1 Bi- directional Path Layout

The bi-directional path layout with N number of pickup and drop-off stations placed along lane L_1 is as shown in Figure 3. An AGV picks up a load from a workstation and drops to another workstation. Once the route is determined, only one AGV can drop the load at any workstation. The bi-directional path layout given by Qiu and Hsu (2001) is given in Figure 2.

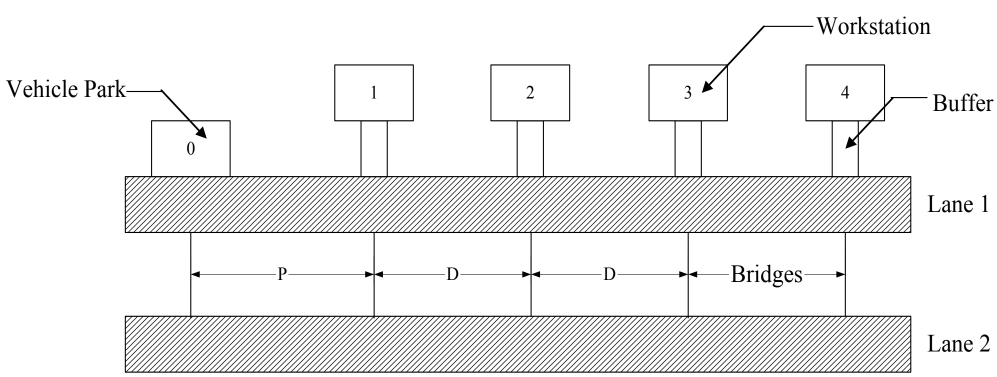


Figure 2. Bi-directional path layout (Qiu and Hsu, 2001)

In the path layout (Qiu and Hsu 2001) as shown in Figure 2, the vehicle park (0), where AGVs rest initially, is provided. As the P/D task is assigned to each vehicle, the vehicle moves from the park to a source workstation. This empty travel trip (deadheading) reduces the routing efficiency, and hence, the system throughput. In the proposed path layout, park can be removed, and the buffer space can be enlarged, where park will be provided for the AGVs. By this modification, empty travel trip time can be saved. Also, the floor space utilization will remain the same, increasing the system throughput. The new bi-directional path layout has been proposed as shown in Figure 3. The proposed path layout is as below:

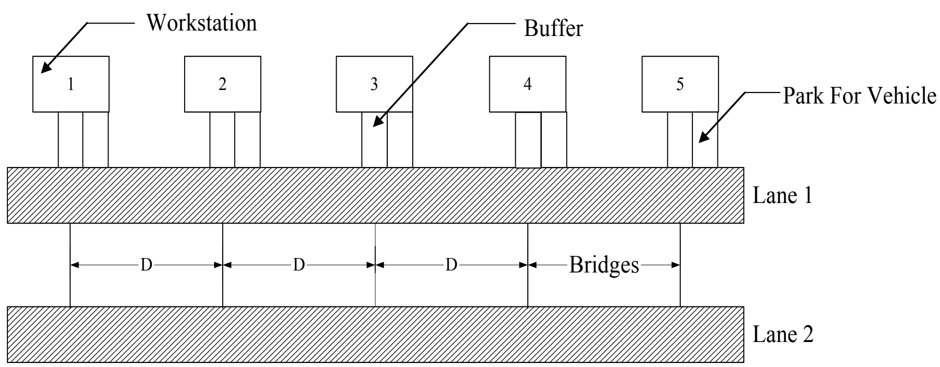


Figure 3. The proposed path layout

(1) There are two parallel lanes L_1 and L_2. Parking space and buffer is provided at each station along lane L_1. For simplicity in presentation, we assume that a workstation lies off the main travel area and is only entered by an AGV, when a pickup or drop-off has to be made. A vehicle can stop at the buffer to either pick up or drop-off the load. A buffer is an area off the main travel space where an AGV can wait, usually to permit another AGV to move on the path.

(2) There is a bridge connecting two lanes at each station. The points, where bridges are connected to lane L_2 are referred as mirror stations, denoted by $N+1, N+2, \cdots, N+N$. Thus, a bridge can be identified by an ordered pair $(i, N+i)$. However, there is no buffer storage at the mirror stations.

(3) The lanes and bridges are bi-directional, and the distance between any two adjacent stations is equal (*D*).

(4) The width of the lanes and bridges is such that the only one vehicle can pass at a time. However, a vehicle can pass by a station, while loading or unloading process of another vehicle is being carried out in the buffer.

(5) The zone length is vehicle length plus twice the safety allowance, which will protect the vehicle from collision.

3.1.1 Definitions

The following definitions are made to formulate the model:

N The number of workstations,

D The distance between the adjacent workstation,

P The distance of the first workstation from the park,

K The number of P/D jobs,

P/D A load to be picked up from the specified workstation (origin) and then to be delivered to another different specific workstation (destination),

(P_i, D_i) An ordered pair that identifies P/D job, where P_i and D_i represents the P/D jobs respectively,

J The set of K P/D jobs, J can be represented as,

$J = \{(P_i, D_i) \mid 1 \le P_i \le N, 1 \le D_i \le N \text{ and } P_i \ne D_i \text{ for } i = 1,2,\cdots,K\}$,

J^+ The set of P/D jobs, $P_i < D_i$, $J^+ = \{(P_i, D_i) \mid P_i < D_i \text{ for } i = 1,2,\cdots,K\}$,

J^- The set of P/D jobs, $P_i > D_i$, $J^- = \{(P_i, D_i) \mid P_i > D_i \text{ for } i = 1,2,\cdots,K\}$,

Where $J^+ \cap J^- = \phi$, $J^+ \cup J^- = J$ and $2 \le K \le N$,

C_{AGV} The set of ordered workstations with two AGVs, $C_{AGV} = \{C_1, C_2, \cdots, C_s\}$,

E_{AGV} The set of ordered workstations with no AGV, $E_{AGV} = \{E_1, E_2, \cdots, E_s\}$,

${C_{AGV}}^{+}$ The set of workstations (pair) where $C_i < E_i$,

$${C_{AGV}}^{+} = \{(C_i, E_i) \mid C_i < E_i \text{ for } i = 1,2,\cdots,s\},$$

${C_{AGV}}^{-}$ The set of workstations (pair) where $C_i > E_i$,

$${C_{AGV}}^{-} = \{(C_i, E_i) \mid C_i > E_i \text{ for } i = 1,2,\cdots,s\},$$

T_p The time for an AGV to pick up a load,

T_D The time for an AGV to drop off a load,

T_{loaded_run} The time required by a job set J when AGVs run with load,

T_{move} The time required for an AGV to move to the nearest station,

$T(P_i, D_i)$ The time for a loaded AGV to run from pickup station P_i to the drop-off station D_i.

Accordingly,

V^{+} = the set of AGVs (vehicles) that carry out jobs in J^{+},

V^{-} = the set of AGVs (vehicles) that carry out jobs in J^{-},

$|J^{+}|$ = the number of jobs in subset J^{+},

$|J^{-}|$ = the number of jobs in subset J^{-},

${W_{AGV}}^{+}$ = the set of AGVs (vehicles) that move along ${C_{AGV}}^{+}$,

${W_{AGV}}^{-}$ = the set of AGVs (vehicles) that move along ${C_{AGV}}^{-}$,

s = the number of AGV movement tasks, $s = |C_{AGV}|$ or $|E_{AGV}|$.

3.1.2 Assumptions

The following assumptions are made to develop the routing algorithm

(1) All of the K P/D jobs are distinct, i.e., no two or more vehicles have the same pickup or a drop-off station.

(2) All the vehicles run with the same velocity V, on either lane L_1 or L_2.

(3) The velocity of the vehicle on the bridge is V/r, where $r > 1$, the velocity slowdown factor.

(4) Only one P/D job is assigned to a single AGV at a time.

(5) Initially, all AGVs will rest at the respective station in a park near the buffer storage.

3.2 Routing Algorithm

The aim of route planning is to achieve maximum throughput for an AGV operations. The focus is to find an optimal (the shortest possible time path) and feasible route for every single AGV. Three aspects are considered while making the routing decision: (a) it should detect whether there exists a route which could lead the vehicle from its origin to the destination, (b) the route selected for an AGV must be feasible, i.e., the route must be congestion, conflict, and deadlock free (Taghaboni and Tanchoco 1995), and (c) the route must be optimal (minimize idling runs of vehicles). The routing algorithm proposed by Qiu and Hsu (2001) was used as a basis for parallel processing of AGVs along the bi-directional. According to the proposed path layout, the new routing algorithm is developed. Based on the path layout (Figure 3) and assumptions, the shortest path routing algorithm is given as below.

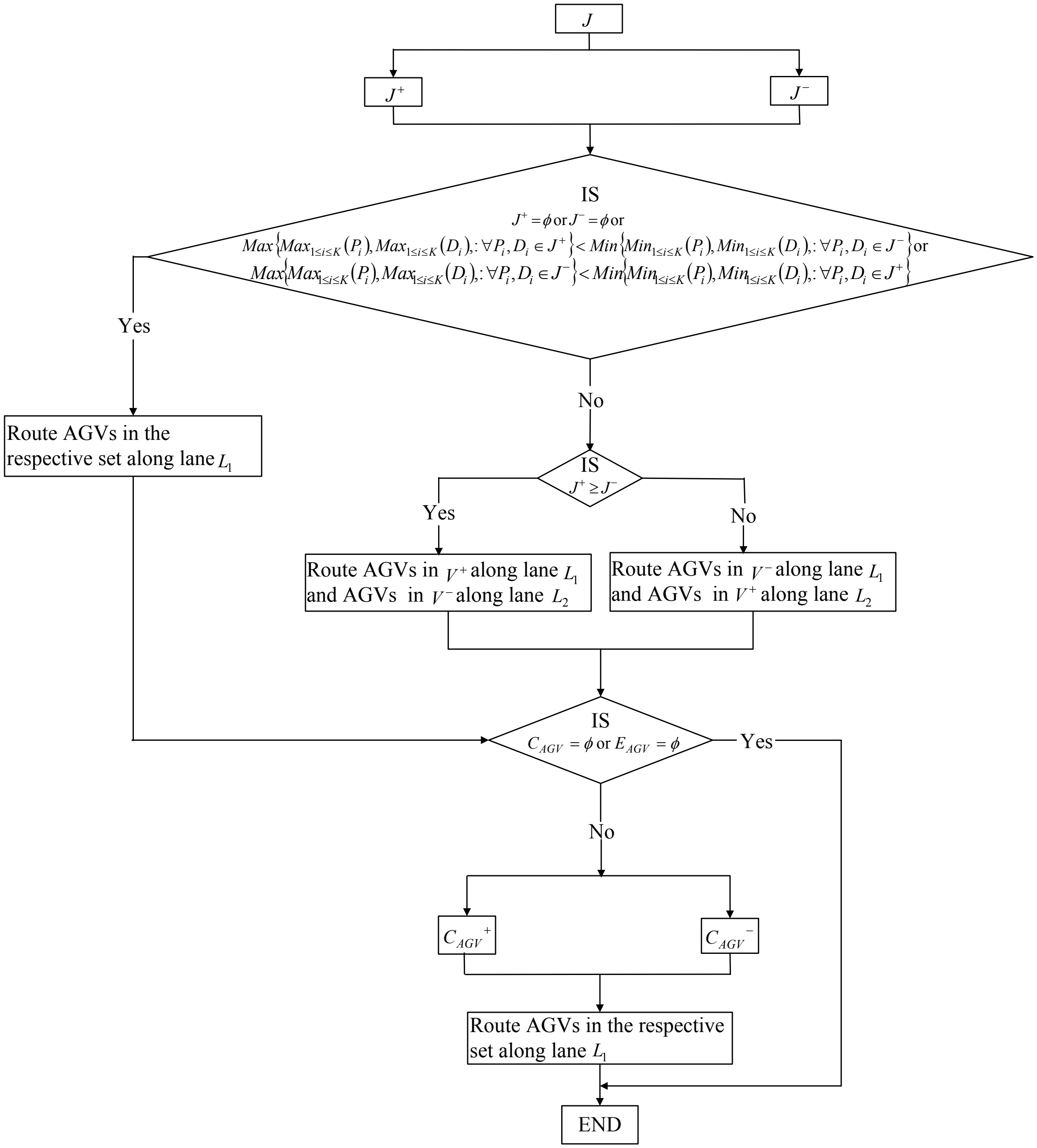


Figure 4. Routing Algorithm Flow Chart

3.2.1 Algorithm 1: Routing Algorithm

(1) Let K be the number of jobs. Initially, all AGVs are at the pickup station. Loading is done according to the requirement at the respective station. After loading, all AGVs are set out to their respective drop-off stations.

2. (a) Check the movement of AGVs in V^+ and V^- set. If

$$Max\{Max_{1\le i\le k}(P_i), Max_{1\le i\le k}(D_i): \forall P_i, D_i \in J^+\} < Min\{Min_{1\le i\le k}(P_i), Min_{1\le i\le k}(D_i): \forall P_i, D_i \in J^-\}$$

or $Max\{Max_{1\le i\le k}(P_i), Max_{1\le i\le k}(D_i): \forall P_i, D_i \in J^-\} < Min\{Min_{1\le i\le k}(P_i), Min_{1\le i\le k}(D_i): \forall P_i, D_i \in J^+\}$

then route all AGVs along lane L_1. Go to step 3.

(b) If $\left|J^+\right| \ge \left|J^-\right|$, all AGVs in V^+ advance along lane L_1 from the *left* side to the *right* side, while AGVs in V^- cross the bridge, reach their mirror-pickup stations, and advance along lane L_2 from the *right* to the *left* side. Go to step 3.

(c) If $\left|J^+\right| < \left|J^-\right|$, all AGVs in V^- advance along lane L_1 from the *right* side to the *left* side, and AGVs in V^+ cross the bridge and reach their mirror-pickup stations, then advance along lane L_2 in opposite directions. Go to step 3.

(3) When AGVs moving on lane L_1 reach their destinations, they immediately start unloading, and stay in buffer after completion. However, AGVs on lane L_2 have to (a) reach their mirror stations (b) cross the bridge to reach their drop-off stations; (c) drop loads off and stay in buffers. The workstations with two AGVs and no AGVs are arranged serially in the sets C_{AGV} and E_{AGV}, respectively.

(4) If $K < N$**,** and $C_{AGV} \ne \Phi$;

(a) Consider C_{AGV} and E_{AGV} sets. If $C_i < E_i$, take the pair of stations (C_i, E_i) in the set $C_{AGV}{}^+$. If $C_i > E_i$, take the pair of stations (C_i, E_i) in the set $C_{AGV}{}^-$.

Update the sets C_{AGV} and E_{AGV} by deleting these number of stations.

(b) Repeat the step (a), while $C_{AGV} \neq \Phi$.

(5) Route AGVs in W_{AGV}^{+} set and W_{AGV}^{-} set along lane L_1 from the *left* side to *right* and from the *right* side to the *left* side, respectively.

(6) Once an AGV moves from a drop-off station to the nearest station (park), AGV will rest at that station until the scheduling for the next operation is done.

- **Computational Complexity**

The computational complexity of the model is calculated as below.

Table 1. Computational complexity of the model

Steps	Algorithm Step Description	Procedure	Operations
1.	Divide the given job set into two disjoint subsets (J^+ and J^-) and list the jobs.	Listing	n
2.	Check $Max\{Max_{1\leq i\leq K}(P_i), Max_{1\leq i\leq K}(D_i); P_i, D_i \in J^+\}$ $< Min\{Min_{1\leq i\leq K}(P_i), Min_{1\leq i\leq K}(D_i); P_i, D_i \in J^-\}$	Finding minimum and maximum	$\{(n/2-1)+(n/2-1)+1\}+$ $\{(n/2-1)+(n/2-1)+1\}$ $=(n-1)+(n-1)$
3.	Find Loaded AGV travel time	Addition	n-1
4.	List AGVs in C_{AGV} and E_{AGV} List AGVs in C_{AGV}^{+} and C_{AGV}^{-} Delete C_{AGVi} and E_{AGVi} from sets C_{AGV} and E_{AGV}	Listing Listing Deletion	n n n
5.	Find AGV movement time to the nearest station	Addition	n/2-1
6.	Find the total AGV time	Addition	2
Total number of operations required = $n + (n-1) + (n-1) + n-1 + n + n/2 - 1 + n + n/2 + 1$ $= (15n/2 - 2) = \Theta(n)$			

Thus, the complexity of the model is Θ (n), where n represents the number of jobs.

- **Numerical Example 1: Routing Algorithm**

AGV routing problem has been conceived to represent the system, which maps the proposed algorithm. Consider a container port system with fourteen serial workstations. Let a set of workstations be placed at an interval of 50 ft. The length of the bridge (L_b) is 2 ft, and the velocity reduction factor (r) on the bridge is taken as 1.2. Let the length of an AGV that protects it from collision be 1.5 ft. Consider a situation, where the load is coming from and going to the stations shown in the following set:

$$J = \{(1,8),(2,11),(5,12),(6,9),(4,1),(12,6),(8,5),(9,2),(7,3),(10,4)\}.$$

The problem is to route AGVs along the bi-directional path layout, so that the distance traveled will be minimum.

- **Solution**

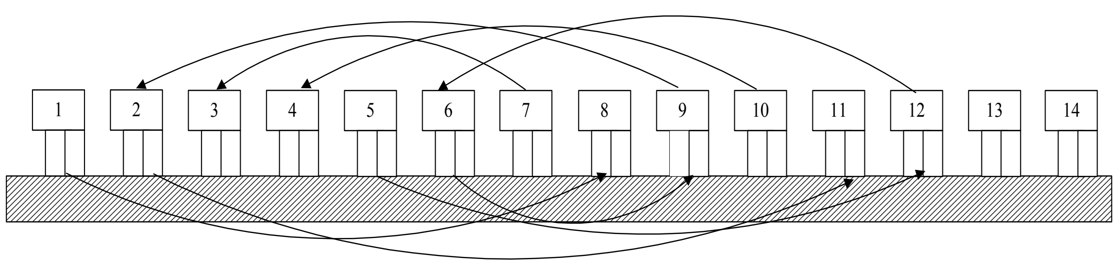


Figure. 5. Bi-directional AGV flow path

The step-by-step procedure of the proposed algorithm for the problem stated in example 1 is given below.

Step 1. Loading is done at the respective station. Based on the positions of P/D jobs, the given set of jobs is classified into two disjoint subsets. Accordingly, AGVs are classified into two disjoint subsets. The two groups of AGV move in the opposite directions as shown in Figure 4.

$J^{+} = \{(1,8),(2,11),(5,12),(6,9)\}$, and $J^{-} = \{(4,1),(12,6),(8,5),(9,2),(7,3),(10,4)\}$.

Step 2. Since $|J^{+}| < |J^{-}|$, all AGVs in V^{-} advance along lane L_1 from the *right* side to the *left* side, while AGVs in V^{+} cross the bridge, reach their mirror-pickup stations, and advance along lane L_2 in opposite directions.

Step 3. When AGVs moving on lane L_1 reach their destinations, they immediately start unloading and stay in buffers after completion. However, AGVs on lane L_2 reach their mirror stations, cross the bridge to reach their drop-off stations; drop loads off and stay in buffers.

Step 4. Since $C_{AGV} \neq \phi$, we have,

$C_{AGV} = \{3,11\}$, and $E_{AGV} = \{4,7\}$.

Accordingly, $C_{AGV}{}^{+} = \{3,4\}$, and $C_{AGV}{}^{-} = \{11,7\}$.

Step 5. Route AGVs in $W_{AGV}{}^{+}$ set along lane L_1 from the *left* side to the right side, and AGVs in $W_{AGV}{}^{-}$ set along lane L_1 from the *right* side to the *left* side. Once an AGV moves from a drop-off station to the nearest station (park), AGV will rest at that station until the scheduling for the next operation is done.

CHAPTER 4

CRITERIA FOR CONFLICT FREE ROUTING

When more than one AGV tries to occupy the same path segment, there may be a collision. The possible conflicts are head-to-head (two AGVs moving in the opposite directions), head to tail (two AGVs moving in the same direction with slow speed of the vehicle moving ahead) or a collision at the junction (two AGVs approaching at the junction from two different directions). To avoid the conflict, there should be some means to arbitrate which AGV has the right of way.

In the bi-directional path layout, two groups of vehicles run in opposite directions along lanes L_1 and L_2. By this means, the head-on collision of the vehicles is eliminated. Since the vehicles move with constant velocity *V,* each vehicle will maintain its distance $mD\left(m=1,2,3,\cdots,N-1\right)$ with another vehicle. Therefore, no vehicle will collide with another from front to rear position. Hence, the possible conflict is at the junction, where one vehicle moves along lane L_1 passing the station, while another vehicle approaches the same station from lane L_2. The speed of vehicle moving on the bridge is reduced by factor *r*. Since the vehicles are moving with same speed along the bi-directional path in opposite directions, with reduced speed (*V/r*) along the bridge, and the distance between adjacent stations is uniform, the claim and proof given by Qiu and Hsu (2001) can be applied to the conflict-free routing condition. Using the claim and proof provided by Qiu and Hsu (2001) for conflict-free routing, it is shown that the designed path layout, and the routing algorithm ensure that the new vehicle will not enter the junction unless and until the previous one has completely left the junction.

4.1 Notations

The following notations are used to define the criteria condition for conflict-free routing.

D The distance between two adjacent stations

V Velocity of the vehicle running along the lane L_1 or L_2.

V / r Velocity of the vehicle running along the bridge

L_b Length of the bridge.

L_v Length of the vehicle including safety allowance that prevents the vehicle from collision.

L_j Half the length of edges of the junction.

r Velocity reduction factor, $r > 1$.

4.2 Assumptions

In order to simplify the claim of conflict-free routing, we will make following assumptions.

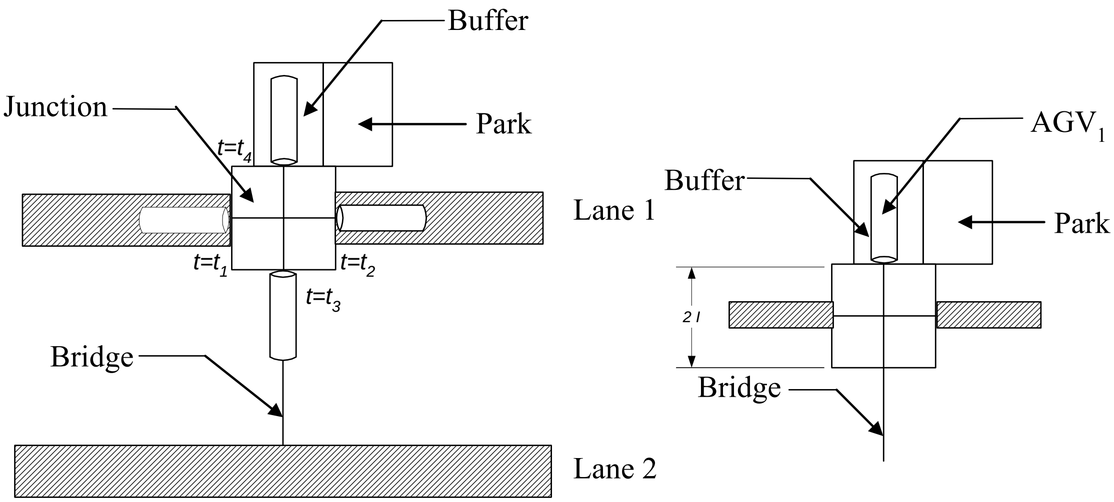
(I) The junction is a square area with $2L_j$ on each side. In order to avoid collision, when an AGV is passing through the junction, the other vehicle is not permitted to enter until the first one leaves the junction completely. Here, $2L_j$ is greater than the width of the vehicle.

(II) The length and width ($2L_j$) of the junction is smaller than the width of the vehicle.

(III) AGV_1 passes through the junction from the left side to the right side within time interval $[t_1, t_2]$ along lane L_1, where t_1 and t_2 represent the time, when an AGV enters and completely leaves the junction.

(IV) Similarly, AGV_2 passes through the junction along lane L_2 within time interval $[t_3, t_4]$, where t_3 and t_4 represent the time, when an AGV enters and leaves the junction completely.

(V) The point from where an AGV with load begins to start is shown in Figure 5.



(a) AGVS Passing through Junction (b) The current location of AGV

Figure 6. Conflict-free AGV junction details

The claim and proof given by Qiu and Hsu (2001, pp. 2182-2184.) for the conflict-free routing of AGV is given as below:

4.3 Claim for Conflict free Routing

Let, L_j be half the length of junction, L_v be the length of the vehicle, and L_b be the length of the bridge; let D be the distance between the adjacent stations, and r be the velocity reduction factor along the bridge. An AGV running on lane L_2 and crossing a bridge to a P/D station will not come into conflict with another vehicle moving on lane

L_1, if:

(1) The distance between two neighboring station satisfies the following:

$$D \geq 2(1+r)L_j + (1+r)L_v. \tag{1}$$

(2) The length of the bridge (L_b) satisfies the following:

$$\frac{1+r}{2r}L_j + \frac{1}{2r}L_v \leq L_b \leq \frac{1}{2r}D - \frac{1+r}{2r}L_j - \frac{1}{2}L_v. \tag{2}$$

Proof: No collision will occur if AGV$_1$ passes the junction before AGV$_2$, or AGV$_2$ passes the junction; i.e., either $t_2 < t_3$, or $t_4 < t_1$. Let m and n represent the workstation numbers. Assume that d_m *(mD)* and d_n *(nD)* are the distances between the junction and the pickup station of AGV$_1$ and AGV$_2$ respectively, where 1≤ i and j ≤ *N-2.*

If V is the velocity of the vehicle, D is the distance between the adjacent stations, L_j is half the length of junction, L_v is length of the vehicle, L_b is the length of the bridge, and r is velocity reduction factor, we have the following relations:

$$V.t_1 = mD - L_j + r.L_j \qquad \text{and} \qquad V.t_3 = nD + 2rL_b$$

$$V.t_2 = mD - L_j + rL_j + 2L_j + L_v \qquad V.t_4 = nD + (2L_j + L_v + 2L_b)r.$$

The possible cases that lead to vehicle conflict (Qiu and Hsu, 2001) are,

Case 1: $d_m < d_n$

In this case, AGV$_1$ is nearer to the junction. To let AGV$_1$ pass through the junction before AGV$_2$ enters, we must have $t_2 < t_3$, or

$$mD - L_j + rL_j + 2L_j + L_v \leq nD + 2rL_b.$$

Rearranging the above equation,

$$(m-n)D + (1+r)L_j + L_v \leq 2rL_b. \tag{3}$$

Consider equation (2),

$$\frac{(1+r)}{2r}L_j + \frac{L_v}{2r} \le L_b.$$

or equivalently,

$$(1+r)L_j + L_v \le 2rL_b.$$

Therefore,

$$(m-n)D + (1+r)L_j + L_v \le 2rL_b.$$

Hence, no AGV will run in conflict with another. □

Case 2: $d_m = d_n$

In this case, let AGV_1 pass through the junction before AGV_2. Similar to Case 1, we have,

$$(1+r)L_j + L_v \le 2rL_b. \tag{4}$$

The equation (4) is the left hand inequality of the equation (2). Hence, AGV_1 can still pass through the junction before AGV_2. □

Case 3: $d_m > d_n$

In this case, AGV_2 is nearer to the junction than AGV_1. Let AGV_2 pass through the junction before AGV_1. We should have $t_4 < t_1$, or:

$$mD + (2L_j + L_v + 2L_b)r \le nD - L_j + rL_j. \tag{5}$$

Rearranging the above equation, we have:

$$2rL_b \le (m-n)D - rL_v - (1+r)L_j.$$

From equation (2) we have:

$$2rL_b \le D - rL_v - (1+r)L_j \le (m-n)D - rL_v - (1+r)L_j. \quad \square \tag{6}$$

Thus, equations (3, 4, and 6) will always hold true and there will not be any conflict. Therefore, if the vehicle travel schedule is generated according to given algorithm and the criteria condition on the proposed path layout, no conflicts will occur and traffic control is not necessary.

- **Example 2: Conflict-free Routing**

Consider a container port system with fifteen serial workstations. When the ship arrives, we assume that the containers are unloaded from the ship within no time. These containers are then transported from shipping area to the yard area and containers from yard area are transported to the shipping area. Let a set of workstations be placed at an interval of 100 ft., and the length of the junction is 1 ft. Length of the bridge (L_b) is 4 ft and the velocity reduction factor (r) on the bridge is taken as 2.4. Let, the length of the AGV that protects it from collision be 3 ft and the speed of AGV be 25 ft/min. Consider a situation where load is going from and to the stations shown in set: $J = \{(1,9),(2,10),(3,13),(11,8),(12,6),(13,7)\}$.

The problem is to show that the system described above accomplishes the given P/D task without conflict.

- **Solution**

According to the positions of the P/D jobs, the given set of jobs is classified into two disjoint subsets:

$J^{+} = \{(1,9),(2,10),(3,13)\}$, and $J^{-} = \{(11,8),(12,6),(13,7)\}$.

From the criteria condition stated in the Chapter 4,

Case 1: $d_m < d_n$

Let a load $(3, 13)$ and $(12, 6)$ be carried out along the bi-directional path. At station 6, the distance traveled by AGV_1 is $d_{AGV_1} = mD - L_j + rL_j + L_v$ = 303.7, and the distance traveled by AGV_2 is $d_{AGV_2} = nD + 2rL_b$ = 619.2. So, AGV_1 will pass the station before AGV_2.

Case 2: $d_m = d_n$

Let AGV_1 and AGV_2 are carrying the jobs $(1, 9)$ and $(13, 7)$, respectively. At station 7, the two AGVs are apart equally, and the distance traveled by AGV_1 is $d_{AGV_1} = mD + L_j + rL_j + L_v$ = 604.7, while the distance traveled by AGV_2 is $d_{AGV_2} = nD + 2rL_b$ = 619.2. Hence, AGV_1 will pass the station before AGV_2.

Case 3: $d_m > d_n$

Let job sets $(2, 10)$and$(11, 8)$ be carried by two AGVs. The AGVs are moving in opposite directions; at workstation 8, the distance traveled by AGV_1 is $d_{AGV_1} = mD + L_j + rL_j + L_v$ = 604.7, and the distance traveled by AGV_2 is $d_{AGV_2} = nD + 2rL_b$ = 319.2. So, AGV_2 will pass the station before AGV_1. In any case, there will not be conflict among the vehicles, while carrying out the P/D jobs.

CHAPTER 5

ROUTING EFFICIENCY

The throughput of system is determined in terms of AGVs travel distance and the time requirement. Thus, the routing efficiency is analyzed in terms of distance traveled and time required to accomplish the given P/D task by an AGV. An incoming vehicle at the junction waits to ensure the availability of space at the buffer or destination. The process of an AGV waiting at the destination due to the removal of load is treated as postprocessing. However, in the model, we are not analyzing the ef fect of an AGV waiting for buffer and post processing operations.

5.1 Distance Traveled by All AGVs

The distance traveled by an AGV is to pick up a load (S_{out}), move with that load to the drop-off station (S_{loaded}), and then to the nearest empty park (S_{move}). For a given P/D job, the distance traveled by an AGV is given as

$$S = S_{out} + S_{loaded} + S_{move}.$$

Since the buffer is placed near the workstation, the distance traveled by an AGV to pick up the load is small, assumed to be negligible for computational purposes. Therefore,

$$S = S_{loaded} + S_{move}.$$

The distance traveled by all AGVs from the pickup to the drop-off station is given as:

$$S_{loaded} = D\sum_{i=1}^{K} |P_i - D_i| + 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right). \tag{7}$$

The distance traveled by all AGVs from a drop-off station to the nearest station (Park) is is given as:

$$S_{move} = D\sum_{i=1,}^{i=s} \left| C_{AGV_i} - E_{AGV_i} \right| . \tag{8}$$

Using equations (7) and (8), the distance traveled by all AGVs is given as:

$$S = D\sum_{i=1}^{K} |P_i - D_i| + 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right) + D\sum_{i=1}^{s} \left| C_{AGV_i} - E_{AGV_i} \right| . \tag{9}$$

5.1.1 Special Cases

Case 1: Lower bound on the distance traveled by an AGV

An ideal case is the best case in which all jobs belong to the same subset (i. e., either J^+ or J^- is empty). All the vehicles at the pickup and drop-off stations are involved in routing. The distance traveled by an AGV is to go from a pickup station to drop-off station. In the ideal case,

$$\left|J^+\right| = K, \text{or} \left|J^-\right| = K .$$

The ideal shortest distance is given as:

$$S = S_{ideal} = D\sum_{i=1}^{K} |P_i - D_i| + D\sum_{i=1}^{s} \left| C_{AGV_i} - E_{AGV_i} \right| . \tag{10}$$

This is the lower bound on the distance traveled by AGVs.

Case 2: Upper bound on the distance traveled by an AGV

In the worst case, both the subsets contain an equal number of jobs, and the distance traveled by AGVs will be the maximum. Hence,

$$\left|J^+\right| = \left|J^-\right| = K/2 \cdot$$

Using equation (9), the total distance traveled by all AGVs is given as:

$$S = S_{worst} = D\sum_{i=1}^{K} |P_i - D_i| + D\sum_{i=1}^{s} \left| C_{AGV_i} - E_{AGV_i} \right| + KL_b . \tag{11}$$

This is the upper bound on the distance traveled by all AGVs.

Thus, the distance traveled by all AGVs to complete any given job set, denoted by *S,* satisfy the following:

$$D\sum_{i=1}^{K}\left|P_i - D_i\right| + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right| \le S \le D\sum_{i=1}^{K}\left|P_i - D_i\right| + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right| + KL_b .$$

- **Example 3: Routing Efficiency in Terms of Distance Traveled**

To explain the routing efficiency of AGVs in terms of the distance traveled, we present a numerical example here.

Consider a container port with twelve serial workstations. Let a set of workstations be placed at an interval of 50 ft. Also, the length of the bridge (L_b) be 2 ft, and the velocity reduction factor (r) on the bridge is taken as 1.2. Let the length of the AGV protecting it from collision be 1.5 ft, and the length of the junction be 1 ft. The length of the vehicle moving on path is 20 ft/min. Consider a situation where load is coming from and going to the stations shown in the following set: $J = \{(1,4), (3,7), (9,2), (5,1), (12,7), (6,10), (11,4)\}$.

The problem is to route AGVs along the bi-directional path layout, so that the distance traveled will be minimum. According to the positions of P/D jobs, the given set of jobs is classified into two disjoint subsets:

$$J^+ = \{(1,4), (3,7), (6,10)\}\text{, and } J^- = \{(9,2), (5,1), (12,6), (11,5)\}.$$

Since the job set containing the stations with two AGVs (C_{AGVs}) is not empty, we have

$$C_{AGV} = \{2, 4, 7, 10\}\text{, and } E_{AGV} = \{3, 9, 11, 12\},$$

from which the number of AGV movement tasks (s) is given as

$$s = |C_{AGV}|, \text{or } |E_{AGV}| = 4.$$

Accordingly, $C_{AGV}^{+} = \{(2,3),(4,9),(7,11),(10,12)\}$, since no set of job is being carried out in a backward direction, $C_{AGV}^{-} = \phi$.

In Table 2, the pickup and drop-off jobs are represented by columns 2 and 3, respectively. The last column gives the cumulative distance traveled by a loaded AGV to carry out the given P/D task.

Table 2. Loaded AGV travel distance

Job set (i)	P_i	D_i	$\lvert P_i - D_i \rvert$	$\sum_{k=1}^{i} \lvert P_k - D_k \rvert$	$D\sum_{k=1}^{i} \lvert P_k - D_k \rvert$
1	1	4	3	3	150
2	3	7	4	7	350
3	6	10	4	11	550
4	9	2	7	18	900
5	5	1	4	22	1100
6	12	6	6	28	1400
7	11	5	6	34	1700

* D = 50 ft.

Table 3. AGV travel distance from drop-off station to the nearest park

Task (i)	C_{AGV_i}	E_{AGV_i}	$\lvert C_{AGV_i} - E_{AGV_i} \rvert$	$\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$	$D\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$
1	2	3	1	1	50
2	4	9	5	6	300
3	7	11	4	10	500
4	10	12	2	12	600

* D = 50 ft.

The workstations with two AGVs and workstations with no AGV are represented by columns 2 and 3, respectively in Table 3. The cumulative distance traveled by AGVs from a drop-off station to the nearest workstation (park) is provided in the last column. Considering data from Example 3, $L_b = 2$ ft., and $K = 7$. Also, from the disjoint subsets, J^+ and J^- ; $Min\left(\left|J^+\right|, \left|J^-\right|\right) = 3$.

Using equation (9) and the values from Tables 2 and 3, the distance traveled by loaded AGVs, and the total distance traveled by all AGVs is given as,

$$S_{loaded} = D\sum_{i=1}^{K} \left|P_i - D_i\right| + 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right) = 1{,}712 \text{ ft.}$$

$$S = D\sum_{i=1}^{K} \left|P_i - D_i\right| + 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right) + D\sum_{i=1}^{s} \left|C_{AGV_i} - E_{AGV_i}\right| = 2{,}312 \text{ ft.}$$

5.2 Time Required Completing the Assigned P/D Task

In this section, the routing efficiency is analyzed in terms of the time required to accomplish the assigned P/D task. AGV travel time represents the time required to pick up a load, move with that load to the drop-off station, and the time required to move to the nearest empty park. The upper and lower bound on the time requirement are calculated. If the pick up time is not constant for all the pickup jobs, the loading time is the maximum time taken by the pickup job. Also, if the unloading time is not uniform for all the jobs, the maximum time taken by the drop-off job is treated as the unloading time. In calculating the routing efficiency, we assume that the loading time (T_p) and unloading time (T_d) are constant for every AGV and P/D job. Therefore, the difference in loading and unloading time creates no conflict, and the time required for a given job set *J*, is determined by the most time consuming job in the set.

Hence, the total time required to carry out the given task of P/D jobs is given as

$$T = T_p + T_D + T_{loaded\_run} + T_{move}.$$

Here, we assume that the time required for loading and unloading is negligible for computational purposes. Therefore,

$$T = T_{loaded\_run} + T_{move}.$$

5.2.1 Loaded AGV Travel Time

Loaded AGV travel time is the time required for an AGV to move from the pickup to the drop-off station. In the equation stated below, the distance traveled by all the AGVs is same as in equation (10), except the distance traveled on the bridge. Since the speed of the vehicle running on the bridge is reduced by the slowdown factor *r,* the distance traveled by the group of vehicles on the bridge is multiplied by the velocity reduction factor. Hence, the time required for the loaded AGVs is given as:

$$T_{loaded\_run} = \frac{D\sum_{i=1}^{K}\left|P_i - D_i\right| + 2rL_b Min\left(\left|J^+\right|,\left|J^-\right|\right)}{V}. \tag{13}$$

We will discuss two cases for AGV travel time:

Case 1: AGV travels to the adjacent station along lane L_1

If $2 \le K \le N$, there may exists a job set *J*, such that $J = \{i, i+1\} \| i = 1,2,\cdots,k.$, where $T(i, i+1) = \frac{D}{V}$, for $1 \le i \le K$. The time required to carry out the given job set is given as:

$$T_{loaded\_run} = \frac{D}{V}$$

Case 2: AGV travels from a pickup station (N) to drop-off station (1) along lane L_2

If $\left|J^+\right| > \left|J^-\right|$, it is clear that the job *(N, 1)* is the most time consuming job and the loaded time required to accomplish the job is given as:

$$T_{loaded\_run} = \frac{(N-1)D + 2rL_b}{V}.$$

In combining the above two cases, we get the conclusion

$T_{loaded\_run} = T_{AGV} - Tp - T_D - T_{move,}$ satisfies

$$\frac{D}{V} \le T_{loaded\_run} \le \frac{(N-1)D + 2rL_b}{V}. \tag{14}$$

5.2.2 AGV Travel Time from a Drop-off Station to the Nearest Park

The time required for AGVs to move from a workstation with two AGVs to the nearest empty workstation (park) is:

$$T_{move} = \frac{D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|}{V}. \tag{15}$$

The total time required to complete the assigned P/D task is the loaded AGV travel time and the time required to move from a drop-off station to the nearest park, given as:

$$T = \frac{D\sum_{i=1}^{K}\left|P_i - D_i\right| + 2rL_b Min\left(\left|J^+\right|, \left|J^-\right|\right) + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|}{V}. \tag{16}$$

5.2.3 Special Cases

Case 1: Lower bound on the time required to carry out the given set of P/D jobs

An ideal case represents the best case in which all jobs belong to the same subset, or all AGVs are routed along lane L_1. In the ideal case, the distance traveled by an AGV to accomplish the given P/D task is to travel from a pickup to drop-off station and one vehicle will move to the nearest workstation. In this case, only one vehicle will travel from the workstation with two AGVs to the workstation with no AGVs. The ideal shortest distance is given as:

$$T_{ideal} = \frac{D\sum_{i=1}^{K}|P_i - D_i| + D\sum_{i=1}^{s}|C_{AGV_i} - E_{AGV_i}|}{V}. \tag{17}$$

This is the lower bound on the time required to accomplish the given P/D task.

Case 2: Upper bound on the time required to carry out the given set of P/D jobs

In the worst case, both the subsets contain the equal number of jobs.

$$|J^+| = |J^-| = K/2.$$

Using the equation (16), the total time required to accomplish the given set of P/D jobs is given as:

$$T = T_{worst} = \frac{D\sum_{i=1}^{K}|P_i - D_i| + D\sum_{i=1}^{s}|C_{AGV_i} - E_{AGV_i}| + KrL_b}{V}. \tag{18}$$

This is the upper bound on the time required to accomplish the given P/D task.

From the equations (15) and (16), the bound on the total time required to accomplish the given P/D task is given as:

$$\frac{D\sum_{i=1}^{K}|P_i - D_i| + D\sum_{i=1}^{s}|C_{AGV_i} - E_{AGV_i}|}{V} \le T \le \frac{D\sum_{i=1}^{K}|P_i - D_i| + D\sum_{i=1}^{s}|C_{AGV_i} - E_{AGV_i}| + KrL_b}{V}, \tag{19}$$

which is the lower and upper bound on the time required accomplishing the given P/D task.

- **Example 4: Routing Efficiency in Terms of Time Required**

To explain the routing efficiency of AGVs in terms of time required to accomplish the given task of P/D jobs, we present an example problem here.

Consider a container port with fourteen serial workstations. Let a set of workstations be placed at an interval of 50 ft. Also, consider the length of the bridge (L_b) to be 2 ft., and the velocity reduction factor (r) on the bridge to be 1.2. Let the length of the AGV protecting it from collision be 1.5 ft., and the length of the junction as 1 ft. The velocity of AGV moving on the path is 20 ft/min. Consider a situation where the load is coming from and going to the stations shown in the set:

$J = \{(1,12),(3,11),(4,14),(7,13),(6,2),(9,1),(13,5),(10,8)\}$.

The problem is to route AGVs along the bi-directional path layout so that the distance traveled will be minimum. According to the positions of P/D jobs, the given set of jobs is classified into two disjoint subsets:

$$J^{+} = \{(1,12),(3,11),(4,14),(7,13)\}, \text{ and } J^{-} = \{(6,2),(9,1),(13,5),(10,8)\}.$$

Since $C_{AGV} \neq \phi$, and $E_{AGV} \neq \phi$, we have

$$C_{AGV} = \{2,5,8,11,12,14\}, \text{ and } E_{AGV} = \{3,4,6,7,9,10\},$$

from which the number of AGV movement tasks (s) is calculated as

$s = |C_{AGV}|$, or $|E_{AGV}| = 6$.

Accordingly, $C_{AGV}^{+} = \{2,3\}$, and $C_{AGV}^{-} = \{(5,4),(8,6),(11,7),(12,9),(14,10)\}$.

The total distance traveled by loaded AGVs to carry out the given job set is displayed in the last column in Table 4.

Table 4. Loaded AGV travel distance

Job set (i)	P_i	D_i	$\lvert P_i - D_i \rvert$	$\sum_{k=1}^{i} \lvert P_k - D_k \rvert$	$D\sum_{k=1}^{i} \lvert P_k - D_k \rvert$
1	1	12	11	11	550
2	3	11	8	19	950
3	4	14	10	29	1450
4	7	13	6	35	1750
5	6	2	4	39	1950
6	9	1	8	47	2350
7	13	5	8	55	2750
8	10	8	2	57	2850

* D = 50 ft.

Table 5. AGV travel distance from drop-off station to the nearest park

Task (i)	C_{AGV_i}	E_{AGV_i}	$\lvert C_{AGV_i} - E_{AGV_i} \rvert$	$\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$	$D\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$
1	2	3	1	1	50
2	5	4	1	2	100
3	8	6	2	4	200
4	11	7	4	8	400
5	12	9	3	11	550
6	14	10	4	15	750

* D = 50 ft.

The cumulative distance traveled by AGVs to move from a drop-off workstation to the nearest empty park is provided by the last column in Table 5.

Considering data from Example 4, $K = 8$, $r = 1.2$ ft., $L_b = 2$ ft., and $V = 20$ ft / min.

From the two disjoint subsets, J^+ and J^-, $Min(\lvert J^+ \rvert, \lvert J^- \rvert) = 4$.

Using equation (16) and the values from Tables 4 and 5, the time required for loaded AGVs, and the time required for all AGVs to accomplish the given P/D task is given by:

$$T_{loaded\_run} = \frac{D\sum_{i=1}^{K} \lvert P_i - D_i \rvert + 2rL_b Min(\lvert J^+ \rvert, \lvert J^- \rvert)}{V} = 2{,}869.2/20 = 143.46 \text{ mins.}$$

$$T = \frac{D\sum_{i=1}^{K}\left|P_i - D_i\right| + 2rL_b Min\left(\left|J^+\right|, \left|J^-\right|\right) + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|}{V} = 3619.2/20 = 180.96 \text{ mins.}$$

CHAPTER 6

COMPARISON WITH PREVIOUS MODEL

The performance of the proposed model was compared with Qiu and Hsu (2001). In order to ascertain whether the proposed model can indeed give better outcome, we compare the routing efficiency in terms of the distance traveled and the time required.

6.1 Theoretical Comparison

Theoretical comparison was performed between Qiu and Hsu (2001) and the proposed model for routing efficiency. In the model proposed by Qiu and Hsu (2001), the park for the vehicle was provided separately at the beginning of the path layout, which causes deadheading to pick up the material. In the proposed model, the empty travel trip of the vehicle is eliminated by providing the park for each vehicle at the workstation. After the loaded AGV travel, when there is a single workstation with two AGVs, the distance traveled will be reduced by more than half, compared to Qiu and Hsu (2001).

In the worst case, when both the subsets contain an equal number of jobs, the distance traveled will be significantly reduced, since the vehicle need not travel to pick up the job, and the vehicle will move to the nearest empty workstation after its unloading operation. Benefits of the proposed model are best realized, when the number of workstations and the distance between the consecutive workstations increase.

The lower and upper bound on the distance traveled and the time required to accomplish the given P/D task is shown in Tables 6 and 7.

Table 6. Comparison of Routing Efficiency between Qiu and Hsu (2001) and the proposed model in terms of Distance traveled.

	Qiu and Hsu (2001)	Proposed Model
S_{AGV}	$2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + 2Lb \text{ Min } (\lvert J^+\rvert, \lvert J^-\rvert)$	$D\sum_{i=1}^{k}\lvert P_i - D_i\rvert + 2L_b Min(\lvert J^+\rvert, \lvert J^-\rvert) + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i}\rvert$
S_{ideal}	$2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1)$	$D\sum_{i=1}^{k}\lvert P_i - D_i\rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i}\rvert$
S_{worst}	$2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + KL_b$	$D\sum_{i=1}^{k}\lvert P_i - D_i\rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i}\rvert + KL_b$
Bound on the Travel distance	$2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) \le S \le 2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + KL_b$	$D\sum_{i=1}^{k}\lvert P_i - D_i\rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i}\rvert \le S \le D\sum_{i=1}^{k}\lvert P_i - D_i\rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i}\rvert + KL_b$

Table 7. Comparison of Routing Efficiency between Qiu and Hsu (2001) and the proposed model in terms of Time required.

	Qiu and Hsu (2001)	Proposed Model
T_{AGV}	$\frac{2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + 2rLb\ \text{Min}\ (\lvert J^+ \rvert, \lvert J^- \rvert)}{V}$	$\frac{D\sum_{i=1}^{k}\lvert P_i - D_i \rvert + 2rL_b Min(\lvert J^+ \rvert, \lvert J^- \rvert) + D\sum_{i=1}^{s}\lvert C_{AGV\,i} - E_{AGV\,i} \rvert}{V}$
T_{ideal}	$\frac{2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1)}{V}$	$\frac{D\sum_{i=1}^{k}\lvert P_i - D_i \rvert + D\sum_{i=1}^{s}\lvert C_{AGV\,i} - E_{AGV\,i} \rvert}{V}$
T_{worst}	$\frac{2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + KrL_b}{V}$	$\frac{D\sum_{i=1}^{k}\lvert P_i - D_i \rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i} \rvert + KrL_b}{V}$
Bound on the Travel Time	$\frac{2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1)}{V} \le T \le \frac{2KP + 2D\sum_{i=1}^{k}(Max(P_i, D_i) - 1) + KrL_b}{V}$	$\frac{D\sum_{i=1}^{k}\lvert P_i - D_i \rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i} \rvert}{V} \le T \le \frac{D\sum_{i=1}^{k}\lvert P_i - D_i \rvert + D\sum_{i=1}^{s}\lvert C_{AGV_i} - E_{AGV_i} \rvert + KrL_b}{V}$

From Qiu and Hsu (Q&H, pp. 2185.), the total AGV travel distance is given as:

$$S_{Q\&H} = 2KP + 2D\sum_{i=1}^{K}\left(Max\left(P_i, D_i\right) - 1\right) + 2rL_b Min\left(\left|J^+\right|, \left|J^-\right|\right).$$

From equation (9), we have:

$$S_{proposed} = D\sum_{i=1}^{K}\left|P_i - D_i\right| + 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right) + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|$$

Let

$$x = D\sum_{i=1}^{k}\left(Max\left(P_i, D_i\right) - 1\right),$$

$$a = KP,$$

$$y = D\sum_{i=1}^{k}\left|P_i - D_i\right|,$$

$$z = D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|,$$

$$q = 2L_b Min\left(\left|J^+\right|, \left|J^-\right|\right).$$

So, the above equations transform to:

$S_{Q\&H} = 2a + 2x + q$, and

$S_{proposed} = y + q + z$

Since all the workstations are arranged serially along the bi-directional path, we have:

$Max\left(P_i, D_i\right) - 1 \geq \left|P_i - D_i\right|$, and

$Max\left(P_i, D_i\right) - 1 \geq \left|C_{AGV_i} - E_{AGV_i}\right|$,

which can be expressed as

$x \geq y$, and $x \geq z$.

Hence, the factor,

$$2x - (y + z) \geq 0,$$

or more strongly,

$$2(a + x) - (y + z) > 0,$$

will always holds true.

Since the vehicle is moving with a uniform speed, the same conclusion can be drawn for the time required to accomplish the given P/D task. Hence, the routing efficiency obtained by the proposed model will always be greater than that of Qiu and Hsu (2001).

6.2 Numerical Comparison

To evaluate the performance, a numerical comparison between the proposed methodology and Qiu and Hsu (2001) model is made. A numerical example problem is presented and the results obtained from the proposed algorithm were compared with those by Qiu and Hsu (2001).

- **Example 5: Routing Efficiency (Ideal Case)**

In the ideal case, there will be only one workstation with two AGVs after the loading operation is done. Consider a container port with twelve serial workstations. Let a set of workstations be placed at an interval of 50 ft., and the distance of the first station from the park to be 60 ft. The length of the bridge (L_b) is 2 ft., and the velocity reduction factor (r) on the bridge is taken as 1.2. Let the length of the AGV that protects it from collision be 1.5 ft., while the length of the junction be 1 ft. The velocity of AGV moving on the path is 20 ft/min. Consider the situation where the load is going coming from and going to the stations shown in the set: $J = \{(1,3),(3,5),(5,7),(7,10),(10,12)\}$.

The problem is to route AGVs along the bi-directional path layout, so that the distance traveled will be minimum. According to the positions of P/D jobs, the given set of jobs is classified into two disjoint subsets as:

$$J^{+} = \{(1,3),(3,5),(5,7),(7,10),(10,12)\}, \text{ and } J^{-} = \phi .$$

The set J^{-} will be empty, as no job is being carried out in backward direction. The numerical calculations for Qiu and Hsu (2001) and the proposed model are as below:

(a) Qiu and Hsu (2001) model:

The P/D job set and the cumulative distance traveled by all AGVs from and to the workstation 1 is shown in Table 8.

Table 8. Total AGV travel distance (From workstation 1)

Job Set (i)	P_i	D_i	$Max(P_i, D_i) - 1$	$\sum_{k=1}^{i} Max(P_k, D_k) - 1$	$2D \sum_{k=1}^{i} Max(P_k, D_k) - 1$
1	1	3	2	2	200
2	3	5	4	6	600
3	5	7	6	12	1200
4	7	10	9	21	2100
5	10	12	11	32	3200

* D = 50 ft.

Considering data from Example 5, $K = 5, P = 60 \text{ ft}$. and $V = 20 \text{ ft} / \text{min}$.

Using equations for the ideal case from Qiu and Hsu (2001, pp. 2185-2187.) and the values from Table 8, the distance traveled and the time required for all AGVs to accomplish the given P/D task is given as:

$$S_{AGV} = 2KP + 2D\sum_{i=1}^{K}(Max(P_i, D_i) - 1) = 3{,}800 \text{ ft.}$$

$$T_{AGV} = \frac{2KP + 2D\sum_{i=1}^{K}(Max(P_i, D_i) - 1)}{V} = 3{,}800/20 = 190 \text{ mins.}$$

(b) The proposed model:

Since we have set of workstations with two numbers of AGVs,

$C_{AGV} = \{12\}$, and $E_{AGV} = \{1\}$,

which gives the number of AGV movement tasks (s) as

$s = |C_{AGV}|$, or $|E_{AGV}| = 1$.

Accordingly, $C_{AGV}{}^{+} = \varphi$, and $C_{AGV}{}^{-} = \{(12, 1)\}$.

Table 9. Loaded AGV travel distance

Job Set (i)	P_i	D_i	$\lvert P_i - D_i \rvert$	$\sum_{k=1}^{i} \lvert P_k - D_k \rvert$	$D\sum_{k=1}^{i} \lvert P_k - D_k \rvert$
1	1	3	2	2	100
2	3	5	2	4	200
3	5	7	2	6	300
4	7	10	3	9	450
5	10	12	2	11	550

* D = 50 ft.

In Table 9, pickup and drop-off jobs are shown in columns 2 and 3, respectively. The total distance traveled by loaded AGV to carry out all the P/D jobs is shown in the last column.

Table 10. AGV travel distance from drop-off station to the nearest park

Task (i)	C_{AGV_i}	E_{AGV_i}	$\left\|C_{AGV_i} - E_{AGV_i}\right\|$	$\sum_{k=1}^{i}\left\|C_{AGV_k} - E_{AGV_k}\right\|$	$D\sum_{k=1}^{i}\left\|C_{AGV_k} - E_{AGV_k}\right\|$
1	12	1	11	11	550

The cumulative distance traveled by AGVs to move from a drop-off station to the nearest workstation (park) is given in Table 10.

Considering data from Example 5, $L_b = 2$ ft, $r = 1.2$ ft, $K = 5$ and $V = 5$ ft/min

Using equations (10) and (17) and the values from Tables 9 and 10, the distance traveled and the time required by all AGVs to accomplish the given P/D task is given as:

$$S = D\sum_{i=1}^{K}\left|P_i - D_i\right| + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right| = 550 + 550 = 1{,}100\,\text{ft.}$$

$$T = T_{ideal} = \frac{D\sum_{i=1}^{K}\left|P_i - D_i\right| + D\sum_{i=1}^{s}\left|C_{AGV_i} - E_{AGV_i}\right|}{V} = 1{,}100/20 = 55\,\text{mins.}$$

- **Example 6: Routing Efficiency (Worst Case)**

In the worst case, both the subsets will contain an equal number of jobs before loading and after the loading operation. Consider a production line with twelve serial workstations. Let a set of workstations be placed at an interval of 50 ft., and the distance of the first station from the park be 60 ft. Consider the length of the bridge (L_b) as 2 ft., and the velocity reduction factor (r) on the bridge as 1.2. Let the length of an AGV protecting it from collision be 1.5 ft., and the length of the junction be 1 ft. Let the velocity of AGV moving on the path be 20 ft/min. Consider a situation where the load is coming from and going to the stations shown in the following set: $J = \{(1,5), (3,9), (10,12), (8,2), (7,4), (11,6)\}$.

The problem is to route AGVs along the bi-directional path layout, so that the distance traveled will be minimum. According to the positions of P/D jobs, the given set of jobs can be classified into two disjoint subsets as:

$$J^{+} = \{(1,5),(3,9),(10,12)\}, \text{ and } J^{-} = \{(8,2),(7,4),(11,6)\}.$$

The numerical calculations for Qiu and Hsu (2001) and the proposed model are as below:

(a) Qiu and Hsu (2001) model:

The pickup and drop-off jobs for the respective job set are shown by columns 2 and 3, respectively in Table 11. The cumulative distance traveled by all AGVs from and to the workstation 1, to carry out all P/D jobs is given in the last column.

Table 11. Total AGV travel distance (From workstation 1)

Job Set (i)	P_i	D_i	$Max(P_i, D_i)-1$	$\sum_{k=1}^{i} Max(P_k, D_k)-1$	$2D\sum_{k=1}^{i} Max(P_k, D_k)-1$
1	1	5	4	4	400
2	3	9	8	12	1200
3	10	12	11	23	2300
4	8	2	7	30	3000
5	7	4	6	36	3600
6	11	6	10	46	4600

* D = 50 ft.

Considering data from Example 6, $K = 6$, $L_b = 2$ ft, $P = 60$ ft., $r = 1.2$ ft., and $V = 20$ ft/min.

Using equations for the worst case from Qiu and Hsu (2001, pp. 2185-2187.) and the values from Table 11, the distance traveled and the time required for all AGVs to accomplish the P/D task is given as:

$$S_{AGV} = 2KP + 2D\sum_{i=1}^{K}\left(Max(P_i, D_i) - 1\right) + KL_b = 5{,}332 \text{ ft.}$$

$$T_{AGV} = \frac{2KP + 2D\sum_{i=1}^{K}(Max(P_i, D_i) - 1) + KrLb}{V} = 5{,}334.40 / 20 = 266.72 \text{ mins.}$$

(b) The proposed model:

Since we have a set of workstations with two numbers of AGVs,

$$C_{AGV} = \{2, 4, 5, 6, 9, 12\} \text{ and } E_{AGV} = \{1, 3, 7, 8, 10, 11\},$$

which gives the number of AGV movement tasks (s) as

$$s = |C_{AGV}|, \text{ or } |E_{AGV}| = 6.$$

Accordingly, $C_{AGV}^{+} = \{(5,7),(6,8),(9,10)\}$, and $C_{AGV}^{-} = \{(2,1),(4,3),(12,11)\}$.

Table 12. Loaded AGV travel distance

Job Set (i)	P_i	D_i	$\lvert P_i - D_i \rvert$	$\sum_{k=1}^{i} \lvert P_k - D_k \rvert$	$D\sum_{k=1}^{i} \lvert P_k - D_k \rvert$
1	1	5	4	4	200
2	3	9	6	10	500
3	10	12	2	12	600
4	8	2	6	18	900
5	7	4	3	21	1050
6	11	6	5	26	1300

*D = 50 ft.

Pickup and drop-off jobs are shown by columns 2 and 3, respectively in Table 12. The cumulative distance traveled by all AGVs to move from a pickup station to the drop-off station is given by the last column in Table 12.

Table 13. AGV travel distance from drop-off station to the nearest park

Task (i)	C_{AGV_i}	E_{AGV_i}	$\lvert C_{AGV_i} - E_{AGV_i} \rvert$	$\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$	$D\sum_{k=1}^{i} \lvert C_{AGV_k} - E_{AGV_k} \rvert$
1	2	1	1	1	50
2	4	3	1	2	100
3	12	11	1	3	150
4	5	7	2	5	250
5	6	8	2	7	350
6	9	10	1	8	400

* D = 50 ft.

In Table 13, the workstations with two AGVs and workstations with no AGV are represented by columns 2 and 3, respectively. The cumulative distance traveled by AGVs to move from a drop-off station to the nearest workstation (park) is given in the last column.

Considering data from Example 6, $L_b = 2$ ft., $K = 6$, $r = 1.2$ ft, and $V = 20$ ft / min.

Using equations (11) and (18) and the values from Tables 12 and 13, the distance traveled and time required by all AGVs to accomplish the given P/D task is given as:

$$S = D\sum_{i=1}^{K} \lvert P_i - D_i \rvert + D\sum_{i=1}^{s} \lvert C_{AGV_i} - E_{AGV_i} \rvert + KL_b = 1{,}712 \text{ ft.}$$

$$T = \frac{D\sum_{i=1}^{K} \lvert P_i - D_i \rvert + D\sum_{i=1}^{s} \lvert C_{AGV_i} - E_{AGV_i} \rvert + KrL_b}{V} = 1{,}714.4/20 = 85.72 \text{ mins.}$$

As the vehicles are placed in a park near the workstation, the AGV need not make a distant trip to pick up material neither does it needs to return the same station after the unloading as it will move to a nearby station. Therefore, the vehicle travel distance from the drop-off workstation to the park is reduced, and the total travel time is also reduced for P/D jobs. The performance of the proposed model is better when the distance between the adjacent workstations increase. The computational results of the two models are reported in Table 14.

The results of theoretical comparison between the proposed model and Qiu and Hsu (2001) are summarized in Tables 6 and 7. The last two columns in Table 14 show that the percent reductions achieved in distance and time are $((3800-1100)/3800 \approx 71.05\%)$ and $((190-55)/190 \approx 71.05\%)$, respectively, accomplishing the given P/D task by the proposed model. It is evident from the table that the proposed model is superior to the Qiu and Hsu (2001) model with an increase in routing efficiency and a decrease in total distance traveled.

Table 14. Numerical Comparison result for Example 5 and Example 6

P = 60, D = 50, N = 12, L_b = 2, L_v = 1.5, r = 1.2, and V = 12							
Routing Path	K	Qiu and Hsu (2001)		Proposed Model		Percent Reduction	
		S_{AGVs} *(ft)*	T_{AGVs} *(mins.)*	S_{AGVs} *(ft)*	T_{AGVs} *(mins.)*	S_{AGVs}	T_{AGVs}
{(1,3), (3,5), (5,7), (7,10), (10,12)}	5	3,800.00	190.00	1,100.00	55.00	71.05	71.05
{(1,5), (3,9), 10,12), (8,2),(7,4),(11,6)}	6	5,332.00	266.72	1,712.00	85.72	67.89	67.86

* P, D, L_b, L_v, r, are in *ft*, and V is in *ft/ min*.

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH

7.1 Conclusion

The aim of this research is to achieve higher transportation efficiencies, thereby driving the logistics cost down. The AGV routing and network design is a key factor in the optimization of material transportation in a container terminal. This thesis has proposed a mathematical model for conflict-free routing of AGVs in a bi-directional path layout. The model offers a trade-off between the network optimization and efficient routing. The path layout and routing algorithm for a specific path topology are presented to route AGVs within the shortest possible time. The time required for the loading and unloading process creates no conflict, because these operations are carried out either at the beginning or at the end of operation. As AGVs are placed at each workstation, the AGV travel time is reduced, and the system throughput is increased. The advantage of the model is best realized when the ideal situation (all the vehicles move along lane L_1 and only one vehicle moves to the nearest workstation after drop-off operation) occurs, and the number of P/D task increases. The model shows that the inclusion of park at the respective stations leads to a large reduction in the travel distance, and ultimately reduces the logistics cost. The proposed model may be regarded as a framework suitable for extension and application to a container terminal.

7.2 Significance of Research

The proposed model incorporates the issue of effective path layout design with an efficient routing algorithm. The criteria condition for conflict-free routing is presented. The model carries a significant contribution in terms of increased routing efficiency of

AGVs in a bi-directional path layout, since the proposed algorithm and the path layout are very efficient. This research can form the basis for routing AGVs on complex path layouts. Further research can exploit the routing algorithm and criteria condition for conflict-free routing. New routing algorithms can be developed to route AGVs on mesh topologies.

7.3 Future Research

The following research may advance current status of the problem.

- **Communication failure and AGV breakdown:** In case of communication failure of an AGV with the central controller, the vehicle will stick in the path. Also, an AGV breakdown during the routing operation is not taken into consideration. As this will block all the vehicles carrying out the P/D task, these failures may be considered for future extension.
- **AGV routing on a non-uniform path layout:** AGVs can be routed along the bi-directional path layout where the distance between the adjacent workstations may not be uniform. In this case, the speed of the vehicle must be synchronized, so that the time required in traveling a distance between any two adjacent workstations will be the same.
- **Continuous AGV Routing:** In the proposed model, if loading and unloading time is not uniform across different P/D jobs, the time taken for these operations is characterized by the most time consuming job. This decreases the routing efficiency. However, with continuous routing, AGVs will be set out immediately after loading and unloading, regardless of the time for these operations across different P/D jobs. Then, the AGVs will be scheduled for the next P/D task.

- **Routing AGVs on mesh topology:** In the model, AGVs are routed along the bi-directional path. The model can be extended to route the AGVs along a complex path layout like mesh topology, which is a rectangular array of linear path.

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