AN OPTIMAL OPERATIONAL POLICY FOR AN INTEGRATED PRODUCTION-DELIVERY SYSTEM UNDER CONTINUOUS PRICE DECREASE

A Thesis

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ABSTRACT

In today's competitive world, the unit cost of a high-tech product declines significantly over its life cycle. An integrated inventory model for products experiencing continuous decrease in unit cost is studied in this research. In this integrated model a manufacturing facility purchases raw material from outside supplier at a fixed size and supplies a fixed quantity of finished products to a buyer periodically after using its production processes. Moreover, buyers demand frequent deliveries of small lots of finished products since the price is continuously decreasing, and this emphasizes the significance of just-in-time (JIT) inventory management for successful companies in technology-related industries. The goal in this study is to minimize the total cost of the supply chain in JIT environment while the price of the high-tech product is linearly decreasing over its life cycle. A cost model composed of manufacturer's raw materials and finished goods and buyer's incoming goods inventory costs is developed here. An efficient algorithm is employed to determine the optimal or near-optimal lot sizes for raw material procurement, manufacturing batch and buyer's ordering policies. It is also shown in the implemented model that the integrated total cost over the planning horizon considers the changing prices at each replenishment for both manufacturer's and buyer' s inventory costs. Consequently, in this article, the traditional integrated inventory model is relaxed by removing the restriction of constant unit cost. Finally, the solution technique for the developed model is illustrated with numerical examples, and compared with the previously developed integrated inventory models to test its accuracy. It is proven that the model is accurate and effective for the inventory systems with decreasing unit cost.

CHAPTER 1

INTRODUCTION

Nowadays, companies can no longer compete solely as individual entities in the constantly changing business world. Globalization of market and increased competition force organizations to rely on effective supply chains to improve their overall performance. A supply chain involves individual entities such as raw material supplier, finished good manufacturer, retailers, wholesalers, buyers/consumers etc. who are responsible for converting the raw material into a finished good and make them available to customers to satisfy their demand in time at least possible cost. Successful supply chain management requires a change from managing distinct function to integrating activities into key supply chain processes. Integration between two different business entities is an important way to gain competitive advantages as it lowers supply chain cost.

1.1 Supply Chain Management of Technology-Related Companies

According to Lambert and Cooper (2000), "supply chain management deals with total process excellence and represents a new way of managing the business and relationships with other members of the supply chain". The challenging issue in the supply chain management is to figure out how to accomplish the cross-functional coordination among the parties of various businesses. Supply chain integration can take place either through intra- or inter-company cooperation or combination of both. The intra-company cooperation considers integrating manufacturer's raw material procurement and its production whereas, the inter-company cooperation includes integration between manufacturer's production and buyer's ordering.

In recent years, one of the approaches that have had a major impact on supply chain is just-in-time (JIT) inventory management. JIT policy forces manufacturer to produce required quantities of goods at the required time. In JIT environment, supplier needs to adjust the production schedule simultaneously with the buyer's demand. At each cycle supplier has to order exact required quantities of raw materials to produce the expected demand of buyers while reducing the inventory cost. For the most effective JIT inventory control, the inventory of the raw materials should be zero at the end of production time of each cycle. Furthermore, as expected the inventory of the finished goods should also be equal to zero at the end of each cycle time in order to minimize the inventory cost of the manufacturing facility. However, it is not easy to be capable of following this most effective JIT inventory policy. Suppliers usually have to keep larger quantities of finished goods since JIT buyers expect their suppliers to deliver products frequently in small quantities at a lower cost. Especially in technology-related market, buyers wait till the last minute to give their orders to the supplier for only small lots since the price of products are linearly decreasing over time.

A defining characteristic of high-tech industries is short product cycle with decreasing component prices. For example, the price of personal computers (PCs) and components each fell at a rate of 1% per week in 1998 (Hansell 1998). The decreasing price leads buyers to place their order at the last minute just for small lots in order not to increase their inventory cost. This is because when they give the order at the last minute instead of a few days earlier, they save money from both paying less for the purchasing cost and less for carrying cost. Therefore, shorter product life cycles and buyers' rapid demands in small lots force manufacturers to respond quicker and to shorten the cycle times in today's competitive market. This explains the heavy emphasis on the JIT policy practiced by the technology-related companies while considering the integrated inventory model to minimize the total supply chain cost.

The main purpose of this research is to develop an integrated inventory model for high-tech industries in JIT environment while effectively and successfully accomplishing supply chain integration so that the total cost of the system is minimal.

1.2 Literature Review

In literature, many researches have been done for both the integrated inventory model under just-in-time (JIT) policy and the economic order quantity (EOQ) and economic manufacturing quantity (EMQ) models under continuous price change. Although many researches have been developed in both areas, not many researches have been directed towards incorporating an integrated inventory model under JIT policy with a product experiencing continuous price decrease. Literature review in both areas is summarized in this section separately, before pointing out the shortcomings of the prior researches.

1.2.1 Integrated Inventory Models

Integrated inventory models can be developed either for an integrated vendor-buyer (IVB) system (inter-firm cooperation) or an integrated procurement-production (IPP) system (intra-firm cooperation) or combination of both IVB and IPP systems. As it is explained by Lee (2005), IVB systems coordinates the buyer and the manufacturer in deciding the quantities of the ordering lot size and production batch size, but do not include the raw material procurement. On the other hand, IPP systems study to determine the raw material procurement lot size and the production batch size to minimize the total cost without considering the buyer's ordering quantity or the inventory holding cost. The models which combines IVB and IPP systems together develops economic raw material procurement lot size, production batch size and buyer's ordering lot size.

In the area of integrated procurement-production systems, Golhar and Sarker (1992), Jamal and Sarker (1993), and Sarker and Parija (1994) implemented various solution methodologies for the integrated model and determined an optimal or near-optimal ordering policy for procurement of raw materials and the manufacturing batch size to minimize the total cost while considering equal shipments of the finished products, at fixed intervals, to the buyers. They considered the JIT philosophy in manufacturing sectors and formulated the model from the point of view of the benefit to the manufacturing firm. First, Golhar and Sarker (1992) developed the solution methodology for this model using a one-directional search procedure to obtain an optimal or near optimal solution iteratively. However, their procedure finds the break points (shipment points) only when the production rate is equal to the demand rate of finished goods inventory, and this is not the case for all the time. Later, their procedure was improved by Jamal and Sarker (1993) in order to get the break points at each iteration when the production rate is also greater than the demand rate of finished goods inventory. Finally, Sarker and Parija (1994) solved the integrated model optimally in a closed form to obtain the optimal solution.

Furthermore, integrated inventory models implemented by Lu (1995), Goyal (1995), Hill (1997), Viswanathan (1998), Hill (1999) and Goyal and Nebebe (2000) cover integrated vendor-buyer systems without taking the raw material procurement into consideration. Lu (1995) developed an optimal policy for a single-vendor single-buyer problem in which the delivery quantity to the buyer is identical at each replenishment. Then Goyal (1995) and Hill (1997) removed the restriction of identical shipments and allows delivering all available vendor inventories to the buyer. Their models showed that 'deliver what is produced' is better than 'identical delivery quantity'. However, Viswanathan (1998) discussed that none of the strategies explained by Lu (1995), Goyal (1995) and Hill (1997) obtains the best results for all possible problem parameters. Hill (1999) and Goyal and Nebebe (2000) kept working on IVB systems to obtain a better optimal solution while considering alternative policies.

More recently, Lee (1995) proposed an integrated inventory model for a single-manufacturer single-buyer supply chain problem by combining IVB and IPP systems together. Therefore, the joint economic lot sizes of manufacturer's raw material ordering, manufacturing batch, and buyer's ordering are generated by the developed model. Lee (1995) discussed that there is no existing literature considering both IPP systems and buyer's ordering quantity and inventory carrying cost together and emphasized the importance of the combined systems in inventory control model to lower total cost of the supply chain.

The common unrealistic assumption of the above integrated inventory models is that the unit cost is constant over the planning horizon. However, especially for the successful companies in high-tech industries this is not a reasonable assumption. The price of the components and finished goods decreases continuously during their life cycle.

1.2.2 Inventory Models under Price Change

In literature the developed models which allow price change based on two criteria: finite horizon vs. infinite horizon and continuous price change vs. single price change. Infinite horizon EOQ models with a single price change have been first studied by Naddor (1966), and he assumed that price change occurs at the end of an EOQ cycle. This assumption is relaxed by Taylor and Bradley by considering a situation where the price increase does not occur at the same time with the end of EOQ cycle. In addition to single price change models, a few continuous price change models are discussed in the inventory management literature. Buzacott (1975) and Erel (1992) implemented a model with continuous unit cost decrease due to inflation. The two models are similar but Buzacott considered increasing price and setup cost in an infinite horizon, whereas Erel considered increasing price during a finite planning horizon.

Khouja and Park (2003) developed an approximate closed-form expression for the optimal cycle time for a product which has a declining unit cost over a finite horizon. They considered a product whose unit cost is decreasing continuously by constant percentage over time. Another EOQ model extension for products whose cost and demand are changing over time has been discussed by Teng and Yang (2004), and they studied a model with partial backlogging during a finite horizon.

More recently, Khouja *et al.* (2005) implemented an efficient algorithm for solving the joint replenishment problem for products that may be experiencing unit cost increase or decrease in EOQ model. They tested their proposed algorithm on a sample of randomly generated problems containing up to 25 items and showed that it identifies the global optimal solutions for most of these problems. In 2005, Teng *et al.* relaxed the traditional economic production quantity model to allow time varying cost and demand, and they proved that the optimal production schedule uniquely exists. They solved the EPQ model by a continuous version with a simple analytical solution for easy understanding and applying.

1.3 Previous Research and Shortcomings

After a literature review it is realized that there are some flaws in the earlier researches. In the area of integrated inventory models under JIT policy, the unit cost of the products is always assumed to be constant even though this is not the case in reality. On the other hand, the change in the unit cost attracts the attention of few researchers in recent years for the inventory models, but they only considered one side of the supply chain, which is either the buyer or the vendor side. As it is mentioned earlier, nowadays integration of entities is really essential in order to be successful in the competitive market in a supply chain system. Unfortunately, the researchers who studied the price change did not concern about this key issue of the supply chain management.

Based on all researches and shortcomings mentioned above, this thesis incorporates the integrated inventory model under JIT policy with products experiencing continuous unit cost decrease for a successful supply chain management of technology-related industries.

CHAPTER 2

RESEARCH OBJECTIVES AND SCOPE

In this chapter, the objective and scope of the research is explained in details to give a general idea about the study presented in this paper. The motivation of the study is discussed with the support of possible applications in various industries. The solution strategy found to overcome the shortcomings of the prior research is briefly addressed and the overview of the whole study is pointed out.

2.1 Motivation

This research aims to minimize the total cost of the supply chain by accomplishing a successful integration among parties in JIT relationship. An effective supply chain management protects an industry from losing its reputation and success in the competitive market. In this research, technology-related industries whose products are experiencing continuous price decrease are taken into consideration.

Dealing with an integrated inventory model under JIT policy is motivated by the success of the Dell Computer Corporation, which achieved inimitable development through efficient inventory management. Dell is one of the most successful technology-related companies that know how to handle the inventory without struggling with either excess inventory or insufficient supply. The main goal of the supply chain management of Dell is to consistently balance the supply and demand in order to meet the customers' delivery expectations while each product is experiencing a short life cycle and a decreasing price (Aston 2001). It is discussed that Dell carries about five days' worth inventory which allowed them to take better advantage of the decreasing costs of components. Dell is one of the high-tech companies that reflect the possible applications of inventory model with decreasing price function, and moreover lots of other companies in either computer area such as Apple Computers and COMPAQ Computers or in other technological areas such as Cisco are also applicable for this inventory model.

2.2 Objective

The objective of this research is to effectively model an inventory system for products experiencing continuous price decrease while enhancing the system performance of the high-tech companies. Integration among the entities of technology-related industries is the key issue for both an efficient system and a better management. Most of the time, the classical economic order quantity (EOQ) and economic production quantity (EPQ) model are applied by companies separately to minimize the total cost of the system. However, the recent literature has proven that the integrated inventory model results in lower mean total cost for the supply chain system than that of considering them independently.

In order to incorporate both integration and decreasing unit cost in an inventory system, two different models are studied in this research. The specific inventory models dealt in this research can be stated as follows:

(a) Single-stage production-delivery model under a finite planning horizon

In single-stage inventory model, only the manufacturer side of the inventory system is taken into consideration. This model is first studied to show the significance of considering price change in the supply chain system. Manufacturer's raw material procurement and its production are integrated for products experiencing continuous price decrease and the optimal lot sizes are determined with the applied solution method. Here the specific goal is to find the optimal ordering size of raw material and production batch size under a finite planning horizon to minimize the manufacturer's total inventory cost.

(b) Integrated production-delivery model under an infinite planning horizon

In order to improve the single-stage model, this integrated production-delivery inventory system is studied afterwards. Instead of considering only the manufacturer side of the supply chain system, buyer side is also included into the integrated model. In addition, two different cases of raw material procurement are demonstrated in the integrated model. The restriction of getting all the required raw materials for one production run in only one shipment may cause higher total cost, so it is relaxed here in two different ways. One of those ways is that each lot size of procured raw material meets the demand of more than one production run whereas the other one is that more than one replenishment is needed for every production run. Therefore, the total cost can be lowered by picking the best raw material procurement policy for a particular system. The specific objective here is to determine an optimal economic lot size model for raw material ordering, production setup, and finished goods delivering under an infinite planning horizon.

In summary, the ultimate intention of this research is to study and model the inventory system of the high-tech companies whose products are experiencing continuous price decrease while integrating both parties; manufacturer and buyer, in JIT relationship. For most of the previously developed integrated inventory models, unit cost is restricted to a constant value; however, herein, this restriction is relaxed in order to demonstrate the supply chain system of the technology-related industries.

2.3 Scope

This paper studies an inventory model to effectively integrate the procurement, production and delivery activities in a supply chain consisting of a single manufacturer and a single buyer. Herein, a manufacturer purchases raw material from outside suppliers in a fixed size, then using its production processes, converts them into finished goods, and finally delivers the finished products to the buyer periodically at a fixed shipment quantity. The significance of this paper is that products experiencing continuous price decrease are considered in this integrated inventory system. This feature causes the developed model to be applicable for the high-tech industries such as personal computer (PC) assembly industries whose components prices decrease continuously. For example in PC industry, cost of some components is declining about 1% per week (Aston 2001). Dell Computer Corporation, one of the technology-related companies, faces this situation every day. Therefore, this implemented integrated model can be applied to Dell in order to enhance the system performance of the supply chain.

Here, this research can be applied to the most of the high-tech industries experiencing price changes for their products. The manufacturing companies that can consider this integrated model should be getting 'ready-to-use raw material' from outside suppliers and convert it into finished goods by using their own production processes. In summary, the inventory model developed in this research is applicable to many technology-related manufacturing firms that order the required raw materials from outside suppliers and transform them into finished goods and then deliver finished goods to the buyers at a fixed interval. The integrated inventory model is the synergy of intra- and inter-company integration under an infinite planning horizon, whereas the single-stage production-delivery model under a finite planning horizon is intra-firm planning.

2.4 Solution Strategy

As it is mentioned earlier, a single-stage production-delivery model under a finite planning horizon is studied first. Then, an integrated model is considered to implement a solution methodology while optimizing the total cost of the supply chain system under an infinite planning horizon. In the single-stage inventory model, manufacturer's raw material procurement and its production are considered and an optimal policy for the system is developed with the help of modifications from the previous work of Jamal and Sarker (1993). This single-stage inventory model is completed by illustrating numerical examples, testing its accuracy and analyzing the sensitivity of its parameters. Next, a more general type of integration is applied to the supply chain system to improve the single-stage production-delivery model. An integrated inventory model for raw material ordering, production batch and buyer's ordering is formulated with the aid of previously published work of Lee (2005). In addition, an algorithm is developed to minimize the total cost of the system by an iterative method and then the integrated inventory model is verified. Finally, numerical illustrations and sensitivity analysis are performed for the integrated inventory model and the operational schedules of both models are examined.

2.5 Overview

An integrated inventory model considering a high-tech product experiencing continuous unit cost decrease under JIT policy is studied in this paper. The goal of the study is to determine the optimal or near-optimal economic lot sizes of the high-tech products in order to minimize the inventory cost of the system. The literature review of this study is presented earlier. The paper is organized as follows after this point. The first section of Chapter 3 discusses the single-stage production-delivery model under a finite planning horizon and the next section shows the notation and the model formulation under continuous price decrease. A specific numerical example is also explained in details in section 3. Computational results for the developed model are illustrated to examine the single-stage inventory model, and the accuracy of the model is tested in the next sections of Chapter 3 with the previously published inventory model. An alternative solution methodology is also discovered and a sensitivity analysis is performed numerically to show the effect of both reducing ordering and setup costs and increasing the length of the finite planning horizon. Furthermore, in the last sections of Chapter 3 the importance of considering price decrease in this study is proven. Next, the integrated inventory problem under an infinite planning horizon is explained in the first section of Chapter 4 and additional notation and the model formulation of the integrated inventory model and the second developed algorithm are presented in the next sections. Then the verification of the integrated model is done by comparing it to the previously developed model. In the last sections of Chapter 4 numerical examples are explained in details and sensitivity analysis is performed for some parameters of the integrated inventory model. In Chapter 5, operational schedules are done for both single-stage production-delivery model under a finite planning horizon and integrated production-delivery model under an infinite planning horizon and empirical tests for 12 different problems are presented in the last section. Finally, the conclusion of the research is discussed in Chapter 6. The significance of the research and possible future extensions are summarized in the last chapter to finalize the thesis.

CHAPTER 3

PRODUCTION-DELIVERY MODEL UNDER FINITE PLANNING HORIZON

A single-stage production-delivery inventory model under a finite planning horizon is developed in this chapter to show the significance of incorporating the price change into the supply chain system. The manufacturer's costs from the lot sizing policies of an economic order quantity (EOQ) for raw material and an economic production quantity (EPQ) for finished goods are combined together to integrate the manufacturer's raw material procurement and its production. The single-stage inventory model in JIT environment for high-tech industries developed in this chapter demonstrates the relaxation of constant unit cost in the previously developed inventory models.

3.1 The Problem

In this paper, it is assumed that a production facility purchases raw materials from outside suppliers in a fixed size and converts them into finished products that are to be delivered to a buyer at a fixed interval of time. The buyer's demand of the finished goods is known and uniform. In order not to allow any shortage of products, the production rate of the production facility, P, is assumed to be higher than the demand rate of the finished products, D. Unlike the increasing inventory built up in a traditional economic manufacturing model with a continuous demand, a saw-tooth fashion inventory model is built up here during the production period, T_P, as shown in Figure 3.1. This is because a demand (fixed quantity) of x units of finished goods is instantaneously consumed at the end of every successive shipment period, L, whereas $L = x/D$ time units due to the fixed-interval batch supply. Therefore, this leaves Y - x units at hand at each L time units, where Y is the quantity produced during each period ($Y = Px/D$). The on-hand inventory is consumed sharply at a regular interval of L time units after the production time till the end of a cycle time.

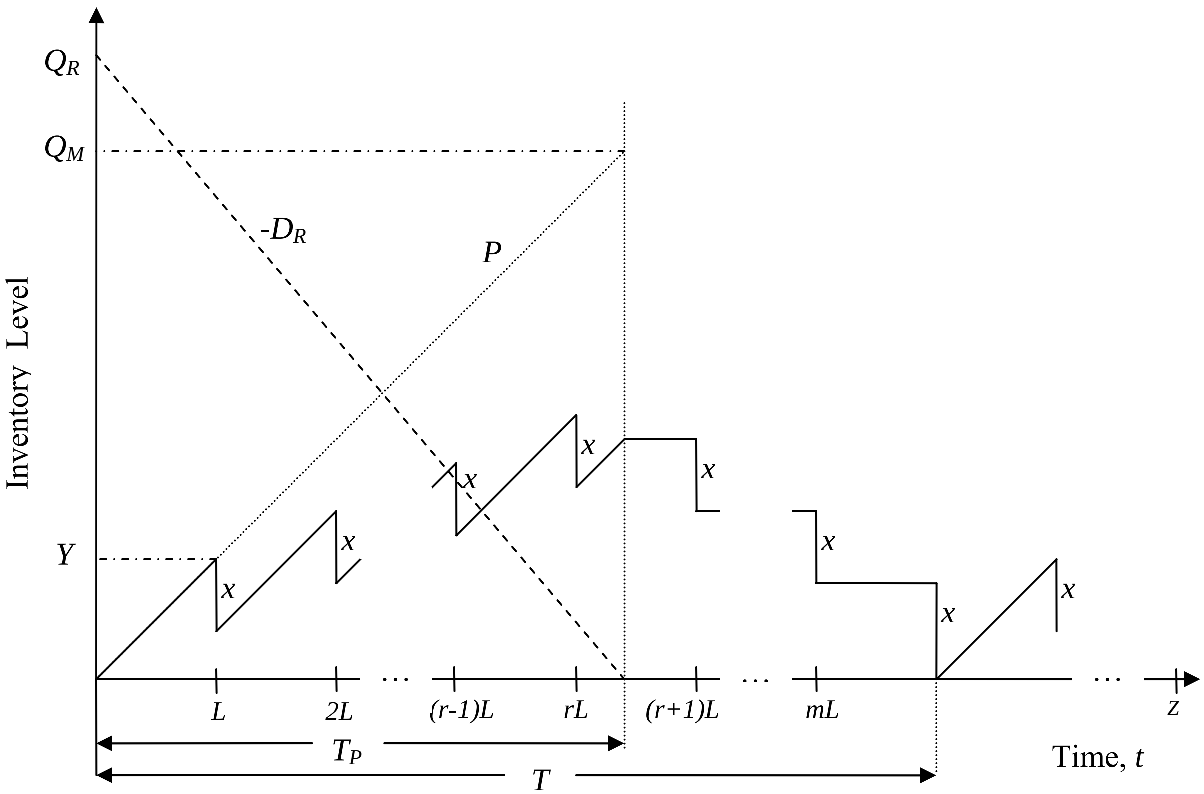

Figure 3.1: Inventory model of raw materials and finished goods.

On the other hand, products experiencing continuous decrease in unit price are considered in this single-stage production-delivery inventory model. This means the unit cost of the raw material and the finished product is decreasing continuously over time. The initial unit cost of raw material, C_O and the expected decrease per unit time, *b*, are given. Therefore, the unit cost of the raw material at time *t* is $C_R(t) = C_O - bt$. In addition, the unit cost of the finished product at time *t*, $C_F(t)$, is equal to the total cost of the number of required raw materials for producing the finished goods plus other fixed manufacturing costs, C_M, which does not change over time [i.e., $C_F(t) = (1/f) \ (C_O - bt) + C_M$]. This shows that the unit cost of the finished goods is also decreasing continuously because of the decreasing cost of raw material. Consequently, the purchasing cost, ordering cost and holding cost of both raw material and finished goods are considered in this model, since the unit cost of raw material and finished goods is different for each cycle over the finite planning horizon. A single-stage production-delivery system under a finite planning horizon is developed to minimize the total inventory costs of raw materials and finished goods. This implemented inventory model can be applied to any technology-related companies whose products are experiencing short life cycles with decreasing unit costs. Especially, in the personal computer assembly industries it could be easily observed that the components' prices decrease continuously and these decreases also reflect on the products' prices.

3.2 Model Formulation

Three types of inventory costs are considered for both an economic order quantity (EOQ) of the raw materials and an economic manufacturing quantity (EMQ) of the finished products. These considered costs for raw materials are: raw material ordering cost, A_o, raw material purchasing cost, $C_R(t)$, and raw material carrying cost, $iC_R(t)$, at time *t*, whereas, the costs for finished goods are: manufacturing setup for each batch, A_S, finished good manufacturing cost, $C_F(t)$, and finished good carrying cost, $iC_F(t)$, at time *t*. A general cost function is also implemented herein to determine an optimum number of cycles during the finite planning horizon by minimizing the total inventory cost of the single-stage production-delivery system.

3.2.1 Notation and Assumptions

The following notation is used to model the single-stage inventory model with continuously declining unit cost:

P Production rate per unit time, (units/year)

D Demand for finished goods per unit time, (units/year)

D_R Demand for raw materials per unit time, (units/year)

f Conversion factor of the raw materials; $f = D/D_R = Q_M/Q_R$

A_O Ordering cost of raw materials, (dollars/order)

A_S Manufacturing set-up cost per batch, (dollars/batch)

i Fraction holding cost of inventory value per unit time, (per year)

Z Length of the finite planning horizon or the duration with fixed cycle time; $Z = nT$, (in years)

T_P Manufacturing period (uptime); $T_P = Q_M / P$, (in years)

T_D Downtime; $T_D = T - T_P = Q_M (1/D - 1/P)$, (in years)

n Number of cycles of equal-lengths during the planning horizon, Z

T Cycle time during the planning horizon; $T = Q_M / D = mL$, (in years)

j *1, 2, …, n,* cycle index

Q_{avg} Average inventory of finished goods manufactured per cycle, (units/cycle)

Q_M Quantity of finished goods manufactured per set up over period *T,* (units/batch)

Q_R Quantity of raw materials required for production in each batch; $Q_R = Q_M / f$, (units/order)

b Decrease in unit cost per unit time, (dollars/year)

C_O Raw material cost at time $t = 0$, (dollars/unit)

$C_R(t)$ Raw material cost per unit at time *t*; $C_R(t) = C_O - bt$, (dollars/unit)

$C_F(t)$ Manufacturer's finished goods cost per unit at time *t*; $C_F(t) = C_R(t)/f + C_M$, (dollars/unit)

C_M Manufacturing cost per unit in cycle (marginal cost), (dollars/unit)

L Time between successive shipments, (in years)

x Fixed quantity of finished goods per shipment at a fixed interval of time; $x = Q_M / m = LD$, (units/shipment)

m Number of full shipments of finished goods during the cycle time; $m = \lfloor T/L \rfloor = \lfloor Q_M / x \rfloor = \lfloor DZ/xn \rfloor$

m' Number of possible shipments of finished goods during the cycle time; $m' = T/L = Q_M / x = DZ/xn$

Y Quantity produced during time L; $Y = LP = Px/D$

$Y - x$ Finished goods inventory built up at the end of each shipment period during the uptime; $Y - x = (P/D - 1)x$

The assumptions of the classical single-stage inventory model being used here to develop the model are:

- All of the raw material order quantity is delivered in one shipment, because large ordering cost force the manufacturing firm to place only one order per cycle from its supplier.
- There is no initial inventory.
- Shortages are not allowed.
- Transportation time between manufacturer and buyer is assumed zero (closely).
- Finite planning horizon is considered.

3.3 General Cost Function

The cost involved in integrating the lot sizing policies of an economic order quantity (EOQ) of raw material and an economic manufacturing quantity (EMQ) of the finished goods in the supply chain system is derived herein. JIT purchasing should be embraced to minimize the total cost by considering frequent deliveries in small lots. The total quantity of finished goods manufactured during the production time, T_P, must be exactly equal to the demand for cycle time, T.

The integrated total cost, $TC_T(n)$, over the finite planning horizon for manufacturer consists of the total cost of the raw materials during the planning horizon (EOQ), $TC_{EOQ}(n)$, and the total cost of the finished products during the planning horizon (EPQ), $TC_{EPQ}(n)$, as shown below.

$$TC_T(n) = TC_{EOQ}(n) + TC_{EPQ}(n) \tag{3.1}$$

3.3.1 Raw Material Costs

The total cost of the raw materials over the planning horizon includes the ordering cost, the purchasing cost and the holding cost. The total ordering cost for n, number of orders over the planning horizon is nA_O for a unit ordering cost of A_O. It is important to note that only one order of raw materials per cycle is allowed in this model. The amount by which the unit cost is decreased in each cycle is calculated by multiplying the given fixed amount of decrease, *b*, with the beginning time of each cycle, $j(Z/n)$, whereas *j* is the cycle index and *Z/n* is the cycle length. This is because the order is given only once at the beginning of each cycle time, and then the amount $bj(Z/n)$ is subtracted from the original price, C_O, at time zero in order to figure out the decreasing unit price. Moreover, the purchasing cost per cycle is the quantity of raw materials required for production in each batch, Q_R, times the unit cost of the raw material at the beginning of that cycle, $\left(C_O - bj(Z/n)\right)$. In order to get the total purchasing cost, the purchasing cost per cycle is summed over the planning horizon for all the cycles (for *n* number of cycles). Q_R is constant during the planning horizon, but the unit cost is decreasing for each cycle. Finally, the holding cost per cycle is the average inventory of raw materials, $Q_R/2$, times the carrying cost. Since the raw materials are only carried during the production time instead of the whole cycle time, the holding cost of the raw materials has to be adjusted accordingly. This can be done by rescaling the holding cost by the ratio of production time, T_P, (uptime) versus the total cycle time, T, (uptime and downtime). Therefore, the carrying cost of the raw material is composed of decreasing unit cost, $\left(C_O - bj(Z/n)\right)$, the interest rate, i, the cycle length, Z/n, and the rescaling factor, T_P/T. The total inventory holding cost for raw material is calculated by summing the holding cost per cycle over the planning horizon for all the cycles. Like the purchasing cost, everything except the unit cost is constant for all the cycles, and the unit cost is again decreased by considering the beginning time of each cycle. The following total inventory cost function of raw material includes all the given information above.

$$TC_{EOQ}(Q_R) = nA_o + \sum_{j=0}^{n-1} Q_R\left(C_O - bj\frac{Z}{n}\right) + \sum_{j=0}^{n-1}\frac{Q_R}{2}\left(\frac{T_p}{T}\right)\left[\left(C_O - bj\frac{Z}{n}\right)i\frac{Z}{n}\right] \tag{3.2}$$

The rescaling factor, T_P/T, considered in the raw material holding cost can also be represented by D/P, since the quantity of finished goods manufactured per cycle, Q_M, is equal to both demand of finished goods per cycle, D, times the cycle length, T, and the production rate, P, times the production time, T_P. On the other hand, raw material can be transformed to a finished product through the manufacturing process at a conversion rate, $f = D/D_R = Q_M/Q_R$. Since the relation between the raw materials and the finished goods is given by f, $Q_R = Q_M/f$, the total cost in equation (3.2) may be written as

$$TC_{EOQ}(Q_M) = nA_o + \frac{Q_M}{f}\sum_{j=0}^{n-1}\left(C_O - bj\frac{Z}{n}\right) + \frac{Q_M}{2f}\frac{D}{P}\sum_{j=0}^{n-1}\left[\left(C_O - bj\frac{Z}{n}\right)i\frac{Z}{n}\right]. \tag{3.3}$$

In order to represent the total cost function in terms of n (number of cycles), Q_M is replaced with DZ/n in equation (3.3). Therefore, the updated total cost function for EOQ model follows as:

$$TC_{EOQ}(n) = nA_o + \frac{DZ}{fn}\sum_{j=0}^{n-1}\left(C_O - bj\frac{Z}{n}\right) + \frac{D^2Z^2i}{2fPn^2}\sum_{j=0}^{n-1}\left[\left(C_O - bj\frac{Z}{n}\right)\right]. \qquad (3.4)$$

The summation of the decreasing unit cost of the raw material over the planning horizon can be simplified as follow:

$$\sum_{j=0}^{n-1}\left(C_O - bj\frac{Z}{n}\right) = nC_O - \frac{bZ(n-1)}{2}. \qquad (3.5)$$

After simplification, equation (3.5) is substituted into equation (3.4) and the total cost of raw material over the finite planning horizon becomes

$$TC_{EOQ}(n) = nA_O + \left[nC_O - \frac{bZ(n-1)}{2}\right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2}\right]. \qquad (3.6)$$

Once the EOQ model for raw material is reached, it is essential to analyze the equation with certain parameters. Figure 3.2 shows how the total cost function behaves with varying number of cycles, *n*. It can be easily concluded that the cost function follows the characteristics of a convex function, and the minimum total cost for raw materials can be obtained at optimal number of cycles, n^*. In the graph the total cost decreases with increasing number of cycles at the beginning, and then it starts to increase again because of convexity property.

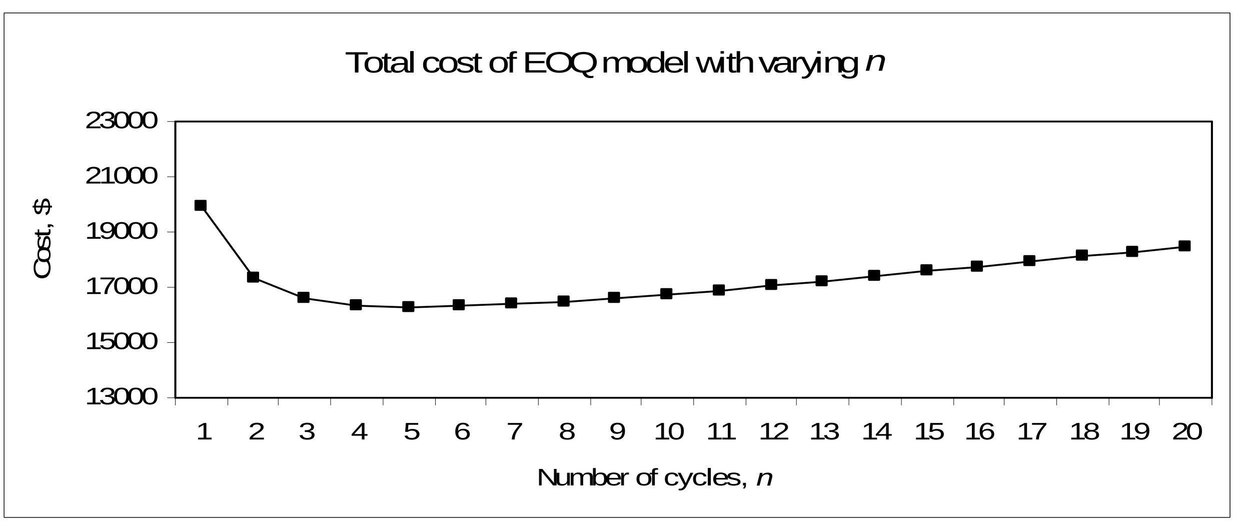

Figure 3.2: Total cost of EOQ model with respect to number of cycles, *n*

3.3.2 Manufacturing Costs

The total cost of manufacturing the finished goods over the planning horizon which includes the setup cost, the production cost and the holding cost of the finished goods is presented here. To model the EPQ of finished goods, the similar idea as in the EOQ of raw material is considered; therefore, the same details will not be repeated. The nA_S value represents the total setup cost for all the cycles over the planning horizon since it is assumed that setup is done once at the beginning of each cycle. Then, the second term gives the total manufacturing cost by multiplying the fixed batch quantity, Q_M, with the changing unit cost of the finished product for every cycle. The manufacturing cost of the finished goods, $(C_O - bj(Z/n))/f + C_M$, includes the cost of the raw material at the beginning of each cycle, $C_O - bj(Z/n)$, multiplied by the number of raw materials required to produce the finished product, $1/f$, plus other fixed manufacturing costs, C_M. The continuous decrease in the cost of the finished good is because of the decrease in the cost of the raw material. Since another fixed manufacturing cost, C_M, is included in the total manufacturing cost of the finished product, the decreasing rate of the raw material's cost is not proportional to the decreasing rate of the finished product's cost. Finally, the third term follows the same idea to calculate the total holding cost of all the cycles over the planning horizon.

$$TC_{EPQ}(Q_M) = nA_S + \sum_{j=0}^{n-1} Q_M \left[\frac{1}{f}\left(C_o - bj\frac{Z}{n} \right) + C_M \right] + \sum_{j=0}^{n-1} Q_{avg} \left[\frac{1}{f}\left(C_O - bj\frac{Z}{n} \right) + C_M \right] i\frac{Z}{n} \tag{3.7}$$

In order to represent the total cost function in terms of *n* (number of cycles), Q_M is replaced with DZ/n in equation (3.7). Therefore, the updated total cost function for EPQ model is as follow:

$$TC_{EPQ}(n) = nA_S + \frac{DZ}{n} \sum_{j=0}^{n-1} \left[\frac{1}{f}\left(C_O - bj\frac{Z}{n} \right) + C_F \right] + Q_{avg}\, i\frac{Z}{n} \sum_{j=0}^{n-1} \left[\frac{1}{f}\left(C_O - bj\frac{Z}{n} \right) + C_M \right]. \tag{3.8}$$

The summation of the decreasing unit cost of the finished goods over the planning horizon can be simplified as follows:

$$\sum_{j=0}^{n-1}\left(\frac{C_O}{f}-\frac{1}{f}bj\frac{Z}{n}+C_M\right)=\frac{nC_O}{f}+nC_M-\frac{bZ(n-1)}{2f}. \tag{3.9}$$

After simplification, equation (3.9) is substituted into equation (3.8) and the total manufacturing cost over the planning horizon becomes

$$TC_{EPQ}(n)=nA_S+\left[\frac{nC_O}{f}+nC_M-\frac{bZ(n-1)}{2f}\right]\left[\frac{DZ}{n}+Q_{avg}i\frac{Z}{n}\right]. \tag{3.10}$$

Using the information of on-hand inventory of finished goods at any time and shipment size *x* at the end of every *L* time units, Sarker and Parija (1994, pp. 893) developed an expression for the average inventory of finished goods per cycle, Q_{avg}:

$$Q_{avg}=Q_M\left[1-\frac{D}{2P}\right]-mx+\frac{m(m+1)x^2}{2Q_M}. \tag{3.11}$$

In order to represent the average inventory, Q_{avg}, in terms of *n* (number of cycles), Q_M is substituted with DZ/n in equation (3.11) and the updated Q_{avg} can be presented as:

$$Q_{avg}=\frac{DZ}{n}-\frac{D^2Z}{2Pn}-mx+\frac{m(m+1)x^2n}{2DZ}. \tag{3.12}$$

Finally, when equation (3.12) is substituted into equation (3.10), the total manufacturing cost function becomes

$$TC_{EPQ}(n)=nA_S+\left[\frac{nC_O}{f}+nC_M-\frac{bZ(n-1)}{2f}\right]\left[\frac{DZ}{n}+\frac{DZ^2i}{n^2}-\frac{D^2Z^2i}{2Pn^2}-\frac{mxZi}{n}+\frac{m(m+1)x^2i}{2D}\right]. \tag{3.13}$$

In Figure 3.3, the behavior of total manufacturing cost is observed with varying number of cycles, *n*. The convexity property also applies to EPQ model while determining the optimal production cost.

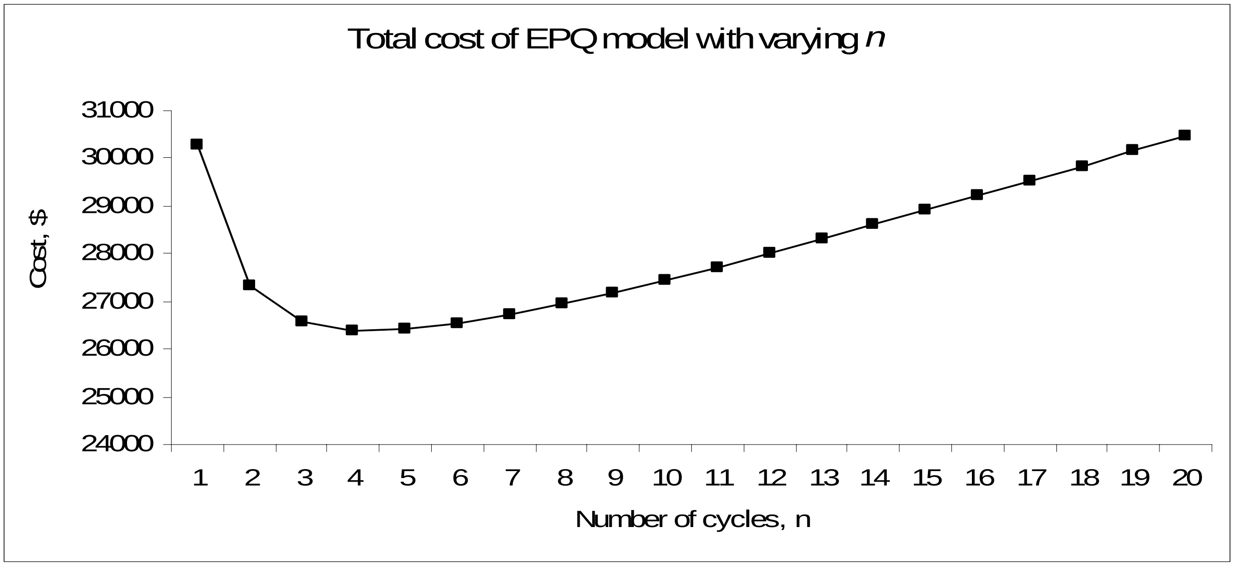

Figure 3.3: Total cost of EPQ model with respect to number of cycles, *n*

Finally, the integrated total cost function, $TC_T(m,n)$, that includes both EOQ model of raw materials and EPQ model of finished goods expressed in terms of the number of cycles, *n*, over the finite planning horizon can be written as

$$TC_T(m,n) = n(A_O + A_S) + \left[nC_O - \frac{bZ(n-1)}{2}\right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2}\right]$$

$$+\left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f}\right]\left[\frac{DZ}{n} + \frac{DZ^2i}{n^2} - \frac{D^2Z^2i}{2Pn^2} - \frac{mxiZ}{n} + \frac{m(m+1)x^2i}{2D}\right]. \qquad (3.14)$$

The first term in equation (3.14) represents the ordering and the setup cost for all the cycles, the next term represent the purchasing and holding cost of the raw materials and the last term represents the manufacturing and holding cost of the finished products for all the cycles over the finite planning horizon. An optimal value of the number of cycle, n^*, needs to be determined to minimize the total raw material and finished goods inventory cost under finite production.

After analyzing the characteristic of the single-stage production-delivery total cost function with certain parameters, it is concluded that the convexity property found in both EOQ and EPQ model, also applies to the integrated inventory model. Figure 3.4 represents the behavior of the single-stage total cost function with varying number of cycles, *n*, and optimal solution is obtained at the minimum total cost of the supply chain.

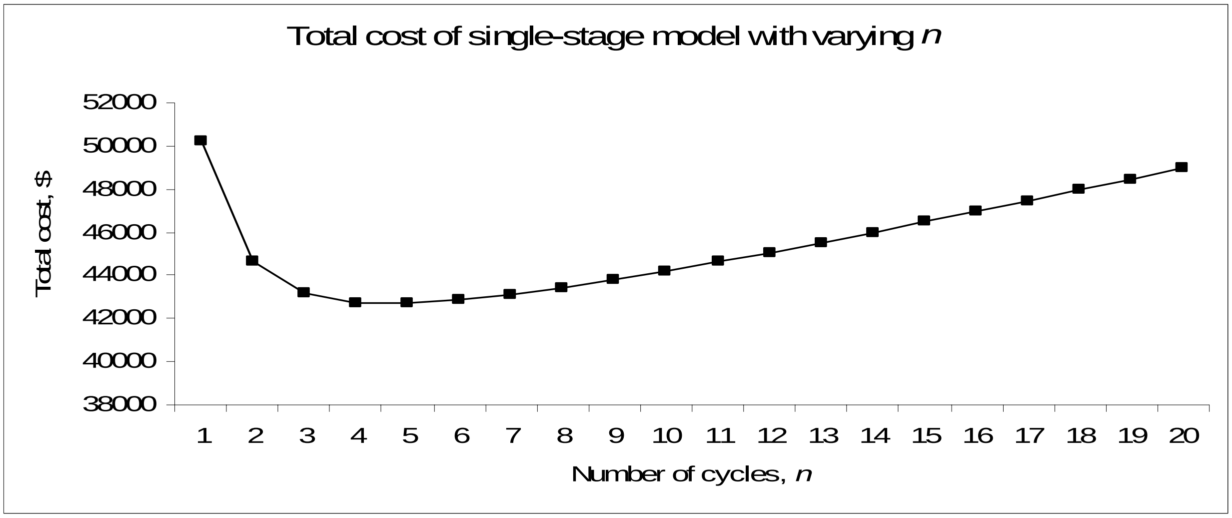

Figure 3.4: Total cost of single-stage model with respect to number of cycles, *n*

3.3.3 Optimal Solution

In the total cost of the single-stage production-delivery system under continuous unit cost decrease, $TC_T(m,n)$, $A_O, A_S, C_O, b, Z, D, f, P, C_M, i,$ and x are the fixed characteristic parameters of the system. In other words, the total cost, $TC_T(m,n)$, is a function of the only decision variable *n*, the number of cycles over the finite planning horizon and *m*, the number of shipments for finished goods.

Golhar and Sarker (1992) discussed that the global minimum value of the single-stage cost function is not easily obtained by simple calculus approach. In order to find the optimal number of cycles, n^*, it is necessary to differentiate $TC_T(m,n)$ equation with respect to *n*, but $TC_T(m,n)$ is a function of integer $m = \lfloor Q_M / x \rfloor$. Hence, $TC_T(m,n)$ is not differentiable, therefore a closed-form solution for n^* cannot be obtained directly. Golhar and Sarker (1992) applied an efficient algorithm which has been proposed by Moinzadeh and Aggarwal (1990) to solve the problem by using a discrete optimization technique. Then, the proposed algorithm which is developed to obtain an optimal or near-optimal solution iteratively is improved by Jamal and Sarker (1993). This improved algorithm is used in this paper to solve the single-stage inventory model under continuously decreasing unit cost. Algorithm 1 is modified only to find the value of number of cycles, *n*, instead of manufacturing batch quantity of the finished goods Q_M, where *n* is the function of Q_M since $Q_M = DZ/n$. This modification is needed because in the developed model the total cost function is expressed in terms of *n* since the unit cost is changing for every cycle.

The proposed algorithm by Jamal and Sarker (1993) is modified below for the developed model.

Algorithm 1: Modified Algorithm for Economic Manufacturing Quantity

Step 1: (a) Initialize $A_O, A_S, C_O, b, Z, D, f, P, C_M, i,$ and x.

(b) Set $n = 0$, $Q_M = 0$, $n^o = 0$, $Q_M^o = 0$, $Y = \frac{Px}{D}$ and $TC_T(m, n^o) = \infty$.

Note: n^o and Q_M^o are number of cycles and batch quantity of finished goods respectively at local minimum total cost where number of shipments, *m*, is fixed and known.

Step 2: (a) $Q_M \leftarrow Q_M + Y$, $n = \frac{DZ}{Q_M}$ and $m = \left\lfloor \frac{DZ}{xn} \right\rfloor$.

Step 3: (a) Compute $TC_T(m,n)$

$$TC_T(m,n) = n(A_O + A_S) + \left[nC_O - \frac{bZ(n-1)}{2} \right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2} \right]$$

$$+\left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f} \right]\left[\frac{DZ}{n} + \frac{DZ^2i}{n^2} - \frac{D^2Z^2i}{2Pn^2} - \frac{mxiZ}{n} + \frac{m(m+1)x^2i}{2D} \right].$$

(b) If $TC_T(m,n) \le TC_T(m,n^o)$, Set $n^o = n$, $TC_T(m,n^o) = TC_T(m,n)$ and go to Step 2.

Otherwise, Set $Q_M^o = Q_M$ and the local minimum $TC_T(m,n^o)$ is obtained.

Jamal and Sarker (1992) proved that by applying modified algorithm, n^o is determined at which $TC_T(m^*, n^o)$ is minimum at the discounted point where $m = \lfloor DZ/xn^o \rfloor$ is a known and fixed integer. They claimed to fix m at m^* so that the total cost equation is differentiable. Therefore, m is replaced with $m^* = \lfloor DZ/xn^o \rfloor$ in equation (3.13). The updated total cost function becomes

$$TC_T(m^*, n) = n(A_O + A_S) + \left[nC_O - \frac{bZ(n-1)}{2}\right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2}\right]$$

$$+\left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f}\right]\left[\frac{DZ}{n} + \frac{DZ^2i}{n^2} - \frac{D^2Z^2i}{2Pn^2} - \frac{m^*xiZ}{n} + \frac{m^*(m^*+1)x^2i}{2D}\right]. \qquad (3.15)$$

Derivative of the single-stage production-delivery total cost function for raw material and finished goods in terms of n (number of cycles) is reached by the software package Mathematica (Wolfram 1991) and is shown below:

$$\frac{dTC(m^*, n)}{dn} = (A_O + A_S) + \left[C_O - \frac{bZ}{2}\right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2}\right] + \left[nC_O - \frac{bZ(n-1)}{2}\right]\left[\frac{-DZ}{fn^2} - \frac{D^2Z^2i}{fpn^3}\right]$$

$$+\left[\frac{C_O}{f} + C_F - \frac{bZ}{2f}\right]\left[\frac{DZ}{n} + \frac{DZ^2i}{n^2} - \frac{D^2Z^2i}{2Pn^2} - \frac{m^*xiZ}{n} + \frac{m^*(m^*+1)x^2i}{2D}\right]$$

$$+\left[\frac{nC_O}{f} + nC_F - \frac{bZ(n-1)}{2f}\right]\left[\frac{-DZ}{n^2} - \frac{2DZ^2i}{n^3} + \frac{D^2Z^2i}{Pn^3} + \frac{m^*xiZ}{n^2}\right]. \qquad (3.16)$$

To simplify the derivative of the total cost function which gives the local minimum cost because of a fixed integer number of shipments, m^*, some constant values are combined as follows:

$$A = (A_O + A_S),\ G = \frac{DZ}{f},\ K = \left(\frac{C_O}{f} + C_M - \frac{bZ}{2f}\right), \text{and } W = \left(C_O - \frac{bZ}{2}\right).$$

After simplifications are substituted into equation (3.15), the derivative function becomes a cubic function as shown below.

$$\frac{dTC(m^*,n)}{dn}=\left[2AP+\frac{PKx^2im^*(m^*+1)}{D}\right]n^3+[2WGP-2GC_OP+bZGP+2KDZP-2Km^*xiZP-\frac{2C_ODZP}{f}$$
$$+\frac{2C_Om^*xiZP}{f}-2C_MDZP+2C_Mm^*xiZP+\frac{bZ^2DP}{f}-\frac{bZ^2m^*xiP}{f}]n^2$$
$$+[WGDZi-2C_OGDZi-GbZP+GDZ^2ib+2KDZ^2iP-KD^2Z^2i-\frac{4C_ODZ^2iP}{f}$$
$$+\frac{2C_OD^2Z^2i}{f}-4C_MDZ^2iP+2C_MD^2Z^2i-\frac{bZ^2DP}{f}+\frac{bZ^3iDP}{f}-\frac{bZ^3D^2i}{f}+\frac{bZ^2m^*xiP}{f}]n$$
$$+[\frac{bZ^3D^2i}{f}-GDZ^2ib-\frac{bZ^3iDP}{f}]. \tag{3.17}$$

In order to solve this cubic function, root-finding formula can be used as explained below. For the general case, if the equation is $f(x)=an^3+bn^2+cn+d=0$, let $q=3ac-b^2/9a^2$ and $r=9abc-27a^2d-2b^3/54a^3$. Then let $s=\sqrt[3]{r+\sqrt{q^3+r^2}}$ and $t=\sqrt[3]{r-\sqrt{q^3+r^2}}$. Finally the solution is $n=s+t-(b/3a)$ (from Wikipedia - the free encyclopedia). The given information for the general case of a cubic function is applied to the derivative of the total cost function. Therefore, *a*, *b*, *c*, and *d*, can be represented as follows from equation (3.17):

$$a=\left[2AP+\frac{PKx^2im^*(m^*+1)}{D}\right]$$

$$b=[2WGP-2GC_OP+bZGP+2KDZP-2Km^*xiZP-\frac{2C_ODZP}{f}+\frac{2C_Om^*xiZP}{f}-2C_MDZP$$
$$+2C_Mm^*xiZP+\frac{bZ^2DP}{f}-\frac{bZ^2m^*xiP}{f}]$$

$$c=[WGDZi-2C_OGDZi-GbZP+GDZ^2ib+2KDZ^2iP-KD^2Z^2i-\frac{4C_ODZ^2iP}{f}$$
$$+\frac{2C_OD^2Z^2i}{f}-4C_MDZ^2iP+2C_MD^2Z^2i-\frac{bZ^2DP}{f}+\frac{bZ^3iDP}{f}-\frac{bZ^3D^2i}{f}+\frac{bZ^2m^*xiP}{f}]$$

$$d = [\frac{bZ^3 D^2 i}{f} - GDZ^2 ib - \frac{bZ^3 iDP}{f}]$$

Since all the variables are given *a, b, c* and *d* can be computed easily. Then *q, r, s,* and *t* values can be obtained in order to determine the value of *n*. On the other hand, a cubic equation solver can also be used to solve the cubic function by only computing the values of *a*, *b*, *c*, and *d*. Even a much easier method of solving the derivative of the total cost function (equation (3.17)) is simply to use the software package Mathematica (Wolfram 1991) to find the roots of the cubic function. Every cubic function contains three roots but only one of them is a real number and the rest is imaginary numbers. Therefore, this determined real *n* value is the optimum value (n^*) to find the minimum total cost $TC(m^*, n^*)$ when $m^* = \lfloor DT/xn^o \rfloor$ is computed into the total cost function as a fixed integer.

- **Example 3.1:** A Numerical Illustration

As an illustration of the single-stage production-delivery model under continuous unit cost decrease, a numerical example is presented for a single product by putting together the given numerical problems in Jamal and Sarker (1993) and Khouja and Park (2003). The numerical example from Jamal and Sarker (1993) is exactly taken and other required information for decreasing unit cost of raw material, annual fraction holding cost and the length of the finite planning horizon is taken from Khouja and Park (2003). In addition to those given variables, since the fixed manufacturing cost per unit is not considered in any of those examples, it is estimated reasonably as $C_M = \$4.00\,/\,\text{unit}$. The variables given by Jamal and Sarker (1993) are as follows: $D = 2400$ units/year, $P =$ 3600 units/year, $A_O = \$200\,/\text{order}$, $A_S = \$300/\text{setup}$, $f = 1$, and $x = 100$ units/shipment. On the other hand, the variables given by Khouja and Park

(2003) are as follows: $i = 8\%$ annually, $Z = 1$ year, the initial price per unit of raw material is $C_O = \$8.00$ and it is decreasing at a rate of 1% per week.

Since the decreasing rate of the raw material is given as 1% per week, this means the total decrease in unit cost of raw material at the end of a year is equal to \$4.16 [$b = (\$8.00)(52\%) = \$4.16/$ year].

After applying algorithm 1, the following results presented in Table 3.1 are obtained. The results shown in the table are calculated by adding the quantity produced during the shipment period, *Y*, to the quantity produced per setup, Q_M, at every iteration and determining the number of cycles, *n*, and the number of shipments, *m*, for each iteration to find the minimum total cost. The number of shipments, *m*, which gives the minimum cost, is selected as an optimal result. Therefore, the optimum number of shipments, m^*, is found at 3rd iteration where $n^o = 5.33$ cycles/year, $Q_M^o = 450$ units /batch with $TC(m^*, n^o) = \$42{,}731.2$ /year from which $m^* = \lfloor Q_M^o / x \rfloor = 4$ shipments/cycle.

Table 3.1: Results of the modified algorithm to obtain the optimum *m*

Iteration	Q_M	n	m	$TC_T(m,n^o)$
1	150	16	1	46,718
2	300	8	3	43,395
3	450	5.3	4	42,731
4	600	4	6	42,740

When $m = m^*$ in the total cost equation, $a = 3838080$, $b = 0$, $c = -86151168$ and $d = -2875392$ for the cubic derivative function. This results in the optimal $n^* = 4.75$ cycles/year, and this is converted to 5 cycles per year since 5 cycles give smaller inventory cost compared to 4 cycles per year. Therefore, the optimal solutions are obtained as: the optimal number of cycles, $n^* = 5$ cycles/year, the optimal batch size, $Q_M^* = 480$ units/batch and the optimal minimum cost, $TC(m^*, n^*) = \$42700.02$ /year. ■

The total cost of the single-stage inventory model is computed for all the possible combinations of number of shipments, *m*, and number of cycles, *n* to check whether the result obtained from the modified algorithm is right or not. All these combinations of *m* and *n* are computed into equation (3.15) to find the total cost values. Table 3.2 presents the total cost values calculated for each possible combinations of *m* and *n* by using Mathematica (Wolfram 1991), and the highlighted value is the minimum total cost of all the calculated costs with varying *n* and *m*. The minimum total cost is reached at *m*= *4* shipments/cycle and *n*= *5* cycles/year, and this is the exact same result found by algorithm 1. This comparison proves the accuracy of the modified algorithm developed in this paper.

Table 3.2: Total cost of the single-stage production-delivery inventory model with all the possible combinations of *n* and *m*

TC (m,n)		*m*									
		1	2	3	4	5	6	7	8	9	10
	1	50456	50368	50284	50204	50128	50056	49988	49924	49864	49808
	2	44852	44779	44713	44655	44603	44560	44523	44494	44472	44457
	3	43364	43300	43247	43204	43173	43151	43141	43141	43151	43173
	4	42880	42824	42782	42754	42740	42740	42754	42782	42824	42880
n	5	42793	42745	42714	42700	42704	42724	42762	42817	42890	42979
	6	42904	42863	42843	42843	42863	42904	42966	43048	43151	43274
	7	43128	43094	43084	43097	43135	43196	43281	43390	43523	43680
	8	43422	43395	43395	43422	43476	43558	43666	43802	43965	44155
	9	43763	43742	43753	43793	43864	43966	44098	44260	44453	44676
	10	44136	44123	44143	44197	44285	44406	44561	44750	44973	45230

3.4 Computational Results

After explaining the solution procedure of a numerical example, more problems are examined to test the developed single-stage production-delivery model under a finite planning horizon, and both the values of the variables and the results of those problems are presented in Table 3.3. All the numerical examples used in this paper are taken from the previously published papers such as problem 1 (a&b) is from Jamal and Sarker (1993) and Khouja and Park (2003); problem 2 (a&b) is from Sarker and Parija (1996) and Khouja and Park (2003), and lastly problem 3 (a&b) is from Khouja and Goyal (2006). The values, which have asterisks (*) next to them, are estimated reasonably, since they are not given in the previously published papers. As mentioned earlier, the reason of estimating some of the values and combining numerical examples of more than one paper is that no previous study has been done in this area to incorporate an integrated inventory model with continuously decreasing price.

Table 3.3: Values and results of the previously published numerical examples

		Problem 1		Problem 2		Problem 3	
		(a)	(b)	(a)	(b)	(a)	(b)
DATA	D	2400	2400	2500	2500	12000	12000
	P	3600	3600	25000	25000	20000*	20000*
	A_O	200	100	100	40	100	50
	A_S	300	150	50	15	150*	90
	x	100	100	100	100	150*	150*
	f	1	1	0.5	0.5	1*	1*
	C_o	8.00	8.00	8.00	8.00	40.00	40.00
	C_M	4.00*	4.00*	4.00*	4.00*	7.00*	7.00*
	i	0.08	0.08	0.08	0.08	0.08	0.08
	Z	1	1	1	1	0.25	0.25
	b	4.16	4.16	4.16	4.16	20	20
SOLUTION	n^o	5.33	8	2.5	2.5	6	12
	$Q_M{}^o$	450	300	1000	1000	500	250
	$TC_T(m^*,n^o)$	42,731	41,395	78,649	78,412	250,267	249,089
	m^*	4	3	10	10	3	1
	n^*	5	7	7	8	8	11
	$Q_M{}^*$	480	343	357	313	375	273
	$TC_T(m^*,n^*)$	42,700	41,334	74,395	73,639	250,115	249,076

* Values not taken from previously published papers
(a) Original costs
(b) Reduced ordering and setup costs

As it can be seen in Table 3.3, each of three problems has two parts: (a) original part and (b) modified part. The difference between (a) and (b) part of each problem is the reduction in the ordering and manufacturing setup costs. The effect of reducing the ordering and manufacturing setup cost under JIT environment is discussed in Section 3.7 (Sensitivity Analysis). The unit costs of $A_O, A_S, C_O, C_M, b, TC_T(m^*, n^O)$ and $TC(m^*, n^*)$ presented in Table 3.3 are in dollars($).

The solution methodology is applied to each of the problems by following the modified algorithm. After obtaining the values of m^*, n^o, Q_M^o, and $TC_T(m^*, n^o)$ through an iterative procedure, the values of *a, b, c, d* within the cubic derivative function of total cost are calculated. Then a cubic equation solver is used to obtain the optimum number of cycles, n^*, over the planning horizon by computing *a, b, c,* and *d*. Finally, by using the optimum number of cycles, n^*, the optimum lot size of the finished goods, Q_M^*, and the total cost, $TC_T(m^*, n^*)$, are determined as shown in Table 3.3. On the other hand, the software package Mathematica (Wolfram 1991) is also used to solve the cubic function to find n^* after m^* is determined by an iterative procedure in order to double check the results in a faster way.

- **Special Case:** Perfect Matching

The developed inventory model expresses a generalized total cost function when matching is imperfect. This imperfect matching means that the last shipment size is less than the fixed quantity of finished goods per shipment, *x*. On the other hand, when the number of cycles, *n*, and the number of shipments, *m*, are integers in the single-stage inventory model or in other words, the production and cycle time are each equal to an integer multiple of shipment periods, then that case is termed as perfect matching. Therefore, the general total cost expression simplifies to a traditional inventory system as discussed in Golhar and Sarker (1992). In perfect matching case, $m' = m = DZ/xn = \lfloor DZ/xn \rfloor$.

For general total cost function under imperfect matching case, the average finished goods inventory, Q_{avg}, is given in equation (3.12) as

$$Q_{avg} = \frac{DZ}{n} - \frac{D^2 Z}{2Pn} - mx + \frac{m(m+1)x^2 n}{2DZ}.$$

By replacing m with DZ/xn, Q_{avg} for perfect matching case is obtained as

$$Q_{avg} = \frac{1}{2}\left(x + \frac{D(1 - D/P)Z}{n} \right). \tag{3.18}$$

When Q_{avg} under imperfect matching case shown in equation (3.12) is updated by equation (3.18), the total cost function under perfect matching case becomes

$$TC_T(n) = n(A_O + A_S) + \left[nC_O - \frac{bZ(n-1)}{2} \right]\left[\frac{DZ}{fn} + \frac{D^2 Z^2 i}{2fPn^2} \right]$$

$$+ \left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f} \right]\left[\frac{DZ}{n} + \frac{DZ^2 i}{2n^2} - \frac{D^2 Z^2 i}{2Pn^2} + \frac{xiZ}{2n} \right]. \tag{3.19}$$

Perfect matching case causes the total cost, $TC_T(n)$, to become a function of the only decision variable *n*, the number of cycles over the finite planning horizon. Moreover, $TC_T(n)$ in equation (3.19) is a differentiable convex function with respect to *n*. The convexity property is proven in Figure 3.5 by the behavior of the total cost function under varying number of cycles, *n*.

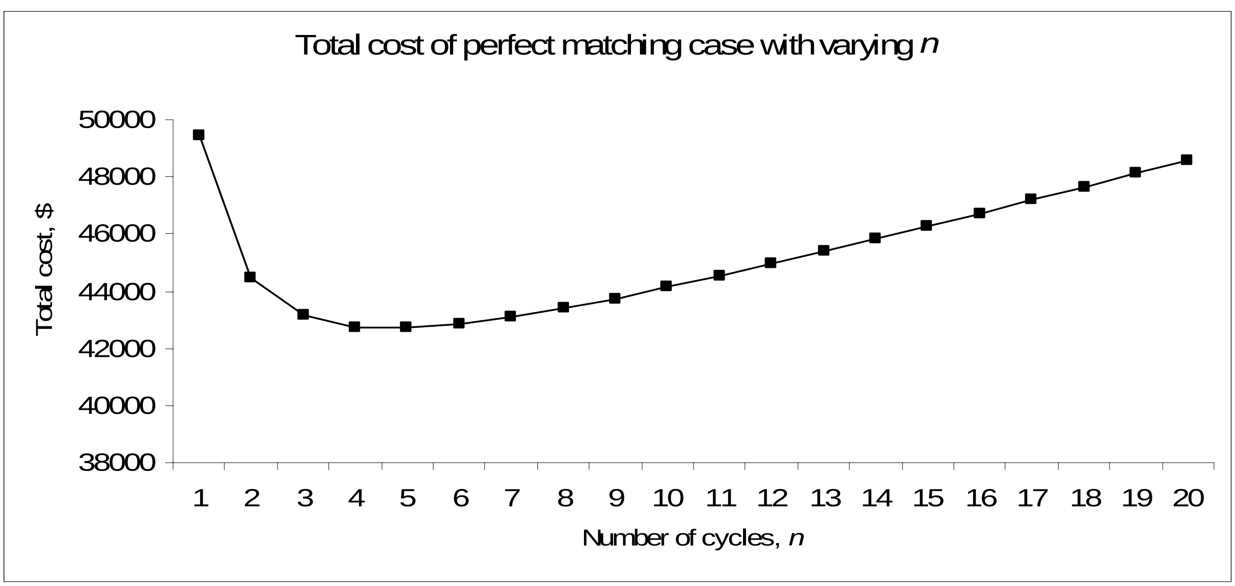

Figure 3.5: Total cost function of perfect matching case with respect to *n*

The optimal number of cycles, n^*, is obtained by equating the first-order derivative of $TC_T(n)$ with respect to n to zero. Therefore, derivative of the total cost function of the perfect matching case is shown below:

$$\frac{dTC(n)}{dn} = (A_O + A_S) + \left[C_O - \frac{bZ}{2}\right]\left[\frac{DZ}{fn} + \frac{D^2Z^2i}{2fPn^2}\right] + \left[nC_O - \frac{bZ(n-1)}{2}\right]\left[\frac{-DZ}{fn^2} - \frac{D^2Z^2i}{fPn^3}\right]$$

$$+\left[\frac{C_O}{f} + C_M - \frac{bZ}{2f}\right]\left[\frac{DZ}{n} + \frac{DZ^2i}{2n^2} - \frac{D^2Z^2i}{2Pn^2} + \frac{ixZ}{2n}\right]$$

$$+\left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f}\right]\left[\frac{-DZ}{n^2} - \frac{DZ^2i}{n^3} + \frac{D^2Z^2i}{Pn^3} - \frac{xiZ}{2n^2}\right]. \quad (3.20)$$

By setting equation (3.20) equal to zero, a cubic function needs to be solved in order to find the optimal solution. The software package Mathematica (Wolfram 1991) is used to determine the solutions of all the problems presented in Section 3.4 under perfect matching case. When the results presented in Table 3.4 are analyzed, it can be concluded that the total cost values obtained from the perfect matching case are very close to the ones obtained from the imperfect matching case. This shows that the developed model under both perfect and imperfect cases provides the same optimal solution.

Table 3.4: Results of the perfect matching case

		n	*n**	*TC(n*)*
Problem 1	(a)	4.64	5	42,701
	(b)	6.56	7	41,337
Problem 2	(a)	12.23	12	72,930
	(b)	20.18	20	71,482
Problem 3	(a)	8.06	8	250,095
	(b)	10.77	11	249,081

3.5 Verification of the Single-Stage Model

In addition to the numerical examples, the developed single-stage inventory model is also compared to the model implemented earlier in Sarker and Jamal's paper (1993) in order to test the accuracy of the model in this study. This comparison is done by finding the relationship between two models and using the exact values of the given numerical example in Sarker and Jamal (1993). The only difference between this paper and Sarker and Jamal's paper (1993) is that here the unit cost is continuously decreasing over the finite planning horizon, whereas Sarker and Jamal assumed a constant unit cost for their model. Therefore, by keeping the unit cost constant in this model, the same results of Sarker and Jamal (1993) should be obtained.

The accuracy is tested by equalizing the differences between two inventory models. Sarker and Jamal (1993) did not consider the purchasing cost of raw material nor the manufacturing cost of finished goods. So, those terms are removed from the new implemented model. Since, Sarker and Jamal (1993) assumed constant unit cost over the planning horizon, decrease in unit cost per unit time, *b*, is set to zero. The holding cost of raw material and finished good are given in Sarker and Parija (1994) as \$1.00 per unit per year and \$2.00 per unit per year respectively. Based on those values, interest rate is assumed 8% annually and the unit cost of raw material and manufacturing cost per unit are both calculated as \$12.5 per unit. Moreover, since all the values are given in yearly base, the planning horizon is taken as 1 year for the developed inventory model. The values of all the variables computed into the single-stage inventory model which are given in Sarker and Parija (1994) are as follows: $D = 2400$ units/year, $P = 3600$ units/year, $A_O = \$200$ /order, $A_S = \$300$ /setup, $f = 1$, $x = 100$ units, $i = 8\%$ annually, $Z = 1$ year, $C_O = \$12.50$, $C_M = \$12.50$ and $b = 0$ (no price decrease). After applying the model developed in this paper, the following results are obtained: $n^o = 1.7778$ cycles/year, $Q_M^o = 1350$ units/batch, $TC(n^o) = \$1{,}887.04$ /year, $m^* = 13$ shipments/cycle and then $n^* = 1.783$ cycles/year, $Q_M^* = 1346$ units/batch, and $TC(n^*) = \$1887.018$ /year.

It can be concluded that the new developed inventory model is accurate since the results obtained from this model are exactly equal to those of Sarker and Jamal (1993). This comparison proves the effectiveness of the single-stage production-delivery model in this paper.

3.6 Alternative Solution Methodology

In addition to the modified algorithm, an alternative methodology is studied for the implemented inventory model, and the results of both methodologies are compared. The alternative methodology applies an interactive solution procedure to get the optimum solution instead of an iterative procedure applied by the modified algorithm.

In this methodology, number of shipments, *m*, is expressed in terms of number of cycles, *n*, and the total cost function of the single-stage inventory model becomes a function of only decision variable *n* over the finite planning horizon. In order to represent *m* in terms of *n*, total cost equation of the single-stage inventory model, $TC_T(m,n)$, (eq. (3.14)) is differentiated with respect to *m* and the following equation is obtained

$$\frac{dTC_T(m,n)}{dm} = \left[\frac{nC_O}{f} + nC_M - \frac{bZ(n-1)}{2f}\right]\left[\frac{imx^2}{2D} + \frac{i(1+m)x^2}{2D} - \frac{ixz}{n}\right]. \tag{3.21}$$

To find optimal number of shipments, *m*, in terms of *n*, equation (3.21) is solved for *m* by setting it equal to zero with the use of the software package Mathematica (Wolfram 1991). Therefore, the optimal number of shipments, *m*, can be written as follow:

$$m = \frac{2DZ - nx}{2nx}. \tag{3.22}$$

In this alternative solution methodology, instead of finding the local minimum total cost and fixing *m* to a value found by an iterative procedure, *m* is updated to a function of number of cycles, *n*, and the derivative of $TC_T(m,n)$, which is equation (3.16) is solved by replacing *m* with $(2DZ - nx)/2nx$. The solutions of all the problems used in Section 3.4 (Computational

Results) are determined according to the interactive solution procedure by using Mathematica (Wolfram 1991). It can be concluded by analyzing the results presented in Table 3.5 that the total cost values obtained from the modified algorithm and the interactive procedure are really close and the difference is insignificant. This shows that the alternative solution methodology is also effective to find the optimum solution. The advantage of this method is that the interactive solution procedure is faster and easier than the iterative solution procedure.

Table 3.5: Results of the alternative solution methodology

		n	n^*	m	$TC_T(m,n^*)$
Problem 1	(a)	4.64	5	4.3	42,699
	(b)	6.56	7	2.9	41,334
Problem 2	(a)	12.41	12	1.2	72,927
	(b)	20.30	20	0.8	71,470
Problem 3	(a)	8.07	8	2.0	250,088
	(b)	10.80	11	1.3	249,072

3.7 Sensitivity Analysis

Sensitivity of ordering and setup costs is discussed by Golhar and Sarker in 1992, and they explained that further reduction in the lots size of finished product is expected in a JIT environment by decreasing the ordering and manufacturing setup costs. This reduction leads to minimize the total cost of the supply chain system by holding low inventory.

Since the total cost function developed for the single-stage production-delivery model is a third-degree polynomial, it is not easy to write the optimum total cost function in a simple form. Therefore, in order to show the effect of reducing the ordering and setup costs in the model, the numerical examples are used. The part (b) of all three problems represented in Table 3.3 includes the same data as part (a) except the ordering and manufacturing setup costs. Reduced costs are estimated for those costs in order to test the effect of these reductions on the model. It can be concluded from the results presented in Table 3.3 that the optimum lot size of the finished goods, Q_M^*, is decreasing while the total number of cycles over the finite planning horizon, n^*, is increasing because of the effect of reduced ordering and setup costs. This result shows the sensitivity of the model to the changes in ordering cost and setup cost. When the setup cost, A_S, and ordering cost, A_O, are reduced to new levels $A_S^{'}$ and $A_O^{'}$ respectively, this results optimal lot size to decrease from Q_M^* to $Q_M^{*'}$ and optimal number of cycles increases from n^* to $n^{*'}$.Consequently, the minimum total cost is $TC_T(m^*, n^*) \geq TC_T(m^*, n^{*'})$.

Table 3.6: Results of reducing the ordering and setup costs (from Table 3.3)

	n^*	$n^{*'}$	Q_M^*	$Q_M^{*'}$	$TC_T(m^*,n^*)$	$TC_T(m^*n^{*'})$
Problem 1	5	7	480	343	42,700	41,334
Problem 2	7	8	357	313	74,395	73,639
Problem 3	8	11	375	273	250,115	249,076

On the other hand, a more general sensitivity analysis can be done by considering a ratio of ordering cost versus manufacturing setup cost. Figure 3.4 presents the effect of increasing the ratio on the total cost of the single-stage inventory model. For each ratio starting from 0.1 to 2, a constant value is picked for the ordering cost, A_O, and the setup cost, A_S, is computed based on the ratio value. Basically, the decreasing total cost in Figure 3.6 proves that as one of the costs reduces with increasing ratio, the total cost value is also decreasing.

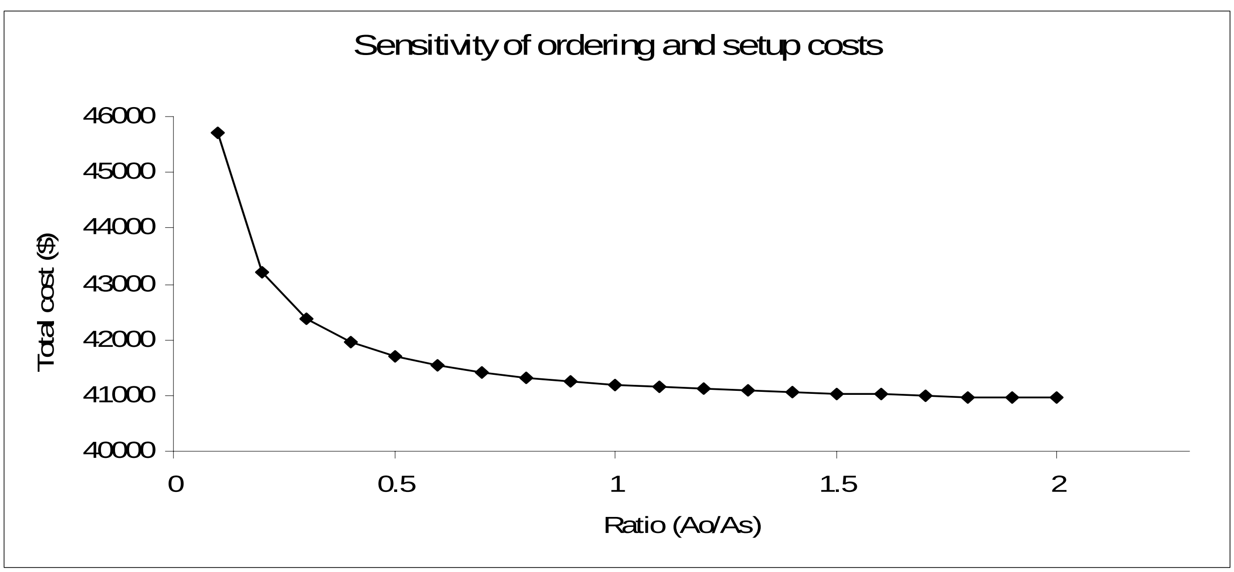

Figure 3.6: Sensitivity of ordering and setup costs

In addition to the sensitivity analysis of ordering and setup cost, the sensitivity of the planning horizon is also tested for the single-stage inventory model developed in this study. As it is shown in Figure 3.7, the total cost raises as the finite planning horizon gets longer. This is an expected conclusion and another proof of the correctness of the developed inventory model.

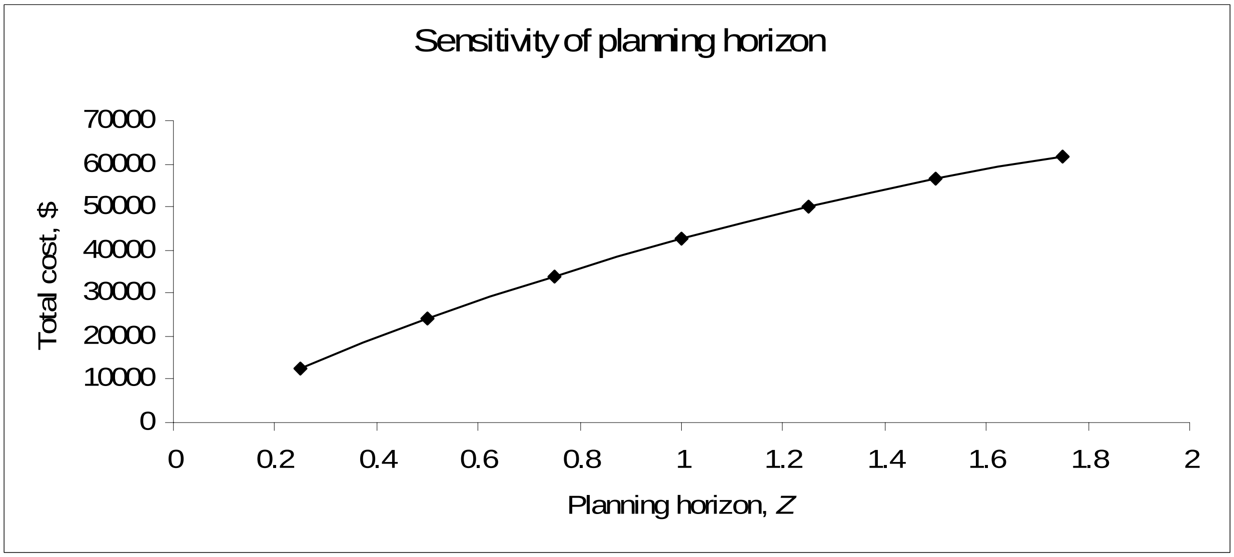

Figure 3.7: Sensitivity of planning horizon

3.8 Discussion

As it is mentioned earlier in this paper, technology-related companies have to consider price decrease in the inventory model in order to be successful in a competitive market, since their products are experiencing continuous price decrease. One of the biggest flaws in the literature is that there are not many researches directed towards an inventory model considering these types of products. To show the effect of incorporating price decrease into the integrated model, the percentage increases in total cost of the classical production-delivery model over the total cost of the inventory model for numerical examples are exposed. Table 3.7 illustrates the cost savings from incorporating the price decrease, *b*, into the classical production-delivery model.

In order the find the cost savings, the optimal results of 3 problems, m^*, n^* and $TC_T(m^* n^*)$ obtained in section 3.4 are referred here for the first half of the discussion where price decrease is not equal to zero. On the other hand, in the second half, price decrease, b, is set equal to zero and the same procedure applied in the numerical illustration (example 3.1) is followed for 3 problems to find the new optimal solutions. Again, first the optimal number of shipments, m^*, is determined iteratively by using algorithm 1 when $b=0$ and then according to that m^* value, the optimal number of cycles, n^*, is calculated by solving equation (3.16). After getting m^* and n^* values, equation (3.15) is used to determine the minimum total cost. These steps are applied to all the problems to find the optimal solutions when the price decrease is not taken into consideration in the inventory system. Finally, the difference between the optimal total cost values calculated for both cases is presented as a percentage in Table 3.7.

Table 3.7: Cost savings from incorporating the price decrease into the model

		Considering Price Decrease [b]	m^*	n^*	$TC_T(m^*,n^*)$	Cost Savings
Problem 1	(a)	No	18	1	49,504	13.74%
		Yes	4	5	42,700	
	(b)	No	13	2	49,004	15.65%
		Yes	3	7	41,334	
Problem 2	(a)	No	10	3	91,226	18.45%
		Yes	10	7	74,395	
	(b)	No	10	3	90,941	19.03%
		Yes	10	8	73,639	
Problem 3	(a)	No	8	2	262,227	4.62%
		Yes	3	8	250,115	
	(b)	No	6	3	261,916	4.90%
		Yes	1	11	249,076	

Costs saving percentages prove why it is that essential for high-tech companies whose products are experiencing continuous price decrease to take this decrease into consideration while implementing the inventory model to minimize the supply chain costs.

CHAPTER 4

PRODUCTION-DELIVERY MODEL UNDER INFINITE PLANNING HORIZON

After demonstrating the effectiveness of the single-stage production-delivery model under a finite planning horizon and proving the importance of considering price decrease into the inventory model, it is worth to improve the developed inventory model by considering the buyer side of the supply chain in addition to the manufacturer side. In order to integrate both parties; manufacturer and buyer, into the supply chain system, an integrated production-delivery model under an infinite planning horizon is studied in this chapter. The differences between the previously developed single-stage inventory model and the integrated inventory model are specified to show the improvements of the model.

4.1 The Problem

In this problem, the costs of buyer/customer such as buyer's ordering, purchasing and holding costs are added to the supply chain system, and the single-stage inventory model under JIT policy is revised accordingly. Here, the manufacturer purchases raw materials from its supplier, then using its production processes, converts the raw material into finished goods, and finally delivers the finished products to the buyer. Once again, high-tech products experiencing continuous unit cost decrease are considered in this problem. Unlike the single-stage inventory problem, the shipment quantity of finished goods, x, is an unknown but fixed parameter herein. On the other hand, in order to improve the developed single-stage inventory model, in this integrated inventory problem the restriction of delivering all the raw materials needed for each production run in only one shipment is relaxed. Two possible raw material ordering situations are examined for a better optimal solution in the integrated inventory model. All the information given in the single-stage inventory problem applies to here unless it is mentioned otherwise. Figure 4.1 shows the inventory of the manufacturer's raw material, finished goods at the top and the inventory of the buyer incoming goods at the bottom. Another additional difference between two problems can be observed from Figure 4.1 that the 1st shipment of finished products from the manufacturer to the buyer takes place as soon as the required shipment quantity, *x*, is produced, and this 1st shipment period is expected to be less than all the other successive shipment periods, L whereas $L = x/D$ due to the fixed-interval batch supply. One last difference between two inventory models is that, single-stage model is considered under a finite planning horizon whereas integrated model is studied under an infinite planning horizon.

Since products considered in the integrated model are experiencing continuous price decrease like the ones in the single-stage model, the purchasing cost of the raw material, the production cost of the finished goods and the selling cost of the finished product are decreasing continuously over time. The initial unit cost of the raw material, C_O, the expected decrease in the raw material price, *b*, manufacturing costs, C_M, and manufacturer's profit, C_P, are given. Therefore, the unit cost of the raw material at time *t* is $C_R(t) = C_O - bt$ and the unit cost of manufacturing the finished product is $C_F(t) = (1/f)(C_O - bt) + C_M$. The cost of the finished goods is determined by adding the total cost of the required number of raw materials based on the conversion factor, *f*, plus other fixed manufacturing costs, C_M. Finally, the profit of the manufacturer is added to the production cost of the finished product to determine the buyer's purchasing price of the product, which is $C_B(t) = (1/f)(C_O - bt) + C_M + C_P$. As it is shown, the manufacturing cost of the finished product and the buyer's purchasing price are also decreasing over time because of the decrease in the unit cost of the raw material.

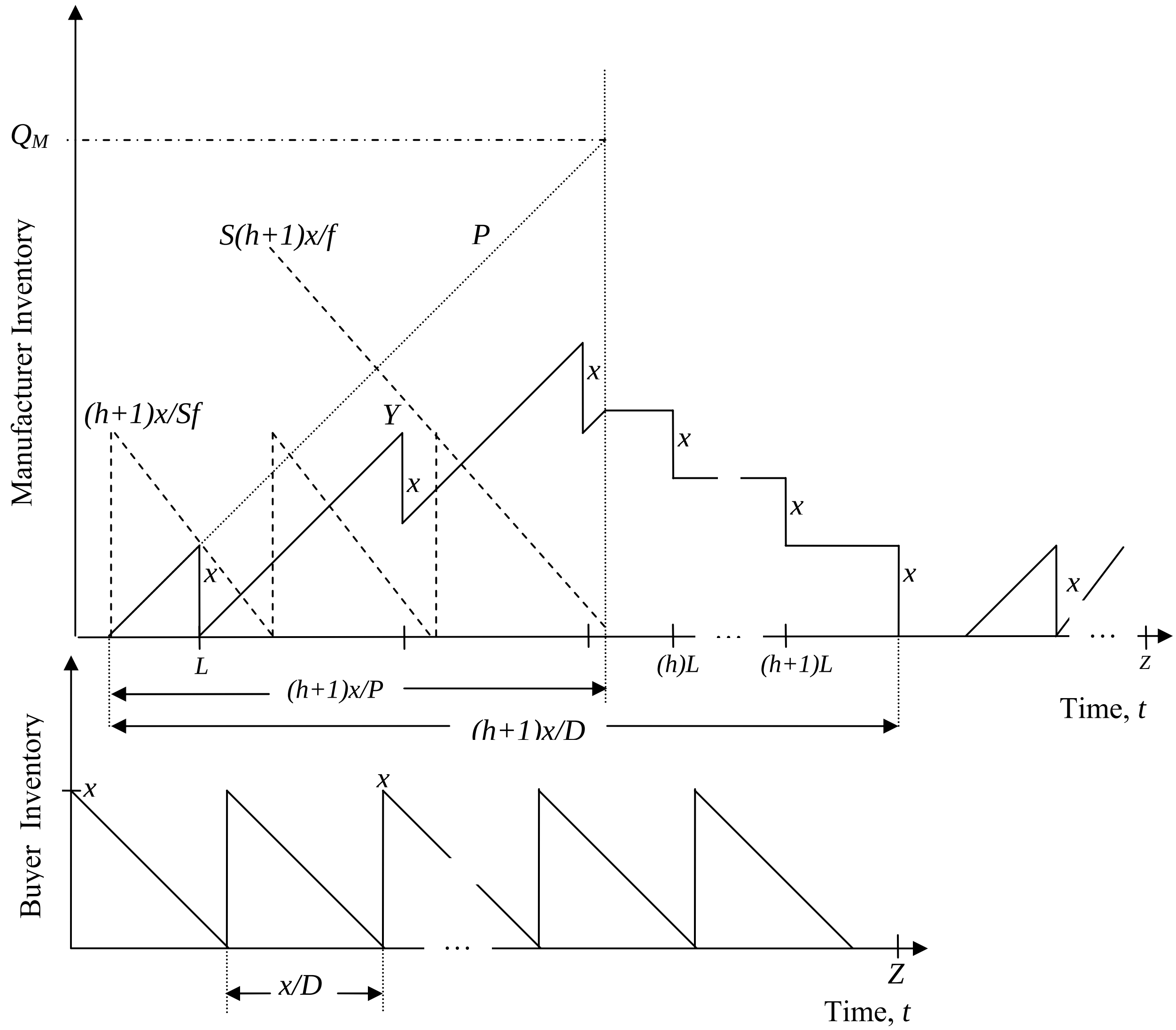

Figure 4.1: Inventory of manufacturer's raw materials, finished goods and buyer's incoming goods.

4.2 Model Formulation

A total of eight costs from manufacturer and buyer are incorporated in the integrated inventory model under an infinite planning horizon. All the considered manufacturer's costs are: raw material ordering cost, A_O, raw material purchasing cost, $C_R(t)$, raw material carrying cost, $iC_R(t)$, production setup, A_S, and finished goods carrying cost, $iC_F(t)$. On the other hand, the costs considered for the buyer side are: ordering cost, A_B, purchasing cost, $C_B(t)$, and carrying cost, $iC_B(t)$.

4.2.1 Additional Notation and Assumptions

The notation and assumptions used in both single-stage and integrated inventory model are not repeated, they can be referred from the notation and assumptions in Section 3.2.1. However, the ones that are not used in the single-stage inventory model are explained below.

A_B Buyer's ordering cost, (dollars/order)

$C_B(t)$ Buyer's purchase price per unit at time *t*; $C_B(t) = (C_R(t)/f) + C_M + C_P$, (dollars/unit)

C_P Profit of the manufacturer per unit, (dollars/unit)

h Number of shipments of finished goods after the 1st shipment; $m = (h+1)$

v Number of production run covered from one procurement of raw material

k *1,2, …,S* , raw material replenishment index

g *1,2, …, (D/x)* buyer's replenishment index

Three of the assumptions from the single-stage inventory model are relaxed in the integrated model. The shipment quantity of finished goods (buyer's delivery lot size), *x*, is assumed as a known parameter in the production-delivery model, but herein it is changed to an unknown but fixed parameter. The second updated assumption in the integrated model is that the order of raw materials for one production run is not restricted to only one shipment anymore. Either lot size of the ordered raw material can meet the demand of more than one production run or more than one replenishment of raw material can be ordered for only one production run. Finally, the planning horizon is assumed to be finite in single-stage inventory model whereas it is infinite in integrated inventory model.

4.3 General Cost Function

In addition to the costs of manufacturer's inventory considered in the single-stage inventory model, in the integrated model the costs of buyer's inventory is also incorporated into the system in order to see a more complete picture of the supply chain system. Therefore, the total cost of the integrated inventory model, TC_T, is composed of the total cost of the manufacturer's raw material, TC_R, the manufacturer's finished goods, TC_F and the total cost of the buyer, TC_B, as shown below.

$$TC_T = TC_R + TC_F + TC_B \qquad (4.1)$$

4.3.1 Raw Material Costs

Manufacturer orders raw materials from the outside suppliers and converts them to finished goods. The total raw material cost is composed of ordering cost, purchasing cost and holding cost. Three different cases of procurement of raw material are examined while implementing the integrated inventory model to determine the optimum total cost of the supply chain. One of these cases, which is the special case of the other two cases, allows only one purchase of raw material for each production run. In another case, Case 1, each lot size of procured raw material meets the demand of more than one production run whereas, in Case 2, more than one replenishment of raw material is needed for every production run. Therefore, none of the raw materials are carried by the manufacturer during the production downtime in Case 2, and this decreases the holding cost but increases the ordering cost. However, some portion of the procured raw material carried during the downtime in Case 1, and this increases the holding cost unlike in Case 2. When the number of procurement of raw material is one in either Case 1 or Case 2, then the model converts into the single-stage inventory model which is called Special Case in the integrated inventory model. A generalized formulation is used while discussing the details of each cost, but the total cost functions are presented separately for the two possible ordering situations.

As it is explained in the single-stage inventory problem, raw material is converted to finished goods by the manufacturer with a conversion factor of *f*, where $f = D/D_R = Q_M/Q_R$. The lot size of the raw material, Q_R, can be represented as vQ_M / f whereas $v = 1$ for the Special Case, $v = \{1, 2, 3, ..., S\}$ for Case 1 and $v = \{1, 1/2, 1/3, ..., 1/S\}$ for Case 2. Therefore, the generalized raw material ordering cost is composed of the total number of procurements for raw material, $[Df / x(h+1)v]$ times the ordering cost, A_O. The number of raw material procurement, $[Df / x(h+1)v]$, is simply $Df / Q_M v$ where $Q_M = (h+1)x$ which is explained in Section 4.3.2. The raw material purchasing cost per each replenishment is the lot size of the raw material for each order, $x(m+1)v / f$, times the cost of the raw material, $C_O - bkx(h+1)v / Df$. As it is discussed in details in the previous problem, the price of the raw material is continuously decreasing during its life cycle. Therefore the fixed amount of decrease, *b*, is multiplied with the raw material replenishment time, $bkx(h+1)v / Df$, whereas *k* is the raw material replenishments index and $x(h+1)v / Df$ is the time between two successive raw material replenishments. Furthermore, the total raw material purchasing cost is calculated by summing the purchasing cost per each replenishment over the planning horizon for all the replenishments. Finally, the holding cost of the raw material is the average inventory of raw material which is different for Case 1 and 2, times the carrying cost. The average inventory for each of the cases is presented below in equation (4.2) and (4.3). The derivation of the average raw material inventories are presented by Lee (2005, pp.167).

$$\text{Case 1:} \quad Q_{Avg} = \frac{x}{2}\frac{(h+1)}{f}\left[\frac{D}{P}v + (v-1)\left(1-\frac{D}{P}\right)\right], \qquad \text{for } v = \{1, 2, 3, ..., S\} \qquad (4.2)$$

$$\text{Case 2:} \quad Q_{Avg} = \frac{x}{2}\frac{(h+1)v}{f}\frac{D}{P}, \qquad \text{for } v = \{1, 1/2, 1/3, ..., 1/S\} \qquad (4.3)$$

The carrying cost also depends on the procurement policy of raw material, but in general it is the interest rate, *i*, times, the replenishment period, $x(h+1)v/Df$, times the decreasing unit cost, $C_O - bkx(h+1)v/Df$. Equation (4.4) represents the total cost of the total inventory cost function of raw material explained above.

$$TC_R(x,h,v) = \frac{Df}{x(h+1)v}A_O + \sum_{k=0}^{\left\lceil \frac{Df}{x(h+1)v} \right\rceil - 1} \left[C_O - bk\frac{x(h+1)v}{Df} \right] \left[\frac{x(h+1)v}{f} \right] +$$

$$\sum_{k=0}^{\left\lceil \frac{Df}{x(h+1)v} \right\rceil - 1} \left[C_O - bk\frac{x(h+1)v}{Df} \right] \left[Q_{Avg} \left[i\frac{x(h+1)v}{Df} \right] \right]. \qquad (4.4)$$

The summation of the decreasing unit cost of the raw material can be simplified by using Mathematica (Wolfram 1991) as follow:

$$\sum_{k=0}^{\left\lceil \frac{Df}{x(h+1)v} \right\rceil - 1} \left[C_O - bk\frac{x(h+1)v}{Df} \right] = \frac{-bDf + 2C_O Df + bvx + bhvx}{2(h+1)vx}. \qquad (4.5)$$

After simplification, equation (4.5) is substituted into equation (4.4) and the total inventory cost of raw material over the finite planning horizon becomes

$$TC_R(x,h,v) = \frac{Df}{x(h+1)v}A_O + \left[\frac{-bDf + 2C_O Df + bvx + bhvx}{2(h+1)vx} \right] \left[\frac{x(h+1)v}{f} + \left[Q_{Avg} \left[i\frac{x(h+1)v}{Df} \right] \right] \right]. \qquad (4.6)$$

4.3.2 Manufacturer's Production Costs

Manufacturer's production cost includes the production setup cost and the finished goods holding cost. The production lot size, Q_M, can be represented by $(h+1)x$ whereas h is the number of shipments after the first shipment and x is the shipment quantity of the finished goods. When h is equal to zero, it means that manufacturer delivers all the finished goods to the buyer in one shipment, which is called lot-for-lot policy. Manufacturer's production setup cost is simply number of setups, $[D/(h+1)x]$, times the setup cost, A_S, since it is assumed that setup processes are done once for each production run. On the other hand, manufacturer's finished goods holding cost varies with the possible procurement situations of the raw material since the cost of the finished products in the manufacturer inventory depends on the purchasing price of the raw material. When a general formulation is discussed, the holding cost for a production run includes the average inventory of finished goods, $[x/2][h[1-(D/P)]+(D/P)]$ derived by Lee (2005) times the carrying cost. The carrying cost is composed of the interest rate, *i*, the time between two successive orders of raw material, $[x(h+1)v/Df]$ and the cost of raw material according to its procurement case, $[(1/f)(C_O - bjx(h+1)v/Df)]$, plus manufacturing cost, C_M. In order to obtain the total inventory holding cost of finished goods, holding cost according to the number of raw material orders are summed. All the explained costs of manufacturing the finished goods are presented in equation (4.5).

$$TC_M(x,h,v) = \frac{D}{(h+1)x} A_S + \sum_{k=0}^{\left\lceil \frac{Df}{x(h+1)v} \right\rceil - 1} \left[\frac{1}{f}\left(C_O - bk\frac{x(h+1)v}{Df} \right) + C_M \right] \left[\left(\frac{x}{2}\right)\left(h\left(1-\frac{D}{P}\right) + \frac{D}{P} \right) \right] \left[i\frac{x(h+1)v}{Df} \right]. \quad (4.7)$$

The summation of the decreasing unit cost of manufacturing finished goods can be evaluated by using Mathematica (Wolfram 1991) as follows:

$$\sum_{k=0}^{\left\lceil \frac{Df}{x(h+1)v} \right\rceil - 1} \left[\frac{1}{f}\left(C_O - bk\frac{x(h+1)v}{Df} \right) + C_M \right] = \frac{-bDf + 2C_O Df + 2C_M Df^2 + bvx + bhvx}{2f(h+1)vx}. \quad (4.8)$$

When equation (4.8) is substituted into equation (4.7), the total cost of manufacturing the finished product becomes

$$TC_M(x,h,v) = \frac{D}{(h+1)x} A_S + \left[\frac{-bDf + 2C_O Df + 2C_M Df^2 + bvx + bhvx}{2f(h+1)vx} \right] \left[\left(\frac{x}{2}\right)\left(h\left(1-\frac{D}{P}\right) + \frac{D}{P} \right) \right] \left[i\frac{x(h+1)v}{Df} \right]. \quad (4.9)$$

4.3.3 Buyer's Costs

The buyer purchases a fixed quantity of finished products from the manufacturer at fixed interval. Since the annual demand rate of the buyer is known and constant, the production rate of the manufacturer is adjusted in such a way that production rate is more than demand rate in order not to have any shortages. The buyer's total ordering cost is $(D/x)A_B$, whereas D/x gives total number of buyer's orders and A_B is the cost for each order. The buyer's purchasing cost for each shipment, which is the selling price of the manufacturer's finished goods, is determined by adding manufacturing cost and profit of the manufacturer to the raw material cost. Since the raw material cost is decreasing continuously over time, the selling price is calculated according to the price of the raw material at the time of each shipment. The purchasing cost decrease for each shipment is calculated by multiplying the given fixed amount of decrease, *b*, with the shipment time, $g(x/D)$ in which g is the shipment index and (x/D) is the time between successive shipments. Therefore, the decrease amount, $bg(x/D)$, is subtracted from the original raw material cost and production cost and manufacturer's profit are added in order to obtain the buyer's purchasing cost per shipment. Moreover the purchasing cost per each shipment, $[(1/f)(C_O - bgx/D) + C_M + C_P]$ times shipment quantity, *x*, is summed over the planning horizon for all the shipments to get the total purchasing cost of the buyer. Finally, the holding cost of the buyer is the average inventory of finished products (shipment quantity), $x/2$, times the carrying cost which is composed of interest rate, *i*, the buyer's purchasing price, $[(1/f)(C_O - bgx/D) + C_M + C_P]$, and time between two successive shipments, x/D. Once again the total inventory holding cost of the buyer is calculated by summing the holding cost per shipment over the planning horizon for all the shipments. The following total inventory cost function of the buyer includes all the given information above.

$$TC_B(x)=\frac{D}{x}A_B+\sum_{g=0}^{\left\lceil \frac{D}{x}\right\rceil -1}\left[\frac{1}{f}\left(C_O-bg\frac{x}{D}\right)+C_M+C_P\right]x+\sum_{g=0}^{\left\lceil \frac{D}{x}\right\rceil -1}\left[\frac{1}{f}\left(C_O-bg\frac{x}{D}\right)+C_M+C_P\right]\left[\frac{x}{2}i\frac{x}{D}\right]. \quad (4.10)$$

The summation of the decreasing selling price of the finished goods (buyer's incoming goods) can be simplified by using Mathematica (Wolfram 1991) as showing below.

$$\sum_{g=0}^{\left\lceil \frac{D}{x}\right\rceil -1}\left[\frac{1}{f}\left(C_O-bg\frac{x}{D}\right)+C_M+C_P\right]=\frac{-bD+2C_OD+2C_MDf+2C_PDf+bx}{2fx}. \quad (4.11)$$

After simplification, equation (4.11) can be substituted into equation (4.10) as shown below to update the total cost of the buyer.

$$TC_B(x)=\frac{D}{x}A_B+\left[\frac{-bD+2C_OD+2C_MDf+2C_PDf+bx}{2fx}\right]\left[x+\frac{x}{2}i\frac{x}{D}\right]. \quad (4.12)$$

Finally, the total cost function of the integrated model over the infinite planning horizon including manufacturer's raw material, finished goods and buyer's incoming goods inventory costs are represented below. For each of the raw material ordering situation, an updated total cost function is written separately. Equation (4.13) presents the special case of total cost function, $TC(x,h)$, which considers that all the needed raw materials for one production run is ordered in only one shipment or in other words, $v=1$ in the generalized total cost function.

$$TC(x,h)=\frac{D}{x}A_B+\left[\frac{-bD+2C_OD+2C_MDf+2C_PDf+bx}{2fx}\right]\left[x+\frac{x}{2}i\frac{x}{D}\right]+$$

$$\frac{D}{(h+1)x}A_S+\left[\frac{-bDf+2C_ODf+2C_MDf^2+bx+bhx}{2f(h+1)x}\right]\left[\left(\frac{x}{2}\right)\left(h\left(1-\frac{D}{P}\right)+\frac{D}{P}\right)\right]\left[i\frac{x(h+1)}{Df}\right]$$

$$\frac{Df}{x(h+1)}A_O+\left[\frac{-bDf+2C_ODf+bx+bhx}{2(h+1)x}\right]\left[\frac{x(h+1)}{f}+\left[\left(\frac{x}{2}\right)\left(\frac{(h+1)}{f}\right)\left(\frac{D}{P}\right)\right]\left[i\frac{x(h+1)}{Df}\right]\right]. \quad (4.13)$$

On the other hand, the following total cost equations show Case 1 and 2 respectively where $v = \{1, 2, 3, ..., S\}$ for Case 1 and $v = \{1, 1/2, 1/3, ..., 1/S\}$ for Case 2. As it is discussed earlier, the differences between two possible raw material ordering situations are the average inventory of raw material, Q_{Avg}, and the number of production run covered by one replenishment of raw material, *v*. Case 1 considers that each lot size of procured raw material meets the demand of more than one production run and the total cost function for this Case, $TC_1(x,h,S)$, is presented in equation (4.14).

$$TC_1(x,h,S) = \frac{D}{x}A_B + \left[\frac{-bD + 2C_O D + 2C_M Df + 2C_P Df + bx}{2fx}\right]\left[x + \frac{x}{2}i\frac{x}{D}\right] +$$

$$\frac{D}{(h+1)x}A_S + \left[\frac{-bDf + 2C_O Df + 2C_M Df^2 + bSx + bhSx}{2f(h+1)Sx}\right]\left[\left(\frac{x}{2}\right)\left(h\left(1-\frac{D}{P}\right)+\frac{D}{P}\right)\right]\left[i\frac{x(h+1)S}{Df}\right]$$

$$\frac{Df}{x(h+1)S}A_O + \left[\frac{-bDf + 2C_O Df + bSx + bhSx}{2(h+1)Sx}\right]\left[\frac{x(h+1)S}{f} + \left[\left(\frac{x}{2}\right)\left(\frac{(h+1)}{f}\right)\left(\frac{D}{P}S + (S-1)\left(1-\frac{D}{P}\right)\right)\right]\left[i\frac{x(h+1)S}{Df}\right]\right]. \quad (4.14)$$

However, in Case 2, every production run contains more than one shipment of raw material and the total cost function of this Case, $TC_2(x,h,S)$, is shown in equation (4.15).

$$TC_2(x,h,S) = \frac{D}{x}A_B + \left[\frac{-bD + 2C_O D + 2C_M Df + 2C_P Df + bx}{2fx}\right]\left[x + \frac{x}{2}i\frac{x}{D}\right] +$$

$$\frac{D}{(h+1)x}A_S + \left[\frac{-bDfS + 2C_O DfS + 2C_M Df^2 S + bx + bhx}{2f(h+1)x}\right]\left[\left(\frac{x}{2}\right)\left(h\left(1-\frac{D}{P}\right)+\frac{D}{P}\right)\right]\left[i\frac{x(h+1)}{DfS}\right]$$

$$\frac{DfS}{x(h+1)}A_O + \left[\frac{-bDfS + 2C_O DfS + bx + bhx}{2(h+1)x}\right]\left[\frac{x(h+1)}{fS} + \left[\left(\frac{x}{2}\right)\left(\frac{(h+1)}{fS}\right)\left(\frac{D}{P}\right)\right]\left[i\frac{x(h+1)}{DfS}\right]\right]. \quad (4.15)$$

4.3.4 Optimal Solution

After formulating the total cost function of the integrated inventory model, it is time to develop a solution methodology to optimize the lot sizes of raw material procurement, production batch and buyer's order under an infinite planning horizon while minimizing the cost. In the production-delivery system under continuous price decrease, $A_B, A_O, A_S, C_O, C_M, C_P$, D, P, i, f, and b parameters of the total cost function are fixed and given. Therefore, the total cost is a function of three decision variables; *x*, shipment quantity, *h*, number of shipments of finished goods after the 1st shipment, and *S*, number of raw material procurement.

Since $TC_r(x,h,S)$ where $r = 1,2$, total cost function for both Case 1 and Case 2, is not differentiable, a closed-form solution cannot be obtained directly. Therefore, an iterative solution methodology is applied to minimize the total cost of the system. An algorithm is developed to find the optimal lot sizes for raw material procurement, Q_R^*, by obtaining S^*, production batch, Q_M^* by obtaining h^*, and buyer's delivery, x^* iteratively.

As it is discussed in Lee (2005), there is no need to examine both Case 1 and 2 simultaneously by algorithm 2; only one of the cases can be picked because of the characteristic of the convex function. Therefore, either of the Cases can be computed by the developed algorithm and as long as the optimal solution of S^*, number of raw material procurement, from the computed Case is not equal to one, then the optimal solution definitely falls into that Case and the computation is sufficient. However, for the computed Case, if S^* is equal to one, then the other Case also needs to be applied to the problem to find out which Case generates a lower total cost. In addition, there is a possibility that the optimal solution for both Cases takes a place at $S^* = 1$ which means that delivering all the required raw material quantity for one production run in only one shipment gives the minimum total cost and this is the Special Case.

The second algorithm developed in this paper for the integrated inventory system is presented below.

Algorithm 2: Optimal Operational Policies

Step 1: (a) Initialize $A_B, A_O, A_S, C_O, C_M, C_P, D, P, i, f$, and b.

(b) Set $r = 1$.

(c) Set $x = 0$, $h = 0$, $S = 0$, $h^{S*} = 0$, $x^{S*} = 0$, $h^* = h$, $x^* = x$, $S^* = S$,

$TC_r(x^{S*}, h^{h*}, S) = \infty$, and $TC_r(x^{(S-1)*}, h^{(S)*}, (S-1)) = \infty$.

Step 2: If $TC_r(x^{S*}, h^{S*}, S) \geq TC_r(x^{(S-1)*}, h^{(S-1)*}, (S-1))$, go to Step 5.

Set $S = S + 1$.

Step 3: If $r = 1$ (Case 1)

1. Solve $dTC_1(x, h, S)/dx = 0$ for shipment quantity, *x*.
2. Compute the total cost $TC_1(x, h, S)$ using equation (4.14). Go to Step 4.

Otherwise, $r = 2$ (Case 2)

1. Solve $dTC_2(x, h, S)/dx = 0$ for shipment quantity, *x*.
2. Compute the total cost $TC_2(x, h, S)$ using equation (4.15).

Step 4: If $TC_r(x, h, S) \leq TC_r(x^{S*}, h^{S*}, S)$, Set $h^{S*} = h$, $x^{S*} = x$ and

$TC_r(x^{S*}, h^{S*}, S) = TC_r(x, h, S)$ and $h = h + 1$, then go to Step 3.

Otherwise, the local minimum $TC_r(x^{S*}, h^{S*}, S)$ is obtained, go to Step 2.

Step 5: If $S = 2$, then set $r = 2$ and go to Step 1(c)

Otherwise, the global minimum total cost is obtained. Set $x^* = x^{(S-1)*}$,

$h^* = h^{(S-1)*}$, $S^* = (S-1)$, $TC_r(x^*, h^*, S^*) = TC_r(x^{(S-1)*}, h^{(S-1)*}, (S-1))$.

Step 6: Stop.

4.4 Verification of the Integrated Model

The model implemented in the integrated inventory problem is verified with the aid of the previously published model in Lee (2005). The integrated model is tested by using the exact values of the given numerical example in Lee's paper (2005). Verification of the model can be done by first finding the relationship between the two models and then removing the differences. The most important difference in this paper compared to the Lee's paper is that the products considered in this system are experiencing continuous price decrease, whereas Lee assumed a constant unit price for the products in the system. This means that if the unit cost is kept constant in this integrated model, the results are expected to be same as the ones from Lee's model (2005), and this can be succeeded by setting the decrease in unit cost, *b*, equal to zero in the integrated inventory model. However, there is one more small difference between two models that should be taken into consideration before testing the accuracy of the integrated model. Lee (2005) did not consider the purchasing cost of the raw material nor the selling price of the finished product. Since the unit cost is decreasing in the implemented model, those costs are incorporated into the model, but this is not the case for Lee's model (2005). So, after removing those costs from the model, it is time to test the accuracy of the developed model.

As it is mentioned by Lee (2005), the following numerical example is a common example used by Banerjee (1986), Goyal (1988, 1995), and Lu (1995). The parameters computed into the integrated model which are common in all the mentioned papers are: $A_S = \$400$ /batch, $A_B = \$25$ /order, $i = 0.2$ annually, $P = 3200$ units/year, $D = 1000$ units /year, $C_M = \$7.5$ /unit and $C_P = \$5$ /unit. Lee (2005) also added three more parameters to these commonly used ones: $f = 0.8$, $A_O = \$2500$ /order, and $C_O = \$10$ /unit. When the developed iterative solution methodology is applied to the integrated model to check the validity of the model, the following optimal results are obtained by using Mathematica (Wolfram 1991): $h^* = 5$ shipments/batch, $v^* = 2$ (Case 1) production run, $x^* = 114$ units, $Q_M^* = 684$ units/batch, $Q_R^* = 1710$ units/ procurement and $TC_1(x^*, h^*, v^*) = \$4{,}528$ /year.

It can be realized that exact same optimal solution is obtained by the integrated inventory model as one in Lee (2005) when the second developed algorithm is applied. This conclusion proves the accuracy of both the integrated inventory model and algorithm 2.

4.5 Computational Results

Here, a common example used by Banerjee (1986), Goyal (1988, 1995) and Lu (1995) is explained elaborately to illustrate the integrated inventory model under continuous price decrease for a single product. The parameters are: $A_S = \$400$ /setup, $A_B = \$25$ /buyer's order, $C_M = \$7.5$ /unit, $C_P = \$5$ /unit, $i = 2\%$ annually, $P = 3200$ units/year, $D = 1000$ units/year. In addition, three more parameters added by Lee (2005) to the example are: $f = 0.8$, $A_O = \$2500$ / manufacturer's order, and $C_O = \$10$ /unit. Moreover, one more parameter is added to the example in this paper to incorporate continuous price decrease into the model: decreasing rate of the unit cost is 1% per week, in another words, the total decrease in the unit cost of the raw material at the end of a year is equal to \$5.2 [$b = (\$10.00)(52\%) = \$5.2$ /year].

For this particular problem, algorithm 2 is first applied to Case 1 and possible combinations of *S* and *h* are computed iteratively to determine *x* by solving the derivative of equation (4.14). Then the total cost function is calculated from equation (4.14) based on the computed *x*, *h* and *S*. First *S* is fixed to 1 and the number of shipments, *h*, which gives the minimum cost for that specified *S* value is determined as 8. When *S* is raised to 2 and the optimal number of shipments, *h*, is found as 4. But the minimum total cost calculated when $S = 1$ is lower than the one calculated when *S=2*. Therefore, it is not necessary to try any other *S* values because the total cost increases with the increasing *S*. Here, since the optimal solution occurs at $S^* = 1$ for Case 1, Case 2 is also computed to find out which Case generates a lower total cost. The same procedure applied for Case 1 is followed for Case 2. The results of the both Cases are presented in Table 4.1.

It can be easily realized that for this particular example, the optimal solution takes place at $S^* = 1$ for both Cases and this means that the total cost is minimum when the order of raw material needed for one production run is delivered in only one shipment. Therefore, the following optimal results are obtained for this particular numerical example: $h^* = 8$ shipments/batch, $v^* = 1$ (Special Case) production run, $x^* = 75$ units, $Q_M^* = 675$ units/batch, $Q_R^* = 844$ units/procurement and $TC(x^*, h^*, v^*) = \$36{,}744$ /year.

Table 4.1: Results of the developed algorithm to obtain the optimal solution

	S	*h*	*x*	*TC(x,h,S)*
CASE 1	1	8	75	36,744
	2	4	75	37,122
CASE 2	1	8	75	36,744
	2	15	72	37,250

On the other hand, Lee (2005) proved that the raw material ordering cost, A_O, is one of the key factors that significantly affects the optimal solution. Therefore, the raw material ordering cost is changed from \$2,500/order to \$50/order and all the other parameters remain unchanged while obtaining the new optimal solution with the use of algorithm 2. The results are shown in Table 4.2 when $A_O = \$50\text{/order}$.

Here, first Case 1 is picked to compute by algorithm 2 and the optimal result is obtained at $S^* = 1$, therefore Case 2 is also computed. As it can be concluded from table 4.2, the optimal solution occurs at $S^* = 6$ and Case 2 generates a lower total cost for the system. This means for one production run, six procurements of raw material give the optimal solution when the ordering cost is reduced. Therefore, the following optimal results are obtained when the raw material ordering cost is reduced: $h^* = 7$ shipments/batch, $v^* = 1/6$ (Case 2) production run, $x^* = 74$ units, $Q_M^* = 592$ units/batch, $Q_R^* = 123$ units/order and $TC_2(x^*, h^*, v^*) = \$31{,}973$/year.

Table 4.2: Results of the algorithm when $A_O = \$50$/order

	S	h	x	TC(x,h,S)
CASE 1	1	3	74	32,752
	2	2	70	33,774
CASE 2	1	3	74	32,752
	2	5	67	32,240
	3	5	76	32,069
	4	6	73	32,000
	5	9	59	31,992
	6	7	74	31,973
	7	8	71	31,984

4.6 Sensitivity Analysis

Lee (2005) mentioned that manufacturer prefers its outside suppliers to have distribution centers near its plant in just-in-time system to lower the raw material ordering cost and the total cost consequently. Based on this perception, it is explained that reducing the ordering cost is one of the key factors that affect the lot size of raw material and the total cost significantly. Therefore, the sensitivity of the raw material ordering cost is performed in this section to show the effect of changing it in the model. Like the single-stage production-delivery system, this integrated inventory model is also a third-degree polynomial, so again it is not easy to express the total cost function in a simple form while performing sensitivity analysis on some parameters.

Sensitivity analysis is done by considering a ratio of raw material ordering cost versus setup cost. Figure 4.2 shows the effect of increasing the ratio on the total cost of the integrated inventory model. For each ratio, a constant value is picked for the manufacturer's setup cost, A_S, and the raw material ordering cost, A_O , is computed based on the ratio. The increasing total cost in Figure 4.2 proves basically that as the raw material ordering cost increases with increasing ratio, the total cost value is also increasing. In addition, this increase in the ordering cost leads the optimal lot size of the raw material, Q_R^*, to increase as well in order to reduce the number of raw material orders and this is presented in Figure 4.3.

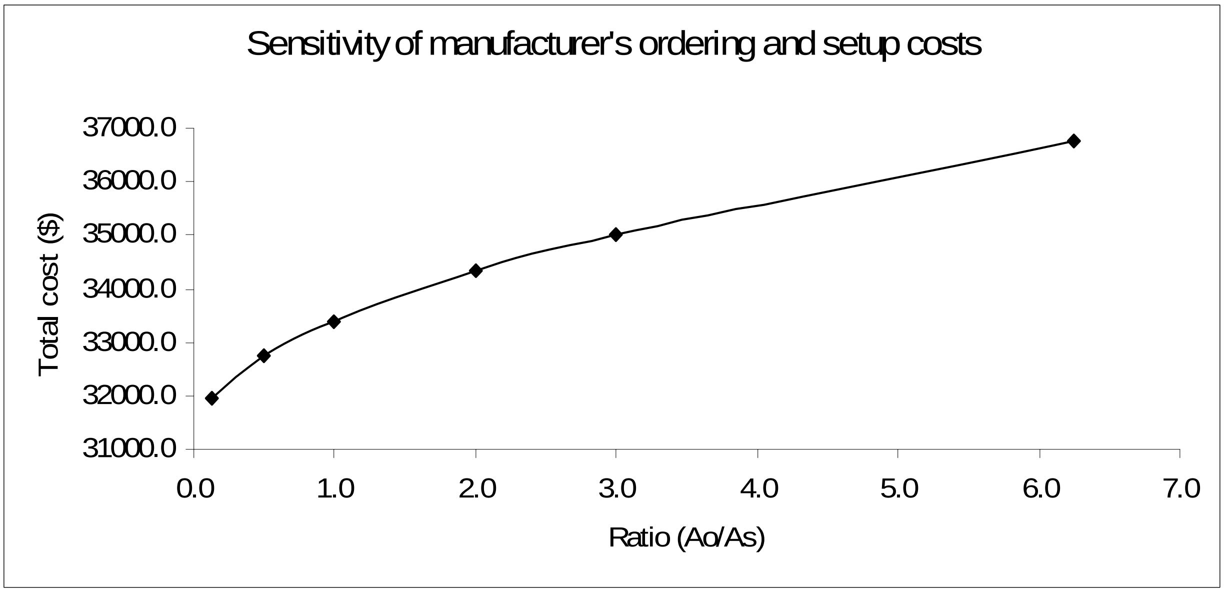

Figure 4.2: Sensitivity of manufacturer's ordering and setup costs

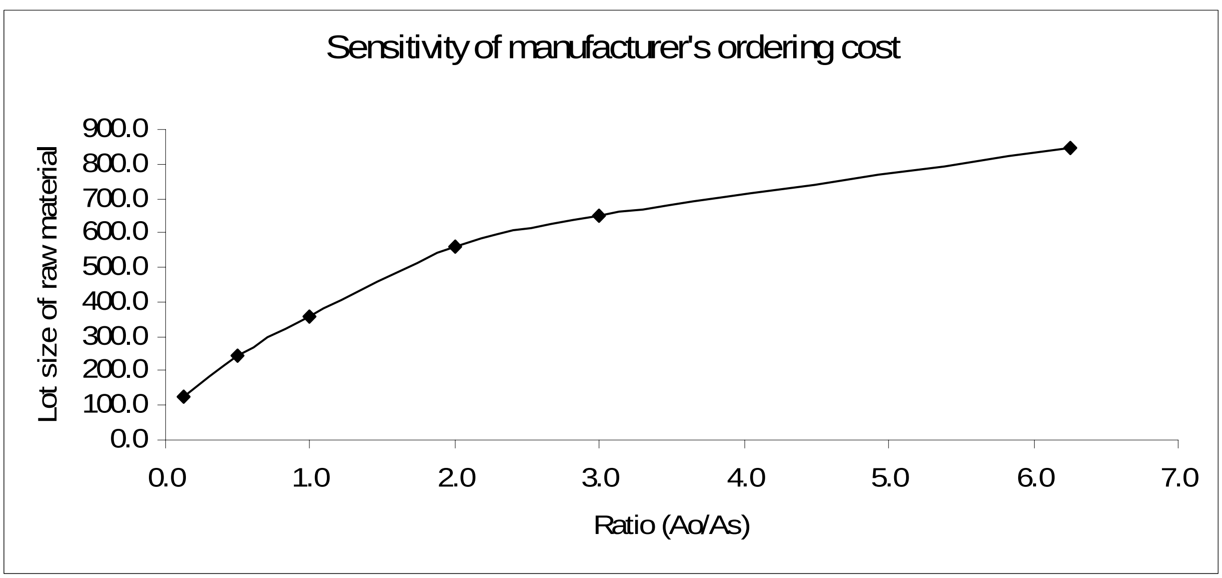

Figure 4.3: Sensitivity of manufacturer's ordering cost

On the other hand, the sensitivity of the raw material ordering cost is also shown in Section 4.5 numerically. Since the numerical example is examined twice by only changing the raw material ordering cost, A_O, the optimal solutions can be compared to show the effect of reducing this cost in the integrated model. These optimal results are represented in Table 4.3, and it can be concluded that when A_O is reduced from \$2,500/order to \$50/order, the number of raw material procurement needed for one production run increases from 1 to 6. Therefore, the lot size of the raw material procurement and the total cost of the inventory system decreases accordingly.

Table 4.3: Optimal solutions for two different A_O

A_O	v^*	Q_R^*	$TC(x^*, h^*, v^*)$
2,500	1	844	36,744
50	1/6	123	31,973

CHAPTER 5

OPERATIONAL SCHEDULE

In this chapter, the developed inventory models are demonstrated with numerical data to show the details of both the developed inventory systems and the applied algorithms. The operational schedule is first applied to the single-stage model and then to the integrated model, and all the procedures followed to obtain the optimal solution is explained elaborately for a specific numerical example. In the last section of this chapter, numerical illustrations are extended to 12 problems to finalize the operational schedule chapter.

5.1 Single-Stage Model under Finite Planning Horizon

In this inventory model, shipment quantity, x, is known and number of raw material procurement needed for one production run, v, is restricted to one. In addition, none of the buyer's costs are incorporated into the system. The first algorithm developed in this study for the single-stage inventory model follows iterative solution method to obtain the optimal solutions while enhancing the system performance. The operational schedule of this system is evaluated by determining the optimum number of shipments and number of cycles during the planning horizon to minimize the total cost of manufacturer under a finite planning horizon. The following values of parameters are used to obtain the operational schedule: $C_M = \$4.00$ /unit, $D = 2400$ units/year, $P = 3600$ units/year, $A_O = \$200$/order, $A_S = \$300$/setup, $f = 1$, and $x = 100$ units /shipment, $i = 8\%$ annually, $Z = 1$ year, $C_O = \$8.00$/unit , and $b = (\$8.00)(52\%) = \$4.16$/year. By using algorithm 1 implemented for the single-stage model the optimal number of shipments, m^*, is obtained and fixed to that value. Therefore, the local minimum total cost is reached iteratively at $m^* = 4$ shipments/cycle. After fixing m^*, equation (3.16) is solved for number of cycles, n^*, to get the global optimum solution. Therefore, when optimal number of cycles is found as $n^* = 5$ cycles/ year, and the optimal minimum cost, $TC(m^*, n^*)$, is calculated as \$42,700.02/year by using equation (3.15). Figure 5.1 represents the calculated optimal solutions during the finite planning horizon. Manufacturer orders 480 units of raw material per production run which takes place once every cycle. 150 units of finished goods are produced during shipment period and 100 units of finished products are shipped every 1/24 year (15 days). 5 non-identical cycles or in other words 5 non-identical production runs take place during the planning horizon to fulfill the demand of 2400 units, since 480 units of finished goods are produced at each setup.

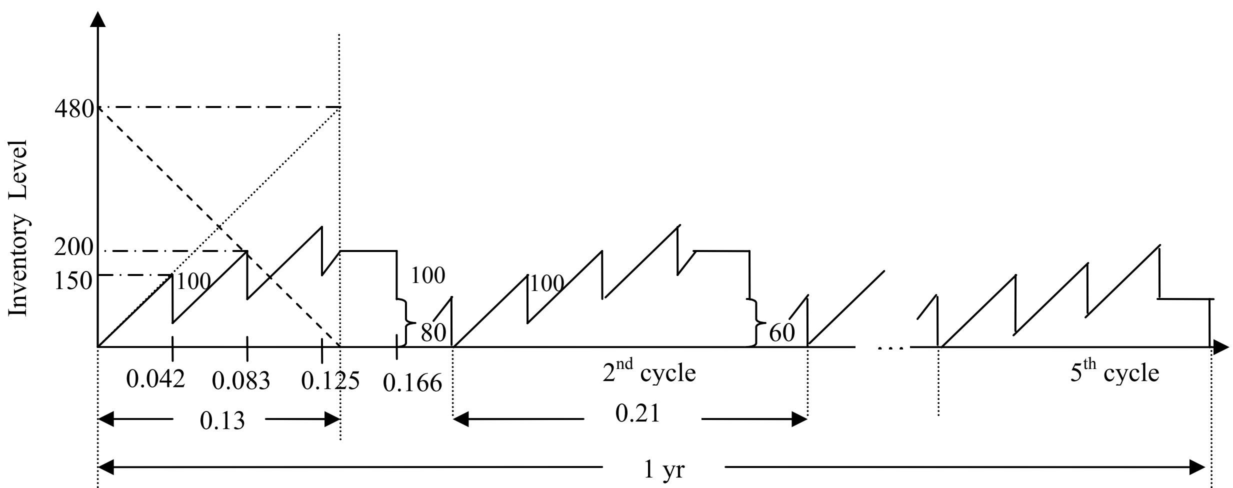

Figure 5.1: Operational schedule for single-stage inventory model

The details of the operational schedule of this particular problem are presented in Table 5.1. The optimal solution consists of 5 non-identical production-delivery cycles and each shown cycle includes the total number of delivery, delivery time of each shipment, production starting time, and ending time, cumulative production quantity and leftover products remained in manufacturer's inventory. A total of 24 shipments with 100 units for each shipment are delivered to the buyer every 0.0417 year (15 days). To fulfill the requirements of these shipments, manufacturer produces 480 units during each production run. Even though the batch quantity is same for each setup, the starting and ending time of each production run differs in order to meet the required shipment quantity at exact delivery time.

Table 5.1: Non-identical production-delivery cycles

Non-identical Cycle	Production Start	Production End	Delivery	Delivery Time*	Cumulative Production	Cumulative Delivery	Leftover Inventory
#1	0	0.1333	1	0.0417	150	100	50
			2	0.0833	300	200	100
			3	0.1250	450	300	150
			4	0.1667	480	400	80
#2	0.2028	0.3361	5	0.2083	500	500	0
			6	0.2500	650	600	50
			7	0.2917	800	700	100
			8	0.3333	950	800	150
			9	0.3750	960	900	60
#3	0.4056	0.5389	10	0.4167	1000	1000	0
			11	0.4583	1150	1100	50
			12	0.500	1300	1200	100
			13	0.5417	1440	1300	140
			14	0.5833	1440	1400	40
#4	0.6083	0.7417	15	0.6250	1500	1500	0
			16	0.6667	1650	1600	50
			17	0.7083	1800	1700	100
			18	0.7500	1920	1800	120
			19	0.7917	1920	1900	20
#5	0.8111	0.9444	20	0.8333	2000	2000	0
			21	0.8750	2150	2100	50
			22	0.9167	2300	2200	100
			23	0.9583	2400	2300	100
			24	1.0000	2400	2400	0

* Delivery interval, $L = 1/24$ year.

The manufacturer produces 150 units every 0.0417 year, therefore in the 1st cycle, 450 units are manufactured in 0.1250 year and the remaining 30 units are manufactured in 0.0084 year (3 days). So, the production run stops before the 4th delivery. From 480 manufactured finished goods, 4 shipments are delivered to the buyer and then 80 units remained in the manufacturer's inventory at the end of 1st non-identical cycle. However, in order to meet the shipment quantity

(100 units/shipment); producer needs to manufacture 20 more units before the 5th shipment. So the 2nd production run starts 0.0055 year (2 days) earlier than the 5th delivery time to produce 20 units. When the 5th delivery takes place at the beginning of the 2nd cycle, the leftover finished products in the inventory goes to zero. Since 20 products of the 2nd batch is already produced, the remaining 460 unit of products are manufactured during 0.1278 year (47 days) and the 2nd production run stops before the 9th delivery. The same procedure is repeated for the remaining non-identical cycles and at the end of the year a total of 2400 units are delivered to the buyer in 24 shipments every 0.0417 year by producing exact 480 units in each production run.

Consequently, for this inventory model, iterative solution is used to find the optimal number of shipments, m^*, and then, by using the obtained m^*, the optimal number of cycles, n^*, and the total cost, $TC_T(m^*, n^*)$, are determined. More numerical examples are evaluated in the last section for the single-stage production-delivery model and the same procedures explained above are followed to get the optimal results.

5.2 Integrated Model under Infinite Planning Horizon

In the integrated inventory model, both manufacturer's and buyer's costs are taken into consideration for an infinite planning horizon. In addition, the restriction of delivering all the raw material required for one production run in only one shipment is relaxed and also shipment quantity, x, is unknown here. Two different ordering policies for raw material procurement are examined in this model to give more flexibility while minimizing the total cost of the system. For each particular example one of the raw material ordering policies gives the optimum solution. However, both cases may give the same optimum solution when all the required raw materials for one production run delivered in only one shipment minimizes the total cost at most.

Another algorithm is implemented for this model to obtain the optimal solution by following an iterative methodology. Both the optimal number of production run covered by one procurement of raw material, v^*, and the optimal number of shipments, $m^* = (h^* + 1)$, are found iteratively. After determining the optimal v^* and m^* values, the optimal shipment quantity, x^*, and the total cost $TC(x^*, h^*, v^*)$ are obtained.

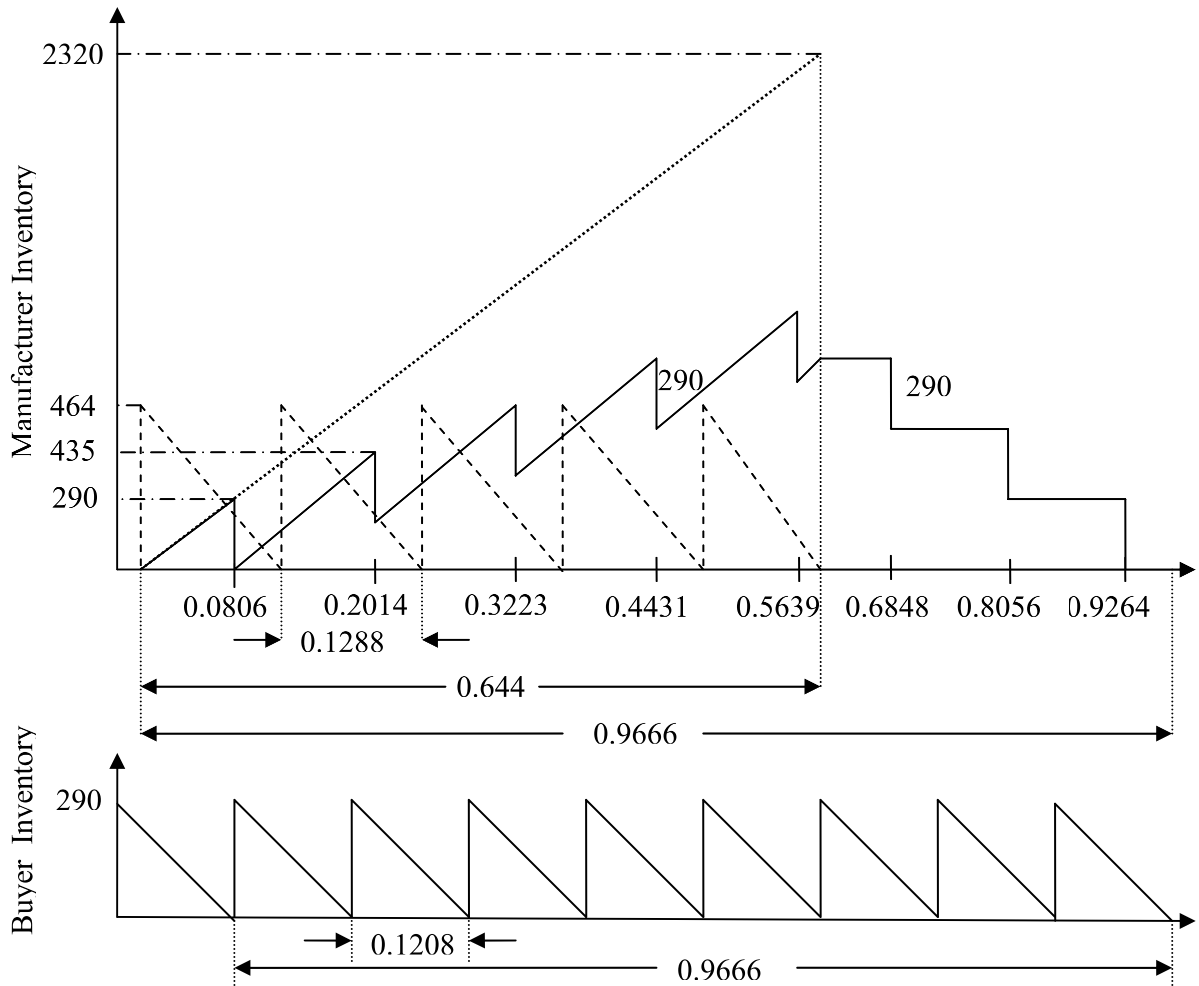

Figure 5.2: Operational schedule for integrated inventory model

The operational schedule of this integrated inventory system is evaluated by using the same values of the parameters given in the operational schedule of the single-stage model. In addition to those parameters, $A_B = \$100$ /order and $C_P = \$3.00$ /unit are included to determine the optimal shipment quantity, x^*, number of shipments after the 1st shipment, h^*, and number of production run covered by one procurement of raw material, v^*. Figure 5.2 shows the optimal solutions obtained by the operational schedule of the integrated inventory model by presenting the manufacturer's and buyer's inventory separately.

By using algorithm 2, various combinations of h and v are checked to find the optimal shipment quantity by solving the derivative of the total cost function, and then either equation (4.14) or (4.15) is used to calculate the total cost according to the selected raw material ordering policy. Therefore, the following results are obtained: $h^* = 7$ shipments/batch after the 1st shipment, $v^* = 1/5$ production run, $x^* = 290$ units, $Q_M^* = 2320$ units/batch, $Q_R^* = 464$ units/procurement and $TC(x^*, h^*, v^*) = \$49{,}547$ /year.

The details of the operational schedule of the integrated model for this particular illustration are represented in Table 5.2. The optimal solution is determined for an infinite planning horizon, and number of raw material procurement, raw material order time, number of shipments or deliveries, manufacturer's shipment time, buyer's delivery time, cumulative raw material, production and delivery quantities, and the leftover finished products in the manufacturer's inventory are all shown in Table 5.2.

Table 5.2: Operational schedule under infinite planning horizon

Raw Material Order	Raw M. Order Time	Cum. Raw Mat.	Delivery/ Shipment	Shipment Time	Delivery Time*	Cum. Production	Cum. Delivery	Leftover Inv.
1	0	464	1	0.0806	0.1208	290	290	0
2	0.1288	928	2	0.2014	0.2417	725	580	145
3	0.2576	1392	3	0.3223	0.3625	1160	870	290
4	0.3864	1856	4	0.4431	0.4833	1595	1160	435
5	0.5152	2320	5	0.5639	0.6042	2030	1450	580
			6	0.6848	0.7250	2320	1740	580
			7	0.8056	0.8458	2320	2030	290
			8	0.9264	0.9666	2320	2320	0

* Delivery interval, L = 0.1208 year.

The optimal raw material ordering policy which gives the minimum total cost value is determined as Case 2, ordering more than once for each production run. The total number of raw material procurement needed to produce 2400 units of finished product per year is 5.172 and the total number of buyer deliveries to get 2400 units of finished goods per year is 8.276. However, in Table 5.2 a total of 5 raw material procurements and a total of 8 deliveries are shown since the planning horizon is infinite. This means instead of covering 2400 units of demand for a year, *(5/5.172)*(2400) = 2320* units of products are considered for *(5/5.172)*(1) = 0.9666* year. Therefore, 464 units of raw material are ordered 5 times (5 procurements) for one production run in order to be converted into a total of 2320 units of finished goods per setup (batch). Producer can manufacture 435 units of finished goods in 0.12083 year. The production run time starts at zero and stops at 0.644 year (235 days) which is before the 6^{th} shipment. The finished products are delivered from the manufacturer to the buyer at a fixed quantity of 290 units per shipment. The 1^{st} shipment occurred as soon as 290 units of products are produced which is 0.0806 year (29 days). After 1^{st} shipment, all the remaining shipments, $h^* = 7$, are delivered periodically every 0.12083 year (44 days). Therefore, the total time for a buyer to receive 8 deliveries is 0.966 year. On the other hand, the manufacturer's shipment times are different than the buyer's delivery times because, the manufacturer's 1^{st} shipment time is 0.0806 year after starting to the production run, but the delivery time is 0.1208 year since periodic shipments are delivered to the buyer every 0.1208 year during the infinite planning horizon. 5 of 8 deliveries take place during the production (uptime) time, whereas the remaining 3 deliveries occur during the downtime. Same procedure is followed for all the numerical examples in empirical tests section while applying algorithm 2 to reach the optimal results.

5.3 Empirical Tests

In this section, numerical illustrations of both single-stage and integrated inventory systems are extended even more by considering 12 problems. The data and optimal solutions for both systems are summarized in Table 5.3. The optimal solutions for each model are specified separately, whereas all the data needed for both models are given together in the table. As it is mentioned earlier in this study, the differences between two inventory models should be taken into consideration while doing these empirical tests. The methodologies applied in the previous sections in this chapter are followed and Mathematica (Wolfram 1991) software package is used to determine the optimum solutions of 12 problems in a faster way.

Table 5.3: Data and optimal solutions of numerical illustrations

			Problem 1		Problem 2		Problem 3		Problem 4		Problem 5		Problem 6	
			(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
DATA	1st & 2nd Model	D	2400	2400	2500	2500	12000	12000	1800	1800	5000	5000	3000	3000
		P	3600	3600	25000	25000	20000*	20000*	2500	2500	8000	8000	4500	4500
		A_O	200	100	100	40	100	50	200	75	2,000	200	5,000	1,000
		A_S	300	150	50	15	$150*	90	275	150	120	60	80	40
		x	100	100	100	100	150*	150*	80	80	75	75	120	120
		f	1	1	0.5	0.5	1*	1*	0.8	0.8	0.7	0.7	0.6	0.6
		C_o	8.00	8.00	8.00	8.00	40.00	40.00	20.00	20.00	30.00	30.00	15.00	15.00
		C_M	4.00*	4.00*	4.00*	4.00*	7.00*	7.00*	6.00	6.00	6.00	6.00	4.00	4.00
		i	0.08	0.08	0.08	0.08	0.08	0.08	0.1	0.1	0.1	0.1	0.05	0.05
		Z	1	1	1	1	0.25	0.25	1	1	2	2	0.5	0.5
		b	4.16	4.16	4.16	4.16	20	20	10.40	10.40	15.60	15.60	0.078	0.078
		A_B	100	50	25	15	50	25	80	40	30	15	250	50
		C_P	3.00	3.00	5.00	5.00	6.00	6.00	8.00	8.00	4.00	4.00	3.00	3.00
SOLUTION	1st Model	n^o	5.33	8	2.5	2.5	6	12	8.1	8.1	13.9	41.7	0.42	0.42
		Q_M^o	450	300	1000	1000	500	250	223	223	720	240	3600	3600
		$TC_T(m^*,n^o)$	42,731	41,395	78,649	78,412	250,267	249,089	84,464	82,439	534,929	496,213	84,592	82,909
		m^*	4	3	10	10	3	1	2	2	9	3	30	30
		n^*	5	7	7	8	8	11	7	10	15	38	0.37	0.52
		Q_M^*	480	343	357	313	375	273	257	180	667	263	4054	2885
		$TC_T(m^*,n^*)$	42,700	41,334	74,395	73,639	250,115	249,076	84,449	82,270	534,777	496,123	84,440	82,700
	2nd Model	h^*	7	7	3	2	7	8	8	9	6	6	1	2
		v^*	1/5	1/5	1/3	1/3	1/5	1/6	1/6	1/7	1	1/2	2	1
		x^*	290	205	122	98	215	156	131	88	108	72	541	301
		Q_M^*	2320	1640	488	294	1720	1404	1179	880	756	504	1082	903
		Q_R^*	464	328	325	196	344	234	246	157	1080	360	3607	1505
		$TC(x^*, h^*, v^*)$	49,547	48,274	69,956	68,943	890,627	886,477	91,099	89,101	342,296	328,402	50,729	48,642

The unit costs of $A_O, A_S, C_O, C_M, b, A_B, C_P, TC_T(m^*, n^o)$, $TC(m^*, n^*)$ and $TC(x^*, h^*, v^*)$ are in dollars.

CHAPTER 6

RESEARCH CONCLUSIONS

In this research, an integrated inventory model for products experiencing continuous decrease in unit cost is developed. Both manufacturer and buyer sides of the supply chain system are considered in the model unlike most of the research done in the area of changing unit cost. In the earlier studies, either EOQ or EPQ models studied individually for those products. As it is mentioned earlier, considering the integrated supply chain is more important than considering individual entities, since individual entities cannot survive solely in today's competitive market. In this last chapter of the thesis, conclusion and significance of this research are discussed briefly, and finally, some suggestions are given for possible future research.

6.1 Conclusions

In this study, first a single-stage production-delivery model under a finite planning horizon for technology-related companies whose products are experiencing continuous price decrease during the life cycle is developed to emphasize the importance of considering price decrease into the supply chain system. Then, an integrated production-delivery model under an infinite planning horizon is studied to gain the advantage of integration among the entities of the supply chain under JIT policy.

In this developed inventory model, since the price is continuously decreasing manufacturing firm both orders the raw material and delivers the finished goods in small quantities frequently and this proves the importance of the JIT policy in inventory control management. Frequent orders and delivers in small lots are really effective to reduce the total cost of the supply chain. Companies in high-tech industries can be successful by following the JIT policy since the price of their products is decreasing continuously. The key for these industries is to reduce the total days of inventory since the component prices are continuously decreasing, and this can only be succeeded by a super-effective supply chain management.

6.2 Research Significance

Nowadays, it is much harder for a company to improve its performance and to survive in a global market. This struggle to be successful in business world can be achieved with the aid of an integrated supply chain management. The developed model incorporates the integrated inventory model under JIT policy and continuously decreasing price of the products. A common assumption of constant unit price for inventory models is not realistic for high-tech products in today's competitive market, so that restriction is relaxed in this study in order to be more applicable for the real practical environment of those industries. Therefore, this research would have a significant contribution for the enhancement of the system productivity as a whole in the supply chain management of technology-related companies.

6.3 Possible Future Extensions

The inventory model developed in this research is limited to certain conditions which can be relaxed in future research. By relaxing some restrictions considered in this study, the problem will become even more complicated but it will be more realistic. Therefore, in order to enhance the supply chain system more, the following possible extensions are worthwhile to be examined:

(a) Varying Cycle Length

The scope of the developed inventory model is limited to a fixed cycle time, and this makes it harder for the companies to take the advantage of the decreasing price function. Therefore, future research may be directed to relaxing this limitation by considering varying cycle length, which would provide more profit to the high-tech industries.

(b) Time Varying Demand

In this research, the demand of the customer is assumed to be constant, however; in a real market, the demand of high-tech products is changing because of the improvement in technology. Every day a new product is introduced to the global market with better features to fulfill the expectations of customers, and this leads to a non-constant demand rate. Consequently, if a time varying demand rate is considered for the integrated inventory system instead of a constant one, the model will be closer to the real situation.

(c) More than One Type of Raw Material

Another possible future extension can be incorporating more than one type of raw material to produce the finished goods. In the developed model, only one type of raw material is considered, however; in reality, different types of raw materials are required to manufacture the finished product. There might be some industries that manufacture the products from one type of raw material, but presumably, this assumption does not apply to most of the industries. So, by improving this restriction of the developed model, the inventory system will become applicable to the industries in various technological areas.

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VITA

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