SUPPLY CHAIN MODELS FOR AN ASSEMBLY SYSTEM WITH PREPROCESSING OF RAW MATERIALS

A Thesis

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ABSTRACT

An assembly line that procures raw materials from outside suppliers and processes the materials into finished products is considered in this research. An ordering policy is proposed for raw materials to meet the requirement of a production facility, which, in turn, must deliver finish products in a fixed quantity at a fixed time interval to the outside buyers. Two different types of raw materials, ‘unfinished’ and ‘ready-to-use’, are procured for the manufacturing system. The ‘unfinished raw materials’ are turned into ‘processed raw materials’ after preprocessing. In the assembly line, the ‘processed raw materials’ and the ‘ready raw materials’ are assembled to convert into the final products. A cost model is developed to aggregate the total costs of raw materials, Work-in-process, and finished goods inventory. Based on the product design and manufacturing requirement a relationship is established between the raw materials and the finished products at different stages of production. A non-linear integer-programming model is developed to determine the optimal ordering policies for procurement of raw materials, and shipment of assembly product, which ultimately minimize the total costs of the model. Numerical examples are presented to demonstrate the solution technique. Sensitivity analysis is performed to show the effects of the parameters on the total cost model. Future research direction is suggested for further improvement of the existing results.

CHAPTER 1

INTRODUCTION

A supply chain (SC) is a network of suppliers, manufacturing facilities, distribution centers, and retailers that performs the functions of procurement of materials, transformation of these materials into intermediate and finished product, and the distribution of these finished product to customers. A supply chain may exist in both service and manufacturing organizations. Many manufacturing operations are designed to maximize throughput and lower costs with little consideration for the impact on inventory levels and distribution. An efficient supply chain system operates under a strategy to minimize costs by integrating the different functions inside the system and by meeting customer demands in time. The supply chain system considered in this research consists of three levels: suppliers, producers and retailers (see Figure 1).

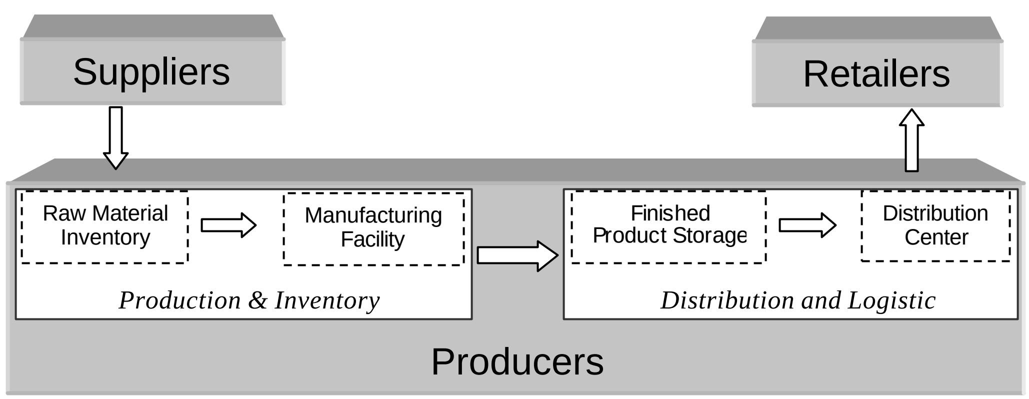

Figure 1.1. Supply chain model overview

The present study considers an assembly production system, where the producer uses several raw materials to convert into finished product. Raw materials are divided into two types, 'unfinished' and 'ready to use'. *Unfinished raw materials* need preprocessing, and *ready raw materials* can directly arrive in the production line for the assembly operation. Finally, finished products are transported to the distribution centers, and ultimately to customers. Figure 2 shows the structure and logical flow of the system.

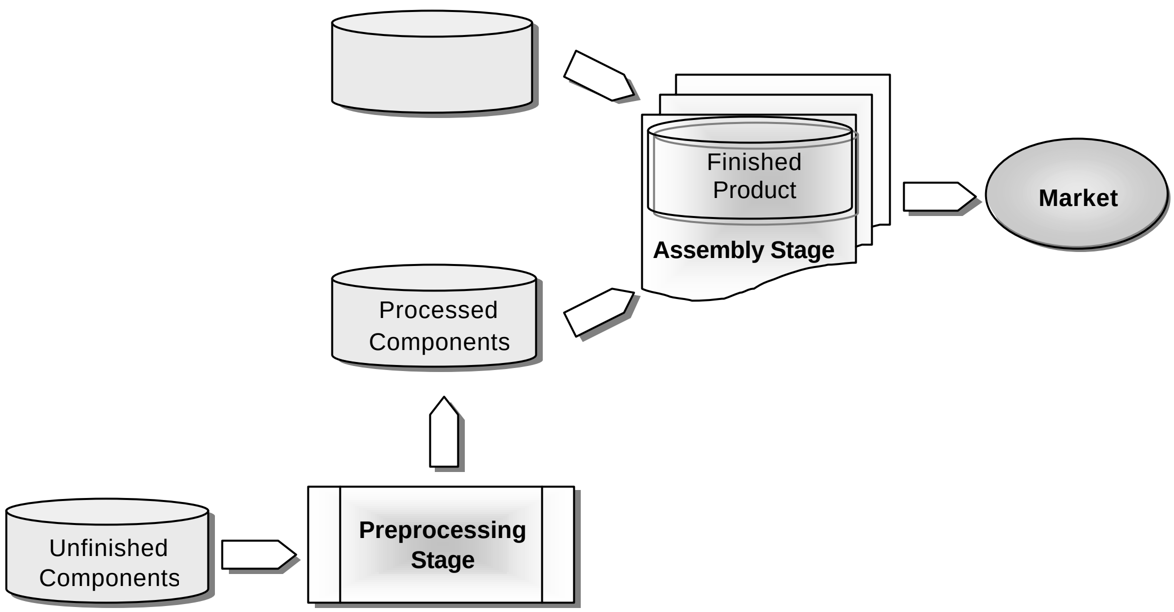

Figure 1.2. Structure and the logical flow of the system

It is well understood that manufacturers do not manufacture (or process) all of the required raw materials during the production of a finished product. For example, computer manufacturers may produce computer cases and motherboards inside the plant, but they may procure the remaining components from outside vendors. Similarly, an air compressors manufacturer may manufacture compressor cabinets, air tanks, and radiators inside the factory from raw materials (e.g., sheet metals) but at the same time it may want to purchase a number of components such as compressor motor from outside suppliers. The raw materials (e.g., sheet metals), which are used to manufacture any subassembly component, are called *unfinished raw materials*. *Unfinished raw materials* that are transformed into manufactured/subassembly parts are termed as *processed raw materials.* The components such as compressor cabinets, and air tanks are preprocessed/sub-assembled inside the facility in the processing stage. The raw materials that do not need any transformation or preprocessing, but are ready to be used in an assembly line to produce final products are categorized as *ready raw materials*.

In the just-in-time (JIT) assembly system, pioneered by Toyota and gradually adopted in North America, raw materials and subassemblies arrived at assembly plant as needed. For example, "Collins and Aikman", a textiles specialty business, produces textiles, wall covering, carpets and textiles for automobile industry, and ships textile raw materials from North Carolina (headquarters) to their assembly plants in Oklahoma and Michigan (local manufacturing factory) for producing finished textile products. "Horn Plastic Inc. (Ontario, Canada)", complying with the proposed model in many aspects, established as a premium tool and dies manufacturer, in 1968 and as a quality injection molder in 1979, and provides customers with a wide range of molded assembly products from two groups of components. The organization procures engineered resins as the raw materials and process them to be prepared for the injection molding operation and/or add liquid Silicon Rubber (LSR), if necessary, in the molding to attain a high temperature resistant and high strength, for the products. The facility procures plastic molded parts as a ready raw material from one of the adjacent plastic molding plants. The plant finally assembles both the resin-molded parts and plastic molded parts to generate finished, assembled products in a flexible manufacturing environment.

In the present research, raw materials are used in production; their ordering policy is dependent on the shipping plan of the finished product. Therefore, it is necessary to determine the optimum-shipping strategy of the finished product and the ordering policy of the associated raw materials as well. A constant quantity of finished product supply in a set interval of time is considered in this research. However, the supply of raw materials and deliveries occurred in multiple lots in which the ordering policy (number of orders) and the delivery frequency are the decision variables.

1.1 Background

The decisions for a supply chain system are classified into two categories, strategic and operational. Strategic decisions are made typically over a longer time horizon, and these are linked to the corporate strategy to guide supply chain policies from a design perspective. On the other hand, the function of operational decisions exists short-term, and focus on activities over a day-to-day basis. The effort in these types of decisions is to effectively and efficiently manage the product flow in a "strategically" planned supply chain. There are three major decision areas in supply chain systems: inventory, production, and transportation (distribution).

Inventory Decision: Inventories exist at every stage of the supply chain as raw material or semi-finished or finished goods. They can also be as Work-in-process between the stages or stations. Since holding of inventories cause a significant cost, their efficient management is critical in supply chain operations.

Production Decision: The planning of a production line and schedule is the explicit decision in a supply chain system. The determination of the exact path(s) through which a product flows to a manufacturing line is included in this decision. These decisions include the construction of the master production schedules, scheduling production on machines, and equipment maintenance.

Transportation Decision: The mode of transportation choice is the primary aspect of these decisions. The best choice of mode is found by trading-off the cost of using the particular mode of transport with the cost of inventory associated with that mode. For example, air shipments may be fast, reliable, and require minimum safety stocks, but they are expensive. Meanwhile, shipping by sea or rail may be much cheaper, but they necessitate holding relatively large amounts of inventory. This research does not include this decision.

1.2 Supply Chain Inventory

It has been an important issue to integrate inventory decisions in supply chain system. A system, which provides excess inventory, reflects lack of planning and poor communication and management.

There are three types of inventories: raw materials inventory, work-in-process inventory, and finished goods inventory. The operational activities of the SC inventories include: (1) Raw materials: inventory planning, purchasing, transportation from vendor to manufacturers and storage. (2) Work-in-process (WIP): processing and/or preprocessing inside the manufacturing unit, (3) Finished goods: warehousing, finished product inventory management, delivery among the wholesalers and retailers. An effective control of inventory systems increases the production efficiency of a system.

1.3 The Problem

A production facility assembles finished product from a group of raw materials in batches. The raw materials are ordered from the suppliers and are procured through shipments. The overall yearly demand of the finished product is consistent and known quantity. There are conversion factors that apply to the raw materials to be converted into finished product. The annual requirements of the raw materials are estimated to produce the demanded amount of the finished product. Assume the facility has the capacity to produce desired quantities of products to maintain satisfactory delivery time. The delivered amount of finished product for each shipment is a constant known time interval. The total amount of raw materials required per cycle and the amount of raw materials per order are procured, accordingly.

The aim of this research is to develop the operation policy of an assembly type, single-stage supply chain system. The finished product is assembled from a group of raw materials, where some of the raw materials require preprocessing before final stage assembling. It also attempts to find a strategic policy to minimize the total cost associated in the process of procuring raw material, transferring and holding raw materials, manufacturing and, finally, delivering the finished product.

1.4 Research Goal

The goal of this research is to effectively model a single stage assembly-type, just-in-time supply chain system and to find operational plans to increase the efficiency of the production process by reducing the level of wasted materials and time and effort involved at each of the production stages. As the demand of the finished product is constant and known value, the raw material requirement quantities are known and match demands. For known quantities of raw materials, their procurement order rates and order sizes are correlated and dependent on the production rate. Again, for a fixed quantity of finished product, the product delivery frequency is also important. The cost is largely dependent on the number of orders of all raw materials, order sizes and the number of deliveries of the finished product to the customers.

Summarizing the above points, the goal of this research is

1. To develop materials procurement and delivery plan to meet demand.
2. To minimize the total cost of an assembly production system.

1.5 Research Objective

The objective of this research is to build an assembly system with a requirement of preprocessing of some components. The efficiency of the system largely depends on the integration of all the cost functions. The associated costs included in the system are the ordering cost of raw materials, transferring cost to send them to buffer inventory, holding cost of raw materials at the stores, holding cost to wait inside the work-in-process area, and, finally, delivery cost to distribute the finished product.

In order to minimize the total cost of the raw materials, transportation cost and inventories in an assembly production line, the objectives of this research are to develop a model that will

1. Determine optimal ordering policy for all raw materials, preprocessed materials and finished goods batches.
2. Schedule the operational policy of different raw materials and general production.
3. Study the sensitivity of the parameters on the cost function.
4. Logistic and operational scheduling of the model.

1.6 Scope of the Research

The model is assumed to function in an environment where the product demands are static and continue for an extended horizon. Products that have a matured life cycle and are assembled from a variety of raw materials in which few of them require preprocessing before the final processing stage are ideal for this model. Products that are transformed from raw materials in a single phase or multi phases such as paper, cement or household instruments, refined oil are the cases where the present research can be implemented.

A manufacturing plant “QSC Audio Products” is cited as an example to demonstrate the present model. This industry assembles amplifiers. Some of the ingredients require initial processing before arriving in assembly line. In the processing stage, raw materials are relocated to the workstations to generate semi-assemble components (such as amplifier case, printed circuited board). In the final stage, ‘semi-processed components’ and ‘ready-to-use raw materials’ (such as screw, bolts, wires) are taken to the assembly line and produce ‘amplifiers’.

This study is expected to have applications in most manufacturing environments where the products are consumed on a regular basis and the demands are stable. Practical examples that illustrate the model may be

- Electric machinery, equipment and supplies industry,
- Metal industry, agricultural products manufacturing industry,
- Light engineered steel fabrications industry,
- Automobile and motor cycle assembly plant,
- Precision and general machineries production plant,
- Paper mills, sugar mills, refinery industries and chemical industry.

1.7 Overview of the Research

This chapter provides an introduction to the present research, background study of supply chain system and the concepts of supply chain inventory. Chapter 2 introduces related literature on EOQ model and JIT model. This chapter ends by reviewing the relevant models of assembly systems indicating the characteristics and constraints considered in the past researches. Chapter 3 presents a general description of the problem of a single-stage supply chain system and the replenishment order placement strategy. The formulation of the inventory cost functions and total cost function is included in this chapter. Test results and computational solutions for the assembly systems with multiple components, two workstations problems are analyzed in Chapter 4. The sensitivity of the input variables and the model parameters on the total cost function are examined in Chapter 5. Chapter 6 describes the logistic operational schedule for raw materials order placement and the corresponding production time and downtime of the processing plant and assembly plant. A complete time-scale synchronized plan of the problem is illustrated in this Chapter. The research conclusion and recommendation for the future research are covered in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

A significant part of the recent literature on supply chain system explores the decisions on controlling inventory, production, and distribution. That literature mostly considers a few important aspects of supply chain production system: the ordering policy that applies to the suppliers, the delivery policy to the buyers and the system that must satisfy demand.

Literature abounds in assembly systems in view of optimal order policies, optimal materials control, production costs minimizing, material requirements planning ordering philosophy, effect of Work-in-process inventory design. Kanban control assembly system, assembly inventory system with back logging, assembly system with stochastic demand etc. are also included in recent supply chain literature. Assembly system integrates a group of raw materials into finished product. Analysis of such integrated system may be complex when the raw materials are different and some need preprocessing before assembly work.

The following sections review the literature related to the economic orders quantity (EOQ) model, just-in-time (JIT) system model, and the assembly of different components.

2.1 Economic Order Quantity (EOQ) Model

Most of the existing EOQ models are focused on lot sizing and material shipping policies. An EOQ model considering joint vendor-buyer replenishment policy (JRP) is modeled by Goyal and Satir (1989). Miyazaki *et al*. (1988) adapted the classical economic order quantity to obtain average inventory under the assumption of instantaneous replenishment. Parlar and Rempala (1992) have presented optimal order and production quantity model for a single-stage production system. Goswami and Chaudhuri (1992) developed a deterministic inventory model allowing shortages and backlogged with two level of storage considering a linear trend in demand. Dan (1995) developed an economic order quantity (EOQ) model where order quantity is set by the maximization of return on investment (ROI), so that the order quantity is fixed regardless of the demand. Drezner *et al*. (1995) presented an economic order quantity (EOQ) model for two raw materials and substituted one for another, if necessary, within a given cost limit. He considered three cases: full substitution, partial substitution and no substitution. Banerjee (1992) developed production lot sizing model to satisfy periodic demand and included Work-in-process. Hariga and Goyal (1995) dealt with the inventory lot-sizing problem with time varying demand having linear trend. Lu (1995) presented a one-vendor multi buyer integrated inventory model. Hill (1996) determined a purchasing and production schedule minimizing total cost for a system in which a single product is manufactured from a single raw material and shipped a fixed quantity to a single customer at fixed intervals.

Fazel *et al*. (1998) discussed an analytical comparison of inventory costs on JIT purchasing vs. EOQ with a price discount. Hariga and Haouari (1999) presented inventory lot sizing model under EOQ framework and then showed the negligent cost penalty of using the EOQ lot size instead of uniform, exponential and truncated distributions. Hill (1996) presented a two-stage lot-sizing model where the production rate of the stages are independent of each other i.e., production at first stage may be higher or lower than the second stage and vice-versa and showed a similar results regardless of production rates between the stages. Viswanathan (1996) considered an algorithm for the joint replenishment problem determining the optimal cyclic policy for all cycles. Viswanathan (1998) concerned an integrated vendor-buyer inventory model for two different strategies and analyzed the relative performance for both the strategies with various problem parameters. Raw material ordering policy and a fixed-interval, fixed-quantity delivery policy to multiple customers for an economic batch size of product was developed to minimize the total cost by Parija and Sarker (1999). Hill (1997, 1999) considers a manufacturing system that produces products at a finite rate and delivers them in fixed intervals. Deriving a global-optimal solution to minimize the total cost that comprises a manufacturing set-up, stock transfer, and stock holding results from his investigation.

2.2 Just-in-time (JIT) Operation Models

In JIT philosophy, researchers investigate the benefit of reducing ordering and setup time to a minimum. Goyal and Gupta (1989), Goyal (1995), and Aderohunmu *et al*. (1995) presented models for joint vendor-buyer policy in a just-in-time environment. Chyr *et al*. (1990) compared between just-in-time system and EOQ system with a view of lot size based on setup times and damage rates. If damage cost is not considered, the unit total cost of JIT system is slightly higher than EOQ system for single stage case; otherwise, EOQ lot-size is equal to JIT lot-size under some conditions. His conclusion shows the lot-size of JIT system (with no damage cost) is better than EOQ system (included damage cost) for a specific range. Otherwise, the latter is better. Baker *et al.* (1994) suggested decision rules in determining the suitability of switching to a JIT model from EOQ model. A two-stage optimum order and fixed quantity model were developed by Ramasesh (1990).

In a fixed quantity, just-in-time delivery system, Golhar and Sarker (1992) tested a generalized inventory model where uptime and cycle time are integer multiples of the shipment interval and match shipment size. Total cost function is piecewise convex and under certain conditions; total cost decreases linearly with reduced shipment size. Jamal and Sarker (1993) estimated the finished product batch quantity in a just-in-time production system. The raw materials ordering policy and the batch size in a regular interval of time within the production cycle was developed in the model. Sarker and Parija (1994, 1996) extended Golhar and Sarker's (1992) model developing multi-order procurement policy for raw material within a single stage-manufacturing batch. The effect of setup cost on the total cost function and an approximate integer optimal solution was adopted. Nori and Sarker (1996) adapted Sarker and Parija's (1996) model including a two-situation case, fixed setup cost and variable setup cost, in a multi-product single-facility system. Sarker and Balan (1996) proposed a single-stage two-station Kanban system for a varying (linear) demand pattern model where the Kanban transports the Work-in-processes from the first station to the second. The number of Kanbans, batch sizes of the Kanbans, the dispatching time, and the schedule for production are illustrated. Sarker and Balan (1998,1999) modified the Sarker and Balan's (1996) model incorporating the optimal number of Kanbans required in two adjacent workstations for both single-stage and multi-stage production line under a just-in-time production system. Their models assumed the demand rate as linear with distinct phases (inception, maturation and declination) of a product's life cycle. Parija and Sarker (1999) addressed multi-ordering policy of procuring raw materials for a single manufacturing system. The model obtained a closed-form solution for minimizing the total cost in a multiple customer system. Betts and Johnston (2001) presented a new analysis of inventory reduction decisions, either by adapting JIT replenishment or component substitutions in a deterministic batch-sizing model in which inventory investment capital is finite and a decision variable. An overview of the related works is shown in Table 2.1.

Table 2.1. Characteristics of the selected papers

Authors	*Stage Type*	**Raw Materials**			**Inventory + cost**			**Finished Product**		*Shortage Cost*	*Solution Method*	*Solution Type*	*f*	*Const-raints*
		Type	*Pre-Processing*	*Purchase / Batch*	*RM*	*WIP*	*FP*	*Demand*	*Product*					
Golhar and Sarker (1992)	Single	Single	No	Single	Y	N	Y	Constant	Single	No	Math model	Optimal	No	No
H. M. Roger (1996)	Single	Single	No	Multiple	Y	N	Y	Constant	Single	No	Math model	Optimal	No	No
Parija and Sarker (1996)	Single	Single	No	Multiple	Y	N	Y	Constant	Single	No	Math model	Optimal	Yes	Integer
Nori and Sarker (1996)	Single	Multiple	No	Single	Y	N	Y	Constant	Multiple	No	Math model	Optimal	Yes	No
Gurnani, Akella and Lehoczky (1996)	Single, Multiple	Multiple	No	Single	Y	N	N	Stochastic	Single	No	Math model	Optimal	No	No
Nori and Sarker (1998)	Single	Single	No	Multiple	N	Y	N	Poisson	Single	No	Math (MP)	Optimal	No	Integer
Powell and Pyke (1998)	Single	Multiple	No	Single	N	Y	N	Stochastic	Single	No	Math model	Heuristic	No	Space
Sarker and Balan (1998)	Single	Single	No	Multiple	Y	Y	Y	Linear	Single	No	Math model	Optimal	No	No
Wilhelm and Pradip (1998)	Single	Multiple	No	Single	Y	N	Y	Stochastic	Single	No	Math (MP)	Optimal	No	Space
Fujiwara and Sangaradas (1998)	Multiple	Multiple	Yes	Single	N	Y	Y	Stochastic	Single	Allowed	Math (MP)	Iterative	No	No
Sarker and Balan (1999)	Multiple	Single	Yes	Multiple	Y	Y	Y	Linear	Single	No	Math model	Optimal	No	No
Parija and Sarker (1999)	Single	Single	No	Multiple	Y	N	Y	Constant	Single	No	Math model	Optimal	No	No
Park and Kim (1999)	Multiple	Single	No	Multiple	N	Y	Y	Constant	Single	No	Math (NL)	Heuristic	No	Due date
Ahmad and Saker (2002)	Single	Multiple	No	Multiple	Y	N	Y	Three-phase	Single	No	Math (L)	Optimal	No	No
Proposed Model	Mixed	Multiple	Yes	Multiple	Y	Y	Y	Constant	Single	No	Math (NL)	Optimal	Yes	Integer

[Note: N = No inventory; Y = Inventory and corresponding cost; MP = Markov Process; L = Linear; NL = Nonlinear; *f* = Conversion factor]

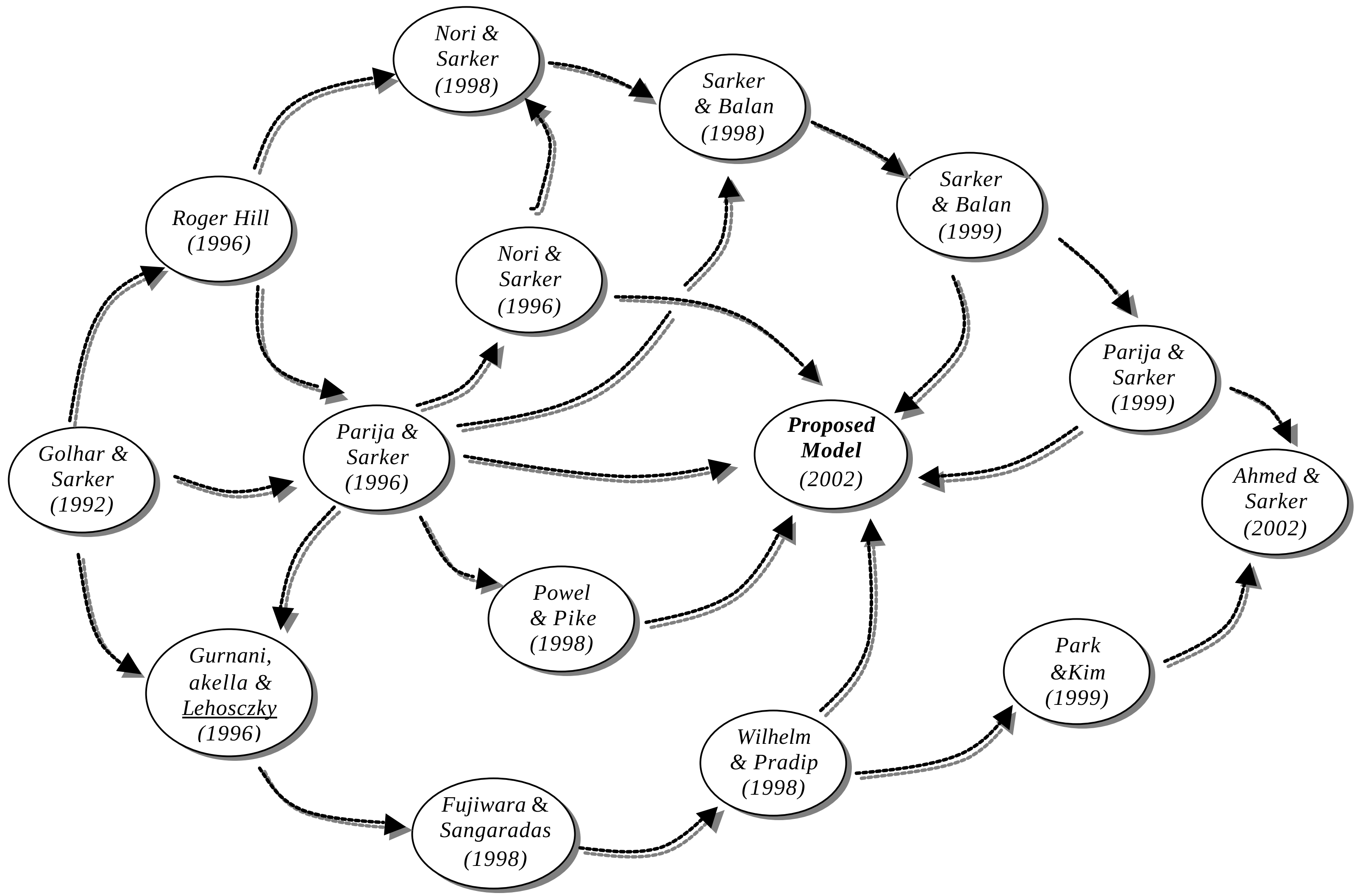

Figure 2.1 Flow pattern of the related research

2.3 Models for Assembly Systems

In supply chain assembly system model, most of the researchers discussed the impact of their inventory decisions on total cost function, and mathematical models are formulated to achieve the cost reductions by optimizing the system parameters and/or the operation sequences. Batch size, order rate, production lead-time, Work-in-process inventory, delivery lead times and development of suitable mathematical models for the solution are the major concern of the models.

Axsäter and Juntti (1996) presented the relative cost difference between the level stock or installation stock reorder policies in a multi-level inventory system for a constant demand. The echelon stock or installation stock policy may be advantageous depending on the structure of the system. Gurnani et al. (1996) considered an assembly problem of two critical components where demand of finished product is stochastic and delivery can be completed in the next cycle. A computational study is conducted to determine the effect of supplier costs and the probability of delivery on the optimal order policy. Rosenblatt and Lee (1996) considered assembly systems of highly expensive components (e. g. aerospace industry) with longer cycle time in which product's value increases the necessity to install additional parts and labor while moving along the assembly line. A branch-and-bound procedure is used to minimizes inventory holding cost and showed sequencing of ascending values of the ratios of the 'value added' to activity duration. Daning and Derek (1997) showed an assembly production/inventory system of constant demanded final product with backlogging allowed. The number of series systems is proportional to, in the extreme case, the factorial of n nodes in the assembly system. The lower bound and the optimal lot-size frequency policy for assembly systems with backlogging are also illustrated. Fujiwara *et al.* (1998) considered a Kanban-controlled, multi-stage production assembly system where raw materials acquisition lead times, reorder points, number of Kanbans, production lead time and demand arrival are the design parameters and variables. Mathematical model and simulation analyses are proposed to evaluate system performance measures. Powell and Pyke (1998) addressed unbalanced assembly systems with limited buffer capacity. Heuristic rules were developed to improve existing operations and to introduce new products. Wilhelm and Pradip (1998) considered the performance measure of a single-stage, single-product, and stochastic assembly system where raw materials are ordered under the material requirement planning (MRP) policy, and the inventory position process is a Markov renewal process and production lead-time is a random variable.

Sarker and Pan (1998) presented a mixed-model supply chain system with a close and open station assembly line format. The minimum total cost was found in the open-station system for a given line length and operation sequences. De Kok and Ton (1999) proposed multi-echelon assembly systems where components are pre-allocated to finished product. The comparison of proposed pre-allocation policies with several commonly used allocation policies was demonstrated in their research. Park and Kim (1999) focused on a make-to-order policy in an assembly system where delivery dates are constraints. A non-linear mathematical model was presented to minimize the holding costs of the inventories and the experimental results are tested. Park and Kim (2000) extend Park and Kim's (1999) model developing a mixed integer linear programming model. They incorporated the 'branch and bound' (B&B) algorithm to find the integer solutions. Agrawal and Cohen (2001) analyzed the cost-service performance and component stocking policies due to shortages and delayed production completion rates of finished product.

2.4 Shortfall in Past Researches

In manufacturing systems, work-in-process (WIP) inventories are required due to the small delivery quantity of the finished product or due to space or capital constraint of the firm. If work-in-process inventory is restricted to zero, i.e., no work-in-process inventories, then the output of the plant will be severely affected. Any manufacturing plant or assembly plant should be designed to have minimal, but not zero, WIP buffer capacity. For example, Gurani *et at*, (1996) considered as assembly problem with stochastic demand of finished product where assembly stage is free, i.e., the firm produces raw materials but sells complete sets. The firm follows a make-to-order basis production policy, so that the production is restricted only to the order(s) from the buyers.

In multi-stage assembly production systems, such as Powel and Pike (1998), where a number of raw materials are acquired from various suppliers and assembled into a single product, procures raw materials once in one cycle period. If the frequency of the raw materials procurement is simplified to a single shipment in each cycle, the raw materials inventory cost are assumed to be greater than the multiple procurement of raw materials.

In many researches, conversion ratio between the raw materials and finished product are not considered. Fujiwara *et al.* (1998) considered an assembly system from a group of raw materials, but did not specify the quantity required for each type of raw materials to produce a single finished product. Agrawal and Cohan (2001) determined optimal stocking policies, but did not consider conversion ratio of raw materials with finished product. In real example, a finished product may require more than unique quantity of each type of raw materials.

Any assembly system requires subassemblies or preprocessing of certain raw materials. If a firm wants to purchase all ready to assemble raw materials from other suppliers and confine the production system only in assembly plant, then the value addition of the product will be significantly low. Thus, the production policy will lead the firm less profitable assembly plant.

2.5 Connection of Past Inadequacies in Present Problem

Considering the above inadequacies, the present problem is to deal with a production facility that assembles finished product from a group of raw materials where some of the raw materials requires preprocessing before emerge into assembly line for production. The raw materials are procured through a number of shipments. The annual finished product demand is constant and known quantity. The finished product delivery order quantity is fixed. The objective of this research is to find the operation policy for raw materials procurement rate and finished product delivery rate. Conversion factors are allocated to each of the raw materials that convert into a single finished product. Once the final product batch size is known, the quantity of the raw materials batch size can be calculated using the corresponding conversion factors. Knowing the batch size and optimal procurement rate of raw materials, the scheduling of the raw materials procurement and order size per shipment can be planned.

The present research is an attempt to find a strategic policy for procurement and delivery rate of an assembly system where raw materials quantities are projected according to their conversion ratio to be converted into the final product. The production system holds work-in-process inventory for both assembly plant and processing plant and some of the raw materials are needed preprocessing before the assembly operation.

CHAPTER 3

INVENTORY OF ASSEMBLY SYSTEMS

This chapter discusses the production planning and the inventory policies of a supply chain system. In the inventory policies, several inventory mechanisms for the raw materials and finished product are described. In the operational policies, raw materials ordering policies to the suppliers, delivery policy to the customers, batch sizes, number of batches, the total quantity of finished goods production and total cost in each production cycle are determined. In the model, raw materials ordering quantity is consistent and dependent on demand.

3.1 General Processing

Consider a manufacturing system, which provides customers with finish products assembled from some feeding materials. The facility procures raw materials from outside suppliers and produces assembled products. The manufacturing facility carries raw materials and finished product inventories. To maintain a minimum inventory level, raw materials are ordered only during the time of production and must be consumed before the production is stopped. The annual demand for finished product is known and fixed. Assume the customer demands are satisfied and deliver the products periodically, the quantity of an equal size with a fixed interval of time. To keep the finished product inventory minimum, the quantity of finished product produced in one cycle is shipped to the customers in the same cycle.

The inventory pattern of an assembling (manufacturing) plant, in a supply chain system is shown in Figure 3.1. In the model, the *unfinished raw materials* arrive during the production time in the processing stage and the *ready raw materials* arrive during the production time in the assembly stage. After processing, the *unfinished raw materials* are converted to *processed raw materials* and transferred to the buffer stock prior to going to the assembly plant. However, the inventory storages and the productions of defective products are not considered. The inventory pattern in a Supply Chain System is shown in Figure 3.1.

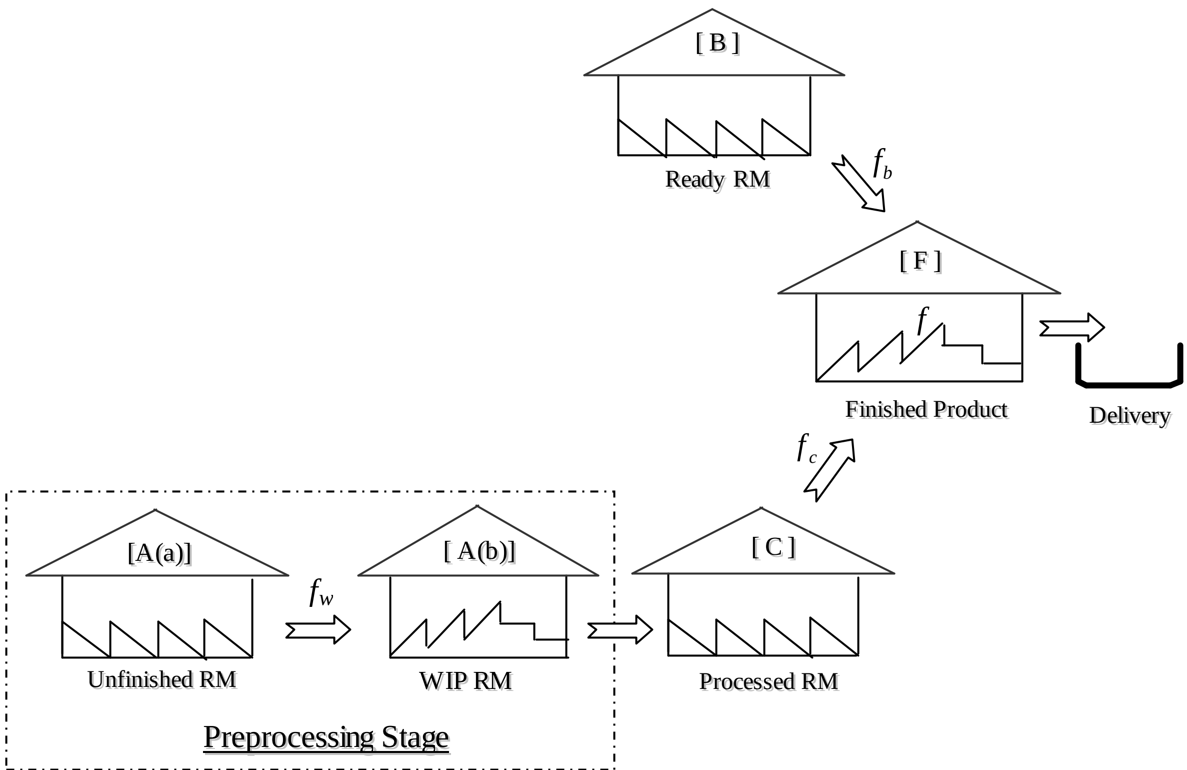

Figure 3.1. Inventory pattern in a supply chain system

3.2 Inventory Systems and Relationship

The production process is described in Figure 3.1. Inventories that build in the system are primarily two kinds: inventory at processing stage and inventory at assembly stage. The inventory at processing stage consists of *unfinished raw materials* inventory

[A(a)], and Work-in-process inventory at processing stage [A(b)]. The inventory at assembly stage is comprised of *ready raw materials* inventory [B], buffer stock inventory [C], and finished product inventory [F]. Production rate at processing stage and assembly stage are assumed to be fixed and higher than the demand rates and thus it builds inventory and avoids shortages in the system.

There are three conversion factors with three conversion processes associated with assembly model. The conversion processes include conversion of, (I) *ready raw materials* to finished product, (II) *unfinished raw materials* to *processed raw materials*, and (III) *processed raw materials* to finished product. The conversion factors determine the relationship between finished product and raw materials. If the yearly demand of finished product are known and fixed, then the yearly demand of raw materials are determined by multiplying the conversion factor with the amount of finished product. Assume the yearly demand for finished product and raw materials are D_F and D_R, then their relationship using the conversion factor, f, is stated as $f = \frac{D_F}{D_R}$.

3.3 Assumptions

The following are the assumptions made to formulate this model

1. No shortages is allowed in the system at any time.
2. Production rate is higher than demand rate so the products are accumulated in the system. Production capacity is fixed.
3. The WIP areas can accommodated the WIP inventory.
4. Initially the system contains no stock.
5. Shipment size (lot) is a known quantity.
6. Finished product demand is nonnegative and constant.

3.4 Notation

The notations used in this model are two kinds, (i) parameters, which are known and given values; (ii) variables, which are unknown. The objective of this model is to determine the variables. The following are the parameters and variables used in this model.

Parameters

A_F = Setup (assembling) cost at assembly stage, $/batch

A_w = Setup (processing) cost at processing stage, $/batch

D_A = Demand of *unfinished raw materials*, in period T, units/year

D_B = Demand of *ready raw materials,* in period T, units/year

D_C = Demand of *processed raw materials,* in period T, units/year

D_F = Demand of finished product, in period T, units/year

f_w = Conversion factor of *unfinished raw materials* to *processed raw materials*,

$$= \frac{D_A}{D_C} = \frac{Q_A}{Q_c}$$

f_b = Conversion factor of *ready raw materials* to finished product,

$$= \frac{D_F}{D_B} = \frac{Q_F}{Q_B}$$

f_c = Conversion factor of *processed raw materials* to finished product,

$$= \frac{D_F}{D_c} = \frac{Q_F}{Q_c}$$

I_{avg} = Average finished goods inventory, units

H_A = *Unfinished raw material* carrying cost, $/unit/year

H_B = Ready Raw material carrying cost, $/unit/year

H_C = *Processed raw material* carrying cost at buffer stock, \$/unit/year

H_F = Finished goods carrying cost, \$/unit/year

H_w = *Processed raw material* carrying cost at processing stage, \$/unit/year

K_A = Setup (ordering) cost of *ready raw materials*, \$/order

K_B = Setup (ordering) cost of *unfinished raw materials*, \$/order

L_1 = Time between shipments of *processed raw materials* to buffer, units

L_2 = Time between successive shipments of *finished product*, units

P_F = Production rate at assembly stage, units/year

P_w = Production rate at processing stage, units/year

S_A = Shipping cost for *unfinished raw materials*, \$/order

S_B = Shipping cost for *ready raw materials*, \$/order

S_C = Shipping cost for *processed raw materials* to buffer stock, \$/order

S_F = Shipping cost for finished products to customers, \$/order

T = Cycle time in assembly stage for finished products, year

T_1 = Production time (uptime) in assembly stage, year

T_0 = Cycle time in processing stage for *processed raw materials*, year

T_w = Production time (uptime) in processing stage, year

TC = Total cost of finished products in a supply chain system, \$/year

TC_A = Cost of *unfinished raw materials* and *work-in-process* inventory, \$/year

TC_B = Cost of *ready raw material* inventory, \$/year

TC_C = Cost of *processed raw materials* inventory at buffer stock, \$/year

TC_F = Cost of finished products inventory, \$/year

x = Quantity of finished goods shipment at a fixed interval T/m, units/shipment

Y_1 = Quantity of *processed raw materials* produced during L period

Y_2 = Quantity of finished products produced during *T/m* period

$Y_1 - Q_0$ = *Processed raw materials* inventory buildup after each shipment at uptime T_w,

= $(P_w/D_c - 1)Q_0$, units/shipment

$Y_2 - x$ = Finished product inventory after each shipment during uptime, units/shipment

Variables

m = Number of finished products delivery to customers, units/ batch

n_a = Number of *unfinished raw materials* shipment, units/ batch

n_b = Number of *ready raw materials* shipment per batch, units/ batch

n_c = Number of *processed raw materials* shipment to buffer stock, units/ batch

Q_0 = Quantity of *processed raw materials* shipped to buffer stock, units/shipment

= Q_C/n_c

Q_1 = Quantity of *unfinished raw materials* ordered each time, units/order

= Q_A/n_a

Q_2 = Quantity of *ready raw materials* ordered each time, units/order

= Q_B/n_b

Q_A = Quantity of *unfinished raw materials* per setup, units/batch

Q_B = Quantity of *ready raw materials* per setup, units/batch

Q_C = Quantity of *processed raw materials* per setup, units/batch

Q_{avg} = Average Work-in-process inventory, units

Q_F = Quantity of finished products shipped per setup, in period T, units/batch

$$= \frac{f_b}{f_c} = \frac{D_F}{D_j} = \frac{Q_F}{Q_j} \quad \text{[for } j = b, c\text{]}$$

$Q_C(t)$ = On-hand *processed raw materials* at time t; $Q_C(t) = Q_P(t) - Q_w(t)$, in units

$Q_F(t)$ = On-hand finished products at any time t; $Q_F(t) = Q_M(t) - Q_S(t)$, in units

$Q_P(t)$ = Quantity of *processed raw materials* at any time t, in units

$Q_w(t)$ = Quantity of *processed raw materials* shipped at time t, in units

$Q_M(t)$ = Quantity of finished products produced at time t, in units

$Q_S(t)$ = Quantity of finished products shipped at time t, in units

3.5 Inventories at Processing Stage

The section discusses two categories of inventory costs at the processing stage. The first category is the cost related to holding and setup of *unfinished raw materials*. The second category is the cost of *work-in-process* inventory at the preprocessing stage. The parameters of the inventories at the preprocessing stage are shown in Figure 3.2.

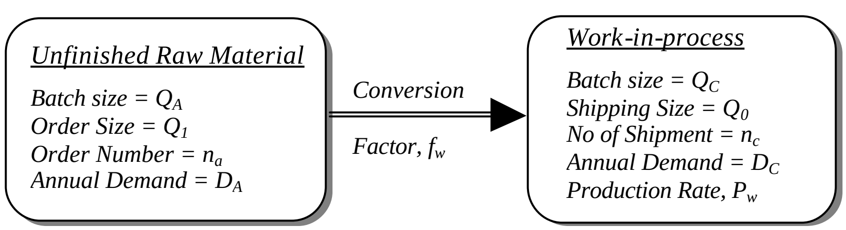

Figure 3.2. Inventory parameters at processing stage

3.5.1 Cost of Unfinished Raw Materials Inventory

In the model, *unfinished raw materials* are converted into finished products in two phases. In the first phase, *unfinished raw materials* are reformed into *processed raw materials* at the processing stage and in the next phase, the *processed raw materials* are converted into finished products at the assembly stage.

Assume a conversion factor, f_w, indicates 1 unit of *processed raw materials* requires f_w units of *unfinished raw materials*. If the *processed raw materials* batch size Q_C and yearly demand D_C were identified then, the relationship of *unfinished raw materials* batch size Q_A and yearly demand D_A would be $f_w = D_C / D_A = Q_C / Q_A$. The pattern of the *unfinished raw materials* inventory is shown in Figure 3.3(a).

If batch size Q_A is ordered in n_a shipments with equal quantity $Q_1 = Q_A / n_a$ at an equal interval T_w / n_a, the total number of order $n_a = D_A / Q_1$. The number of cycles over the year is D_A / Q_A. Since the production time at the preprocessing stage is T_w and the cycle time is T_0, for n_a orders of *unfinished raw materials*, the average inventory held per cycle is $Q_{ave} = (Q_1/2)(T_w/T_0)$. Let the carrying cost be H_A; the total inventory carrying cost over the year is $(Q_1/2)(T_w/T_0)H_A$ dollars/year. If the ordering cost is K_A dollars/shipment, then total ordering cost is $(D_A/Q_1)K_A$ dollars/year. Similarly, if the shipment cost is S_A dollars/order, then the shipment cost in one year is $(D_A/Q_1)S_A$ dollars/year.

The total cost associated with *unfinished raw materials* inventory, TC_1 is stated as

$$TC_1 = \left(\frac{D_A}{Q_1}\right)K_A + \left(\frac{D_A}{Q_1}\right)S_A + Q_{avg}H_A = \left(\frac{D_A}{Q_1}\right)K_A + \left(\frac{D_A}{Q_1}\right)S_A + \frac{Q_1}{2}\frac{T_w}{T_0}H_A \qquad (1)$$

in which $Q_{ave} = (Q_1/2)(T_w/T_0)$ and substituting $Q_1 = Q_A/n_a$, the Equation (1) may be written as

$$TC_1 = \left(\frac{D_A}{Q_A}\right)\{n_a(K_A + S_A)\} + \frac{1}{2}\frac{Q_A}{n_a}\frac{T_w}{T_0}H_A \tag{2}$$

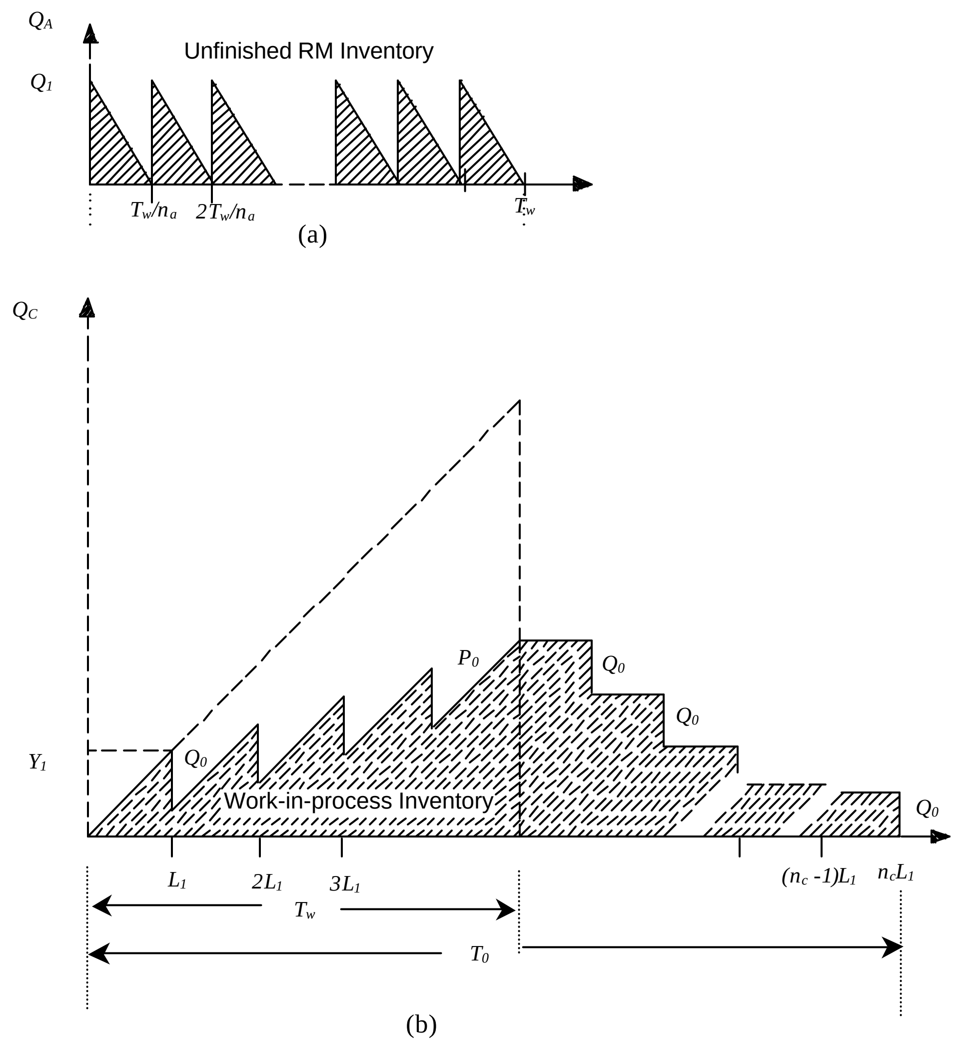

Figure 3.3. Unfinished raw materials and work-in-process inventory

3.5.2 Cost of Work-in-process Inventory at the Processing Stage

The *unfinished raw materials* are processed at the processing stage and then, shipped to the buffer inventory prior to the assembly stage. The production at the processing stage stops when the *processed raw materials* can satisfy the requirement at the assembly stage.

The characteristic of the *processed raw materials* inventory at the processing stage is shown in Figure 3.3(b). In the processing stage, the production time is denoted as T_w and the cycle time is T_0. The amount of *processed raw materials* are produced in a batch at a rate P_w, is Q_C. The quantity Q_C is transferred to buffer inventory by n_c number of shipment with a size $Q_0=Q_C/n_c$ and at a fixed interval $L=T_0/n_c$. Let the manufacturing setup cost for the preprocessing stage be A_w, then, the total manufacturing setup cost per year is $(D_C/Q_C)A_w$. Assume, the carrying cost in the processing stage is H_w. Hence, the total cost associated with the average Work-in-process inventory in the processing stage, TC_2 is stated as

$$TC_2 = Q_{avg}H_w + \frac{D_C}{Q_C}A_w = \frac{Q_C}{2}\left(1+\frac{1}{n_c}-\frac{D_C}{P_w}\right)H_w + \frac{D_C}{Q_C}A_w \qquad (3)$$

in which $Q_{avg} = Q_C/2\left(1+1/n_c - T_w/T_0\right)$ as obtained from Equation (A.7) in Appendix A.

Thus, combining Equation (2) and Eq. (3), the combined cost of *unfinished raw materials* inventory and the 'Work-in-process raw materials' inventory, TC_A, is

$$TC_A = TC_1 + TC_2$$

$$= \left(\frac{D_A}{Q_A}\right)\{n_a(K_A+S_A)\} + \frac{1}{2}\frac{Q_A}{n_a}\frac{T_w}{T_0}H_A + \frac{Q_C}{2}\left(1+\frac{1}{n_c}-\frac{D_C}{P_w}\right)H_w + \frac{D_C}{Q_C}A_w. \qquad (4)$$

By substituting $Q_A=Q_C/f_w$, Equation (4) can be expressed as

$$TC_A = \left(\frac{D_A}{Q_A}\right)\{n_a(K_A + S_A)\} + \frac{1}{2}\frac{Q_C}{n_a f_w}\frac{D_C}{P_w}H_A + \frac{Q_C}{2}\left(1 + \frac{1}{n_c} - \frac{D_C}{P_w}\right)H_w + \frac{D_C}{Q_C}A_w. \quad (5)$$

3.6 Inventories at Assembling Stage

The costs associated with the assembly stage inventory are clustered into three classes. First, the inventory costs due to holding and set up (comprises of ordering and shipping) of *ready raw materials*. Second, the inventory costs for shipping and holding of *processed raw materials*, and the third category contains the cost of *finished products* inventory. Figure 6 shows the inventory parameters at assembly stage.

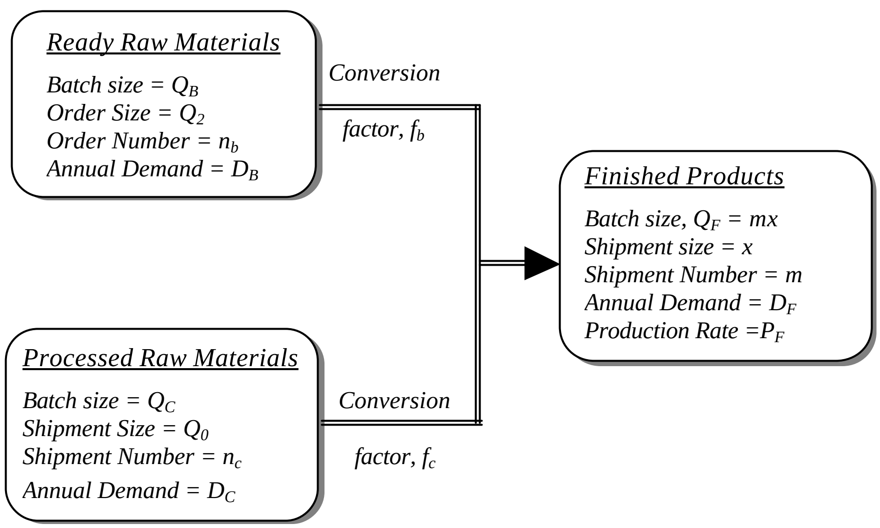

Figure 3.4. Inventory parameters at assembly stage

3.6.1 Cost of Ready Raw Materials Inventory

In the model, *ready raw materials* are converted to finished products and arrived in batches during the production in the assembly stage. Assume a conversion factor, f_b, indicates 1 unit of finished product requires f_b units of *ready raw materials*. If the finished products batch size Q_F and yearly demand D_F is known then, the relationship of *ready raw materials* batch size Q_B and yearly demand D_B would be $f_b = D_F / D_B = Q_F / Q_B$.

The pattern of the ready raw material inventory is shown in Figure 3.5(a). The procured *ready raw materials* batch quantity, Q_B, in cycle time T, is assumed exactly match with the quantity required to meet the demand of finished products. The batch size, Q_B, is ordered in n_b shipments with equal quantity $Q_2 = Q_B / n_b$ at an equal interval T_1 / n_b . The number of cycle over the year is D_B / Q_B and the number of shipments over the year is D_B / Q_2 . Let the carrying cost be H_B (\$/unit/year). For n_b orders, the average inventory for *ready raw materials* per cycle is $Q^B_{avg} = (Q_2/2)(T_1/T)$, the total inventory carrying cost over the year is $(Q_2/2)(T_1/T)H_B$ dollars/year. If the ordering cost is K_B dollars per shipment, then total yearly ordering cost is $(D_B/Q_2)K_B$ dollars/year. Similarly, if the shipment cost is S_B dollars per order, then the shipment cost in one year is $(D_B/Q_2)S_B$ dollars/year.

Hence, the total cost associated with *ready raw materials* inventory, TC_B is given by

$$TC_B = \left(\frac{D_B}{Q_2}\right)K_B + \left(\frac{D_B}{Q_2}\right)S_B + Q^B_{avg}H_B = \left(\frac{D_B}{Q_2}\right)K_B + \left(\frac{D_B}{Q_2}\right)S_B + \frac{Q_2}{2}\frac{T_1}{T}H_B \qquad (6)$$

in which $Q^B_{avg} = (Q_2/2)(T_1/T)$ and substituting $Q_2 = Q_B / n_b$, then Equation (6) may be written as

$$TC_B = \left(\frac{D_B}{Q_B}\right)\{n_b(K_B + S_B)\} + \frac{1}{2}\frac{Q_B}{n_b}\frac{T_1}{T}H_B . \qquad (7)$$

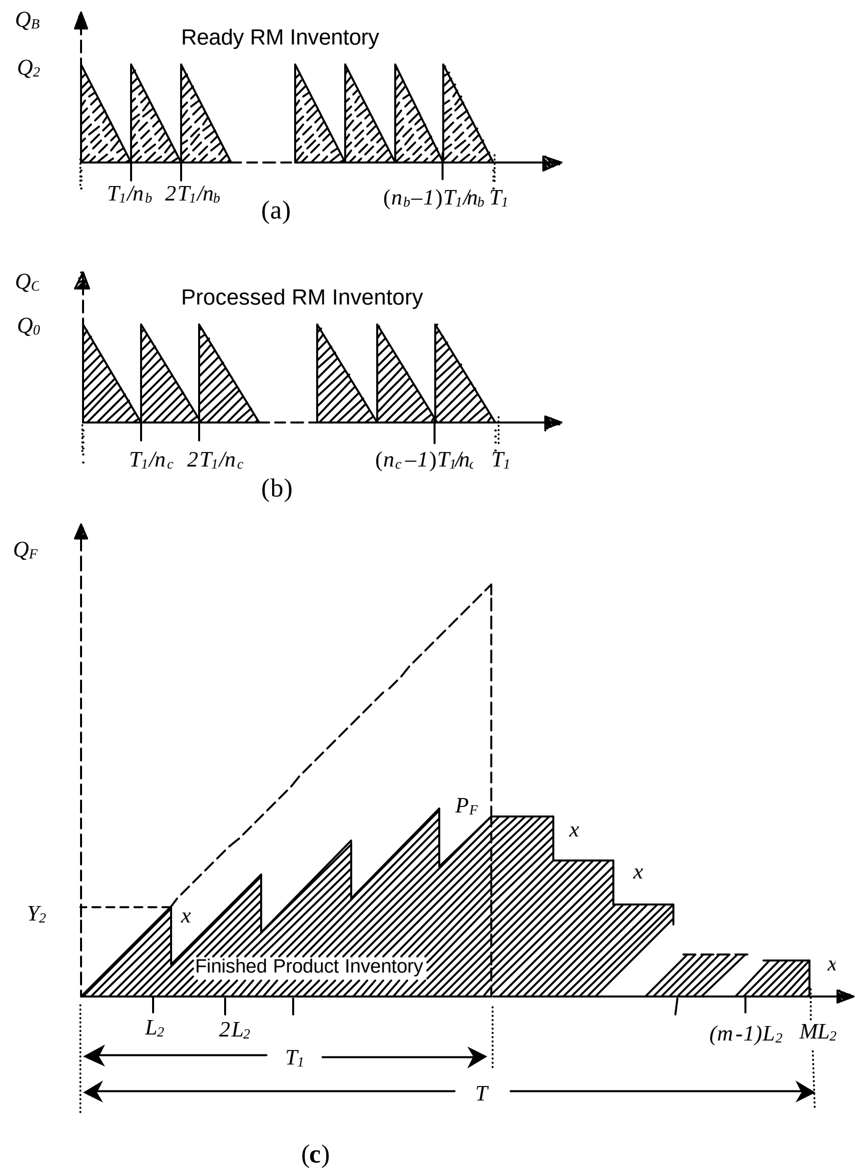

Figure 3.5. Different types of inventories at assembly stage

3.6.2 Cost of Processed Raw Materials Inventory

In this stage, the *processed raw materials* are shipped in a fixed quantity from processing stage to the buffer inventory during the production time at assembly stage, T_1. The arrival quantity is directly related to finished products delivery quantity. To meet the finished products yearly demand D_F, batch quantity Q_F, the *processed raw materials* yearly demand D_C and batch size Q_C, can be determined by assuming a relation of a conversion factor. Conversion factor, f_c, signifies the production of a single unit of finished product which requires f_c units of *processed raw materials*. The relationship can be established as $f_c = D_F / D_C = Q_F / Q_C$. The pattern of the *processed raw materials* inventory is shown in Figure 3.5(b).

The batch of Q_C units is transferred in n_c full shipments with a quantity of Q_0 units per shipment at an interval of T_1/n_c years. If the demand of *processed raw materials* is D_C units/year, the total number of shipments over the year is D_C/Q_0 and the number of cycles/year is D_C/Q_C. Hence, average inventory for n_c orders of *processed raw materials* $Q_{avg}^C = (Q_0/2)(T_1/T)$ and yearly inventory carrying cost is $(Q_0/2)(T_1/T)H_C$ dollars/year, where H_C is the carrying cost in dollars/unit/year. Similarly, denoting S_C as the shipment cost in dollars/order, for D_C/Q_0 shipments, the yearly shipment cost is $\left(\frac{D_C}{Q_0}\right)S_C$ dollars/year. The total cost associated with *processed raw materials* inventory, TC_C is

$$TC_C = \left(\frac{D_C}{Q_0}\right)S_C + Q_{avg}^c H_C = \left(\frac{D_C}{Q_0}\right)S_C + \frac{Q_0}{2}\frac{T_1}{T}H_C, \qquad (8)$$

in which $Q_{avg}^C = \left(\frac{Q_0}{2}\right)\frac{T_1}{T}$, and substituting $Q_0 = Q_C/n_c$, TC_C may be written as

$$TC_C = \left(\frac{D_C}{Q_C}\right)\{n_c S_C\} + \frac{Q_C}{2n_c}\frac{T_1}{T}H_C. \qquad (9)$$

3.6.3 Cost of Finished Product Inventory

In the assembly stage, finished products are continuously produced in batches at a constant production rate P_F. When sufficient quantities of finished goods are produced enough to meet the demand, production stops. There is no additional inventory buildup during downtime, $(T - T_1)$.

The behavior of finished goods inventory over the cycle time T is shown in Figure 3.5(c). The yearly demand for finished product is constant and is denoted by D_F, and the batch size at each cycle is Q_F. There are x units of finished products delivered in each shipment. If the finished product batch, Q_F, is distributed completely by m integer number of full shipment during the corresponding cycle, then $Q_F = mx$.

The total number of shipments per year is D_F / x and there are D_F / Q_F number of cycles in one year. If the shipment cost is S_F dollars/shipment and manufacturing setup cost is A_F, then the total shipment cost for (D_F / x) shipments and manufacturing setup cost over the year is $(D_F / x)\, S_F$ and $(D_F / Q_F) A_F$ dollars/year, respectively.

The total cost associated with finished product inventory TC_F is stated as

$$TC_F = \left(\frac{D_F}{x}\right) S_F + \left(\frac{D_F}{Q_F}\right) A_F + I_{avg} H_F \qquad (10)$$

where $I_{avg} = \frac{mx}{2}\left(1 + \frac{1}{m} - \frac{D_F}{P_F}\right)$, which is obtained from Equation (B.7) in Appendix B.

Thus the Equation (9) is expressed as

$$TC_F = \frac{D_F}{x} S_F + \left(\frac{D_F}{Q_F}\right) A_F + \frac{mx}{2}\left(1 + \frac{1}{m} - \frac{D_F}{P_F}\right) H_F \,. \qquad (11)$$

3.7 Total Inventory Cost

A general expression for the total yearly cost function, TC , may be obtained by combining Equations (5), (7), (9) and (11):

$$TC = TC_A + TC_B + TC_C + TC_D$$

$$= \left(\frac{D_A}{Q_A}\right)\{n_a(K_A + S_A)\} + \frac{1}{2}\frac{Q_C}{n_a f_w}\frac{D_C}{P_w}H_A + \frac{Q_C}{2}\left(1 + \frac{1}{n_c} - \frac{D_C}{P_w}\right)H_w + \frac{D_C}{Q_C}A_w$$

$$+ \left(\frac{D_B}{Q_B}\right)\{n_b(K_B + S_B)\} + \frac{1}{2}\frac{Q_B}{n_b}\frac{T_1}{T}H_B + \left(\frac{D_C}{Q_C}\right)(n_c S_C) + \frac{Q_C}{2n_c}\frac{T_1}{T}H_C + \frac{D_F}{x}S_F$$

$$+ \frac{D_F}{Q_F}A_F + \left\{\frac{mx}{2}\left(1 + \frac{1}{m} - \frac{D_F}{P_F}\right)\right\}H_F . \qquad (12)$$

Upon substituting $\frac{D_A}{Q_A} = \frac{D_B}{Q_B} = \frac{D_C}{Q_C} = \frac{D_F}{Q_F} = \frac{D_F}{mx}$, Equation (12) becomes

$$TC(n_a, n_b, n_c, m) = \frac{D_F}{mx}\{n_a(K_A + S_A) + n_b(K_B + S_B) + n_c S_C + A_w + A_F\} + \frac{Q_C}{2n_a}\left(\frac{H_A D_C}{f_w P_w}\right)$$

$$+ \frac{Q_C}{2n_c}\left(\frac{T_1 H_C}{T} + H_w\right) + \frac{Q_B}{2n_b}\left(\frac{T_1 H_B}{T}\right) + \frac{mx}{2}\left(1 - \frac{D_F}{P_F}\right)H_F + \frac{Q_C H_w}{2}\left(1 - \frac{D_C}{P_w}\right)$$

$$+ \frac{xH_F}{2} + \frac{D_F S_F}{x} . \qquad (13)$$

For the assembly stage, at a production rate, P_F, the quantity of finished products produced in uptime, T_1, must satisfy the customers demand during the cycle time, T . Hence, $T_1 = T\frac{D_F}{P_F}$ and substituting $Q_C = Q_F / f_c$, $D_C = D_F / f_c$ and $Q_B = Q_F / f_b$. Since $Q_F = mx$, the Equation (13) can be modified as

$$TC(n_a, n_b, n_c, m) = \frac{D_F}{mx}\{n_a(K_A + S_A) + n_b(K_B + S_B) + n_c S_C + A_w + A_F\}$$

$$+\frac{mx}{2n_a}\left(\frac{H_A D_F}{f_w f_c^2 P_w}\right) + \frac{mx}{2n_b}\left(\frac{D_F H_B}{f_b P_F}\right) + \frac{mx}{2n_c f_c}\left(\frac{D_F H_C}{P_F} + H_w\right) + \frac{mx}{2}\left(1 - \frac{D_F}{P_F}\right)H_F$$

$$+\frac{mx}{2f_c}\left(1 - \frac{D_F}{f_c P_w}\right)H_w + \left(\frac{D_F S_F}{x} + \frac{xH_F}{2}\right). \quad (14)$$

In total inventory cost, Equation (14), the quantity of finished goods produced at each cycle over a period T, $Q_F = mx$, and the raw materials batch sizes are $Q_A = n_a Q_1$, $Q_B = n_b Q_2$, and $Q_C = n_c Q_0$. The shipment size of all raw materials and finished products are set and consistent. The batch quantities of raw materials and finished products are completely transferred by the end of last shipment and by the last delivery. The number of delivery, m, and raw materials ordering frequency n_a, n_b and n_c are the critical variables, and D_F, K_A, S_A, K_B, S_B, S_C, S_F, P_F, P_w, f_w, f_b, f_c, H_A, H_B, H_C, H_w, H_F and x are the constraints.

Simplifying, the Equation (14):

$$TC = \frac{1}{m}(An_a + Bn_b + Cn_c + f) + m\left(\frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y\right) + g\ , \quad (15)$$

in which

$$A = \frac{(K_A + S_A)D_F}{x}, \quad (15a)$$

$$B = \frac{(K_B + S_B)D_F}{x}, \quad (15b)$$

$$C = \frac{S_C D_F}{x}, \quad (15c)$$

$$D = \frac{xH_A D_F}{2f_w f_c^2 P_w}, \quad (15d)$$

$$E = \frac{xD_F H_B}{2 f_b P_F}, \tag{15e}$$

$$F = \frac{x}{2 f_c}\left(\frac{D_F H_C}{P_F} + H_w \right) , \tag{15f}$$

$$y = \frac{x}{2}\left\{ \left(1 - \frac{D_F}{P_F}\right) H_F + \frac{1}{f_c}\left(1 - \frac{D_F}{f_c P_w}\right) H_w \right\}, \tag{15g}$$

$$g = \frac{D_F S_F}{x} + \frac{x H_F}{2}, \text{ and} \tag{15h}$$

$$f = \frac{D_F}{x}(A_w + A_F). \tag{15i}$$

3.8 Problem Formulation for Single-Stage Supply Chain Model

A general cost model consists of a number of supplied raw materials, processing plant, assembly plant and finished products in a single stage assembly production line, is developed. The objective of this problem is to minimize the total expected cost combining all the individual cost functions of the model. The total cost is a function of outgoing shipment number, m, and the number of raw materials procurement, n_a, n_b, n_c, per cycle. Substituting the term (15a-i) in Equation (15), the problem is formulated as a non-linear integer programming for a fixed delivery supply system.

The single stage supply chain system problem is as follows: Find m, n_a, n_b, n_c, so as to

minimize $$TC(m, n_a, n_b, n_c) = \frac{1}{m}\left(An_a + Bn_b + Cn_c + f\right) + m\left(\frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y\right) + g \tag{16}$$

subject to $m, n_a, n_b, n_c \geq 1$ and integers.

The constraint assures that the optimum solution is the integer value. Unlike the linear programming problem, the nonlinear programs, in general, have more difficulties in finding the solution. The simplex-type algorithm is capable of examining only finite set of extreme points that are highly efficient for linear problem. For nonlinear programs, the optimum point may not lie at an extreme point; thus a simplex-type algorithm could not solve this problem. Furthermore, it could be possible to find an algorithm that would search over all boundary points of any given feasible region. The local optimum may not be the overall optimum of a nonlinear problem, which is an additional difficulty in finding the optimal solution for the problem. The next section will develop the solution methodology.

3.9 The Solution Methodology

The total cost function that is developed for a single-stage assembly type supply chain system is a nonlinear integer-programming (NLIP) problem. To solve the NLIP, the most effective general-purpose optimal algorithm is the branch and bound technique. It is a way of systematically enumerating the feasible solution of a nonlinear integer-programming (NLIP) problem such that the optimal integer solution is found. The problem that made the solution difficult is the fact that the variables are restricted to integers.

To obtain the solution of the present problem, the procedure starts relaxing the integer restriction of NLIP problem. If the solution satisfies the integer constraints, then the optimal solution of the NLIP problem is attained and the procedure stops. If the result is not integer, the relaxation solution provides a lower bound (minimization problems) to the optimal solution. Then, the problem needs a new approach to find the optimal integer solution for the variables of the original problem. The present problem generates two types of variables, *basic variables* and *non-basic variables*. The results of the unconstraint solution reveal that the *non-basic variable(s)* is the function of the *basic variable(s)*. Thus, the *basic* and *non-basic variable(s)* need to solve separately.

If the *basic variables* are integers, the solution procedure begins to find the integer values for the *non-basic variables*. When integer solutions are achieved that provides an upper bound to the optimal solution of NLIP. Any sub nodes that exceed this bound are considered fathomed or dropped from further investigation. The search procedure is continued until all the nodes are considered. If the *basic* are not integers, the solution starts by setting lower bound and upper bound to the non-integer value and then solve the problem one-by-one taking the lower bound case and upper bound case.

Shaojun Wang (2001) presented B&B solution technique in six steps to obtain the integer values of the solutions for MINLP problem. Upon modifying Wang (2001) technique, the algorithm for solving the current NLIP problem is presented in eight steps. A flow chart is presented following the algorithm to set NLIP solution. The flow chart is illustrated in Figure 3.6.

3.9.1 Algorithm 3.1: Branch and Bound Algorithm for Solving NLIP

Step 1. Solve the original single-stage assembly type supply chain NLIP problem by relaxing the integer constraints. Verify whether the variables are unique or dependent on other variables. Identify independent variables as *basic variables* and dependent variables as *non-basic variables*. Set the result of the optimal solution of NLIP problem as the lower bound, $Z_L = TC$;

Step 2. If the solutions are integer, process stops and the optimal solution is obtained; if not, set the upper bound $Z_U \to \infty$;

Step 3. If the *basic variable(s)* is not integer, set the upper bound and lower bound integer value. Solve the *non-basic variables* taking one-by-one the upper bound integer values and lower bound integer values of each of the *basic variable(s)*. If the *non-basic variable(s)* is integer, procedure stops. The optimal solution is obtained and set $Z_U \leftarrow Z_L$.

Step 4. If the *non-basic variable(s)* is not integer, establish alternative integer solutions from the nearer point. Form sets with the lower values and upper integer values of the *basic variable(s)* and solve each set individually. Form the subsets by adding constraints one at a time and then solve the resulting NLIP relaxed subsets one-by-one; verify the feasibility of the solutions and continue the feasible solutions to form the nodes.

Step 5. Compare Z_L with Z_U; If $Z_L \leq Z_U$, update Z_U by $Z_U \leftarrow Z_L$;

Step 6. Search for the most potential node for further fathoming;

Step 7. Repeat Steps 2 – 4. Find the most promising node of integer values giving the minimum cost from each parent set.

Step 8. Compare the results of the promising nodes from each of the parent set and choose the node of a set that gives the minimum cost.

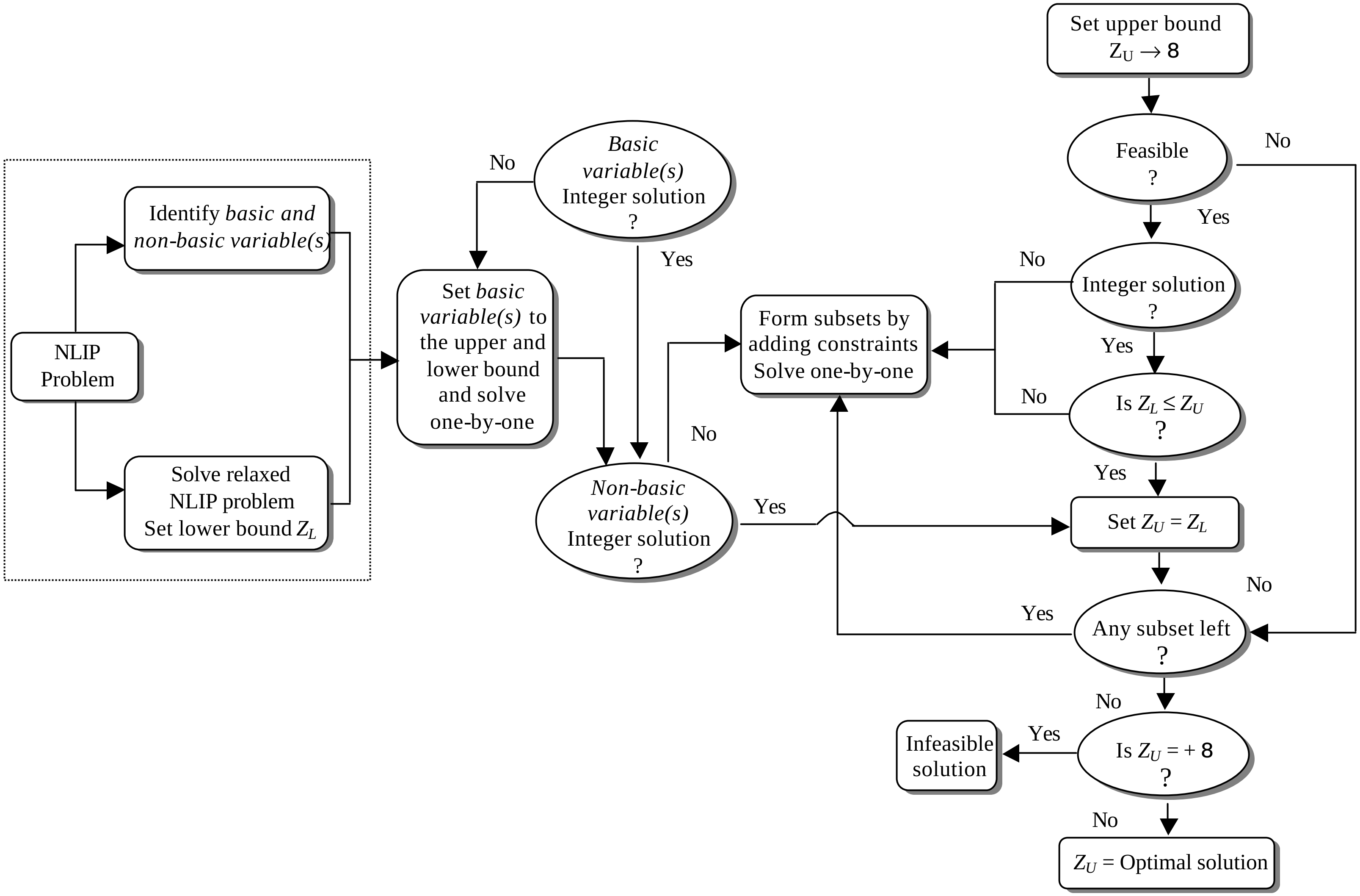

Figure 3.6. Flowchart for branch-and-bound technique

3.9.2 Total Cost Function

For a single-stage assembly type supply chain system, the total cost by the relaxed NLIP (nonlinear integer programming) problem is stated as

$$TC(n^*) = 2\left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \sqrt{f\,y}\right) + g \tag{17}$$

where $n^* = (n_a^*, n_b^*, n_c^*, m^*)$ and $A, B, C, D, E, F, f, y, g$ are given in Equations (15a-i).

Proof: By replacing TC with Z, Equation (16) can be rewritten as

$$Z(m, n_a, n_b, n_c) = \frac{1}{m}\left(An_a + Bn_b + Cn_c + f\right) + m\left(\frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y\right) + g \tag{18}$$

subject to $m, n_a, n_b, n_c \geq 1$ and integers.

From Equation (17), the partial derivatives with respect to m, n_a, n_b, n_c yields

$$\frac{\partial Z}{\partial n_a} = \frac{1}{m}A - m\frac{D}{n_a^{\ 2}} = 0 \quad \Rightarrow \quad n_a^{\ *} = m\sqrt{\frac{D}{A}}, \tag{19a}$$

$$\frac{\partial Z}{\partial n_b} = \frac{1}{m}B - m\frac{E}{n_b^{\ 2}} = 0 \quad \Rightarrow \quad n_b^{\ *} = m\sqrt{\frac{E}{B}}, \tag{19b}$$

$$\frac{\partial Z}{\partial n_c} = \frac{1}{m}C - m\frac{F}{n_c^{\ 2}} = 0 \quad \Rightarrow \quad n_c^{\ *} = m\sqrt{\frac{F}{C}}, \text{ and} \tag{19c}$$

$$\frac{\partial Z}{\partial m} = -\frac{1}{m^2}\left(An_a + Bn_b + Cn_c + f\right) + \frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y = 0\ . \tag{19d}$$

By substituting the values of n_a^*, n_b^*, n_c^* into Equation (19d) is given by

$$m^* = \sqrt{\frac{f}{y}} \tag{20}$$

Substitution of the value of Equations (15g) and (15i) in Equation (19d):

$$m^* = \frac{1}{x}\sqrt{\frac{2D_F(A_w + A_F)}{\left(1-\frac{D_F}{P_F}\right)H_F + \frac{1}{f_c}\left(1-\frac{D_F}{f_c P_w}\right)H_w}} \tag{21}$$

Replacing the values of A, B, C, D, E and F, Equations (19a), (19b) and (19c):

$$n_a^* = mx\sqrt{\frac{H_A}{2 f_w f_c^2 P_w (K_A + S_A)}}, \tag{22a}$$

$$n_b^* = mx\sqrt{\frac{H_B}{2 f_b P_F (K_B + S_B)}}, \text{ and} \tag{22b}$$

$$n_c^* = mx\sqrt{\frac{1}{2 f_c S_C D_F}\left(\frac{H_C}{P_F} + H_w\right)} \tag{22c}$$

Here $n^* = (n_a^*, n_b^*, n_c^*, m^*)$ is a stationary point. Substituting n^* into Equation (18), the Z^* value is obtained as

$$Z(m, n_a, n_b, n_c) = \frac{1}{m}\left(Am\sqrt{\frac{D}{A}} + Bm\sqrt{\frac{E}{B}} + Cm\sqrt{\frac{F}{C}} + f\right) + m\left(\frac{D}{m\sqrt{\frac{D}{A}}} + \frac{E}{m\sqrt{\frac{E}{B}}} + \frac{F}{m\sqrt{\frac{F}{C}}} + y\right) + g$$

$$= \left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \frac{f}{m}\right) + \left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + my\right) + g \tag{22d}$$

in which $m = \sqrt{\frac{f}{y}}$ and the value of $A, B, C, D, E, F, f, y, g$ are known; therefore

$$Z(m, n_a, n_b, n_c) = \left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \sqrt{fy}\right) + \left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \sqrt{fy}\right) + g$$

$$= 2\left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \sqrt{fy}\right) + g\ . \tag{22e}$$

It can be shown that Equation (18) has a unique stationary n^* that is convex, corresponding to the global minimum (see Appendix C).

The parametric values of $A, B, C, D, E, F, f, y, g$ in Equation (22e) are illustrated in Equation (15a) to (15i). If the values of $A, B, C, D, E, F, f, y, g$ are replaced using the Equation (15a) to (15i), the function (22e) becomes

$$Z(m, n_a, n_b, n_c) = \sqrt{\frac{2H_A D_F^{\ 2}(K_A + S_A)}{f_w f_c^2 P_w}} + \sqrt{\frac{2H_B D_F^{\ 2}(K_B + S_B)}{f_b P_F}} + \sqrt{\frac{2S_C D_F}{f_c}\left(\frac{D_F H_C}{P_F} + H_w\right)}$$

$$+ \sqrt{2D_F (A_w + A_F)\left\{\left(1 - \frac{D_F}{P_F}\right)H_F + \frac{1}{f_c}\left(1 - \frac{D_F}{f_c P_w}\right)H_w\right\}} + \frac{D_F S_F}{x} + \frac{x H_F}{2}. \qquad (22f)$$

In Equation (22f), the values of the parameters are known and are non-variables. Once the values of the parameters are given, the total cost (Z-value) can be obtained using the Equation (22f).

CHAPTER 4

TEST PROBLEMS AND COMPUTATIONAL RESULTS

A single-stage assembly type supply chain system with a preprocessing plant and an assembly plant is considered in this model. A detail analysis of the solution procedure and calculations for a test problem are covered in this chapter. The computational result finds the raw materials procurements rates, finished product delivery rate and the total cost of a supply chain system for the relaxed NLIP problem. Branch and bound technique is used to obtain the integer solutions of the problem. A comparison between the relaxed and the integer solution and the alternate results of the problem are shown in a tabular form. Finally, the restricted and unrestricted results for a set of nine case problems are presented.

4.1 Numerical Example

The system parameters of this model includes the demand rate (D_F), production rates (P_w, P_F), transportation costs (S_A, S_B, S_C, S_D), setup (production) costs (A_w, A_F), setup (order) costs (K_A, K_B), holding costs (H_A, H_B, H_w, H_C, H_F), shipment size per order (x), and conversion factors (f_w, f_b, f_c). The parametric values of the test problem are given in Table 4.1.

In this problem, the first step is to find the optimum total cost of supply chain system for the relaxed NLIP problem using the Equation (22e). The next step is to determine the number of procurements, order size, batch size for each raw materials, WIP, and finished product delivery rate for the given test problem. The solutions of the model are in closed forms, so the total cost, procurement rate and delivery rate can be obtained directly by using the parametric values of the test problem.

Table 4.1. The parametric values of the model

Demand (units/year)	D_F	5000
Production rate (units/year)		
Processing Plant	P_w	5700
Assembly Plant	P_F	5400
Shipping cost (dollars/order)		
Unfinished RM shipping	S_A	220
Processed RM shipping	S_B	250
Ready RM shipping	S_C	200
Finished Product shipping	S_F	300
Setup (production) costs (dollars/setup)		
Setup processing plant	A_w	110
Setup assembly plant	A_F	120
Setup (order) costs (dollars/setup)		
Unfinished RM order cost	K_A	90
Ready RM order cost	K_B	95
Holding costs (dollars/unit/year)		
Unfinished RM inventory	H_A	48
Ready RM inventory	H_B	45
Processed RM at processing stage	H_w	35
Processed RM at inventory	H_C	35
Finished product at inventory	H_F	40
Conversion factors		
Unfinished RM to processed RM	f_w	.7
Ready RM to finished product	f_b	.65
Processed RM to finished product	f_c	.9
Shipment per order (units/order)	x	30

4.2 Optimal Total Cost

The optimal total cost of the problem by using the Equation (17) is $TC(n^*) = 2\left(\sqrt{AD} + \sqrt{BE} + \sqrt{CF} + \sqrt{fy}\right) + g$.

The values of $A, B, C, D, E, F, f, y, g$ are given in Equations (15a) to (15i). Using the parametric values, the optimal total cost is

$$= 2\left(\sqrt{57551285.62} + \sqrt{55288461.54} + \sqrt{37448559.67} + \sqrt{2270359.54}\right) + 50600$$

$TC(n^*)$ = \$95,896.34.

4.3 Policy for Procurement and Delivery Rate

To determine the raw materials procurement rate and the finished product delivery rate in one cycle period of time, the parameters values in Table 3.1 are substituted in to Equation (18)

$$MinZ(n_a, n_b, n_c, m) = \frac{1000}{m}(51.67n_a + 57.5n_b + 33.3n_c + 38.3)$$

$$+ m\left(\frac{1113.9}{n_a} + \frac{961.5}{n_b} + \frac{1123.4}{n_c} + 59.2\right) + 50600\,, \qquad (23)$$

where $n_a, n_b, n_c, m \geq 1$ and integers.

The procurement rate (n_a, n_b, n_c) and delivery rate (m) are the input variables in the above function. Branch and bound (B&B) approach is used to solve this NLIP problem. From Equations (19a)-(19c) and Equation (20), it is observed that n_a, n_b and n_c are the function of m. So delivery rate m is the *basic variable* and procurement rates n_a, n_b and n_c are the *non-basic variables*. In this integer approximation solution, if the *basic variable(s)* are non-integer, the *non-basic variables* are solved by taking the integer lower and upper values of the *basic variable(s)*. The presented numerical example is solved in eight steps.

Step 1. Using the Equations (19a)-(19c), (20) and (23), solve the NLIP relaxation of the original problem. The solution of the NLIP relaxation problem is

n_a = 3.73, m = 25.44,

n_b = 3.29, Z = \$95,896.34.

n_c = 4.67,

Step 2. Since m is a *basic variable*, set the value of m to integer, m = 25 and m = 26. As the value of m is changed to an integer, the value of n_a, n_b and n_c will also be changed. Considering m = 26, solve the NLIP relaxation problem using the Equations (19a)-(19c) and (23). Set the result as the lower bound of the optimal solution of NLIP problem, $Z_L = Z$; the solution of the problem is

n_a = 3.82,

n_b = 3.36, Z = \$95,897.06.

n_c = 4.77,

The Z value obtained in solving NLIP relaxation problem is slightly higher than that obtained by optimal cost function in Step (i). Thus, set the current Z value as the lower bound of the of NLIP problem, Z_L = \$95,897.06. Let the procedure continue to obtain the integer value of the other variable.

Step 3. The values of *non-basic variables* n_a, n_b, n_c are not integers, the process continues to obtain integer values. The upper bound is set to $Z_U \rightarrow 8$.

Step 4. To obtain the integer solution, the additional constraints, $n_a \leq 3$, $n_a \geq 4$, $n_b \leq 3$, $n_b \geq 4$, $n_c \leq 4$, $n_c \geq 5$, are added one-by-one into the Equation (23). There are six subsets formed at the first level. All of the subsets are to solved one by one using Equation (19a)-(19c) and (23). The solutions of each subset are verified for the feasibility. The feasible subset(s) form the nodes for further investigation. The node formation continues until all the variables are integers. Verify the integer solutions with minimum Z value. When integer solution is obtained, update Z_L by setting Z_L equal to the integer solution.

Step 5. The solution procedure for m = 26 is shown in Figure 3.8. As the subsets are not integer, the search for the integer solution is continued for the most promising node. All Z values of the six subsets are compared. It is observed that node (2) with Z value of \$95,852.73 is the smallest. Then, Z_L = \$95,897.06 is updated by Z_L (Z_L = \$ 95852.73).

Step 6. The search continues at the node (2). Four sub-nodes are formed in the second level with the constraint $n_b \leq 3$, $n_b \geq 4$, $n_c \leq 4$, $n_c \geq 5$. Sub-nodes are compared. The most promising node at this stage is node (10). Verify the Z value of the node (10) with the nodes at first level, which are nodes (1), (3-6). It is observed that node (6) in the first level is less that Z= 95,926.77. Node (6) is considered for further investigation.

Step 7. Node (10) is now formed into two subsets with the constraint $n_b \leq 3$, $n_b \geq 4$. The two sub-nodes are investigated for the most promising node with integer values. The Z value at node (15) is found the minimum. Repeat Step 2 – 6 for node (6). The Z value at node (14) is found the minimum and same as node (10). The integer values obtained in node (15) and node (18) are same, $n = (n_a, n_b, n_c, m) =$ (4, 3, 5, 25), Z = \$96,023.48.

Step 8. Considering m = 25, solve the NLIP relaxation problem using the Equations (19a)-(19c) and (23). The solution of the problem is

$n_a = 3.67$,

$n_b = 3.23$, $Z = \$95,896.80$

$n_c = 4.59$,

To find the integer solutions repeat Step 2 – 7. The solution procedure for m = 25 is shown in Figure 3.7. The promising node obtained is node (12) in which $n = (n_a, n_b, n_c, m) =$ (4, 3, 5, 25), Z = \$96,039.29. To determine the minimum Z value, verify the values obtained at m = 25 and m = 26. Comparing the values obtained at m = 25 in node (12) in Figure 3.7 and the values obtained at m = 26 in node (15) or node (18) in Figure 3.8, the most promising node obtained with minimum Z value is node (15) in Figure 3.8. The integer values are $n = (n_a, n_b, n_c, m) = (4, 3, 5, 26)$, for minimum Z = $96,023.48.

The optimal values of the variables, integer values of the variables corresponding to lower bound (m = 25) and upper bound (m = 26) of the independent variable are shown in Table 4.2.

Table 4.2. Lists of optimal and integer values of variables

Variables	Optimal Value (Z_{opt})	Integer approximation (Z_H)	
		$m \leftarrow Lower\ Approx.$	$m \leftarrow Upper\ Approx.$
m	25.44	25	26
n_a	3.74	4	4
n_b	3.29	3	3
n_c	4.59	5	5
TC	$ 95,896.34	$ 96,039.29	$ 96,023.48

Here, h is defined as the efficiency measure to estimate the relative error between the solution obtained by optimization and B&B integer approximation. The efficiency can be estimated as

$$h = \frac{Z_H - Z_{opt}}{Z_{opt}} \times 100\%, \text{ so}$$

$$h = \frac{96{,}023.48 - 95{,}896.34}{95{,}896.34} \times 100\% = 0.13\% \approx 0.1\%.$$

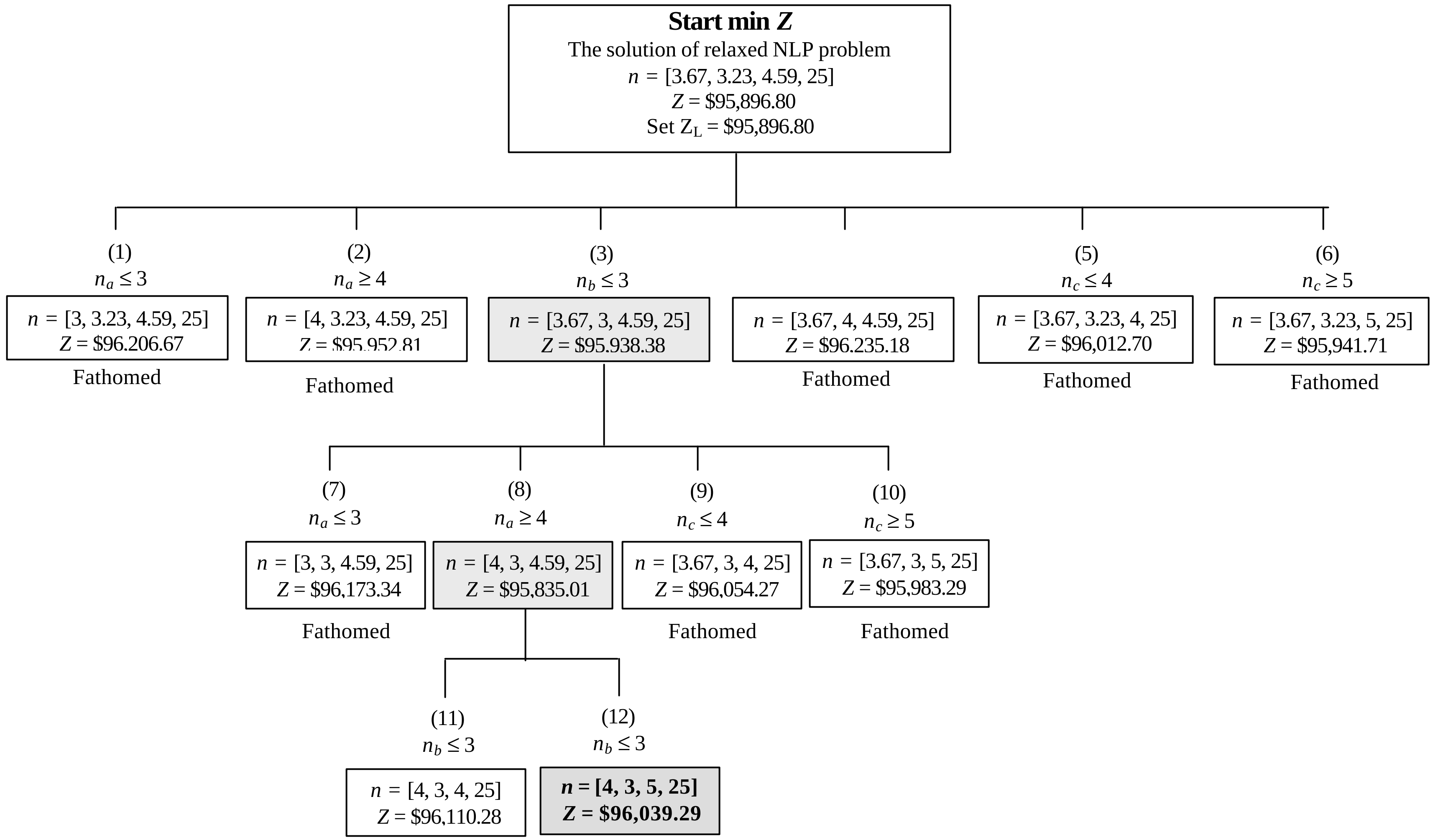

Figure 3.7. Solution technique by branch and bound algorithm (Considering delivery frequency = 25)

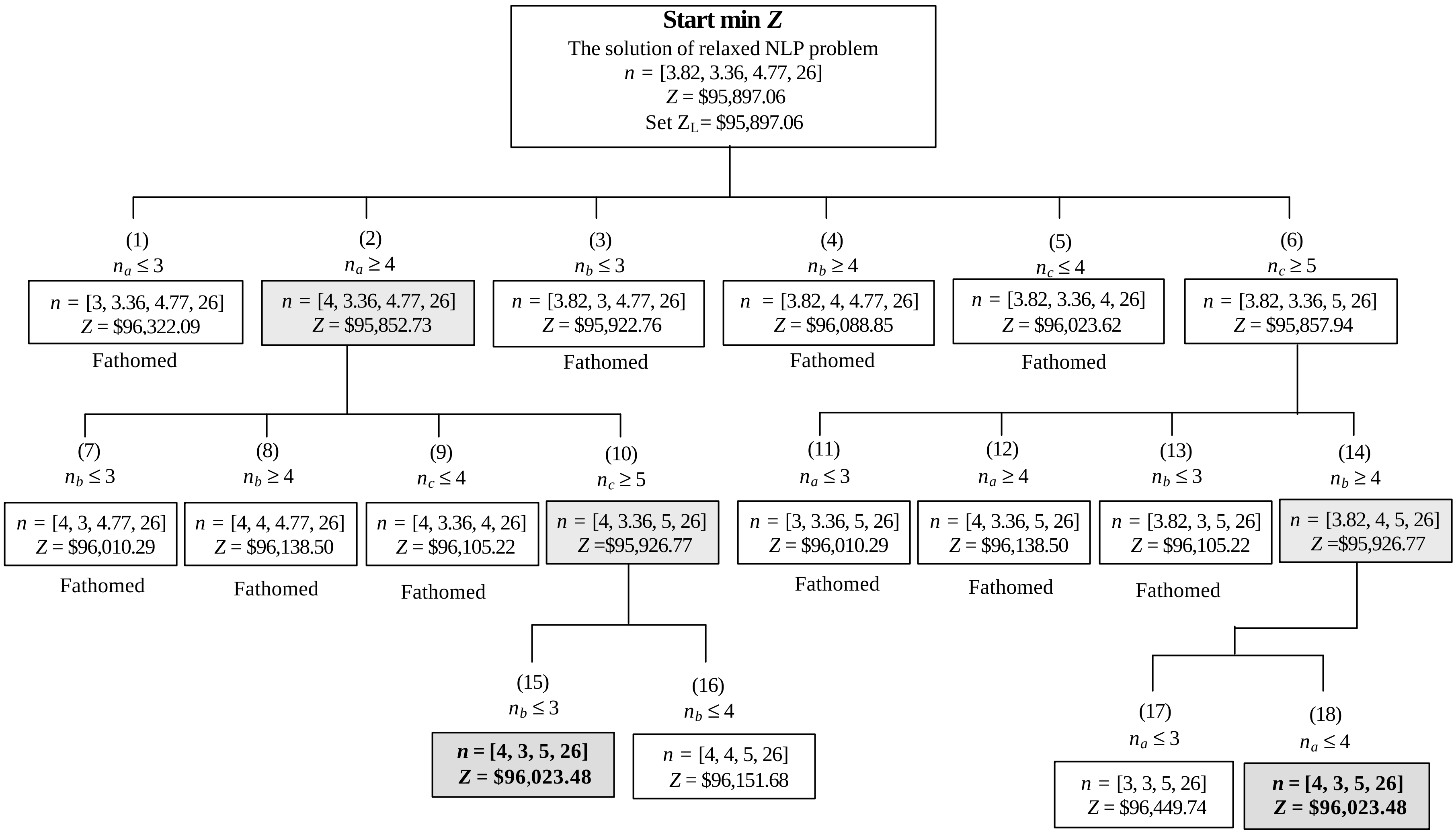

Figure 3.8. Solution technique by branch and bound algorithm (Considering delivery frequency = 26)

4.4 Alternate Results

It is observed that the result of the test problem can be obtained from six nodes, node (11) and (12) in Figure 3.7 and node (15), (16), (17), (18) from Figure 3.8. The alternate solution series, including the best solution, are listed in Table 4.3.

Table 4.3. Solution sequence

Node (Figure)	$n^* = (n_a, n_b, n_c, m)$	Z value
11(Figure 3.7)	(4, 3, 4, 25)	$96,110.28
12(Figure 3.7)	(4, 3, 5, 25)	$96,039.29
15(Figure 3.8)	**(4, 3, 5, 26)**	**$96,023.48**
16(Figure 3.8)	(4, 4, 5, 26)	$96,151.68
17(Figure 3.8)	(3, 3, 5, 26)	$96,449.74
18(Figure 3.8)	**(4, 3, 5, 26)**	**$96,023.48**

4.5 Supplementary Case Studies

A number of supplementary case studies are made to test the solution of the model. A series of nine cases of SSSCS are generated randomly. Both the unconstrained and constrained optimization of the test problems are solved. From the results of the nine case studies, it can be observed that the procurement rate for all the raw materials are within the range of (3–7) and the finished product delivery rate is on the range of (18-33). The results of the given numerical example fall within the range of the results obtained from these case studies. The parametric values of all cases for the SSSCS model are shown in Table 4.4.

Table 4.4. The parametric values for case studies

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
D_F	3500	3900	4300	4700	5100	5300	5500	5800	6000
P_w	4000	4500	5000	5300	5700	6000	6300	6500	6700
P_F	3700	4100	4500	4900	5200	5600	5800	6100	6300
S_A	150	160	170	180	190	200	230	240	250
S_B	200	210	220	230	240	250	260	270	280
S_C	170	180	190	210	220	230	240	250	260
S_D	250	260	270	280	290	300	310	320	330
A_w	100	103	107	110	112	115	120	125	127
A_F	110	113	116	118	119	121	125	128	130
K_A	70	75	80	85	90	95	100	105	110
K_B	65	70	75	80	85	90	95	100	105
H_A	38	41	44	47	50	53	56	59	65
H_B	35	38	41	45	49	55	58	62	65
H_w	31	33	35	39	43	45	50	53	58
H_C	31	33	35	37	38	41	46	49	53
H_F	36	41	44	48	52	57	63	65	67
f_w	.71	.73	.75	.77	.79	.8	.81	.83	.85
f_b	.61	.63	.65	.67	.69	.7	.71	.73	.75
f_c	.9	.89	.88	.89	.91	.89	.9	.91	.93
x	25	27	29	33	35	37	39	40	41

The unconstrained NLIP is solved using the Equations (19a)-(19c) and Equation (20). The unrestricted solutions of the nine cases are shown in Table 4.5. The Z values obtained in relaxed integer solutions serve as the lower bound, Z_L, in B&B technique. Identifying the basic and non-basic variables from the relaxed solutions, the heuristic results for the best integer solution of the cited cases are determined by using the B&B algorithm. The supplementary tests results for the constrained NLIP are shown in Table 4.6.

Table 4.5. The supplementary test results for optimal solution

Test Cases	m	n_a	n_b	n_c	**Total cost, $**
Case 1	28.47	4.36	3.85	5.34	65,916.57
Case 2	27.89	4.36	3.86	5.40	72,232.65
Case 3	28.24	4.51	3.99	5.65	78,488.82
Case 4	30.49	5.27	4.73	6.55	82,402.98
Case 5	32.76	5.61	5.26	7.19	88,461.44
Case 6	23.07	4.15	3.88	5.31	93,679.37
Case 7	18.97	3.35	3.29	4.64	99,000.09
Case 8	20.58	3.60	3.57	5.06	105,204.78
Case 9	18.26	3.21	3.16	4.58	110,884.65

Table 4.6. The supplementary test results for integer solution

Test Cases	m	n_a	n_b	n_c	**Total cost, $**
Case 1	28	4	4	5	65,965.87
Case 2	27	4	4	5	72,285.63
Case 3	28	4	4	6	78,589.23
Case 4	30	5	5	6	82,480.06
Case 5	33	6	5	7	88,519.42
Case 6	23	4	4	5	93,721.38
Case 7	19	3	3	5	99,226.84
Case 8	21	4	4	5	105,358.42
Case 9	18	3	3	5	111,025.28

CHAPTER 5

SENSITIVITY ANALYSIS

In this chapter numerical sensitivity of the system parameters and the input variables are evaluated. The analysis shows the general behavior of the system and illustrates the characteristics of the parameters through the nature of the curvature. The result provides the sensitivity of the model parameters on total cost and demonstrates the critical point for cost minimization and shows the area of modification of the cost functions for effective improvement of the system.

The important input parameters in the total cost function are raw materials procurement rates n_a, n_b, n_c and product delivery rate m. Equation (19a) to (19c) and (21) shows that n_a, n_b, n_c are the function of m and delivery rate m is a function of shipment order size x. Taking the basic cases such as the effect of finished product batch size, shipment order size, raw materials procurement rates and delivery rate on total cost are considered in the analysis. To observe the influence of other parameters on total cost, the analysis continues varying each of these parameters separately.

5.1 Effect of Batch Size and Shipment (order) Size on Total Cost

The impacts of changing the batch size, shipment order size at various conditions are studied in this section. Finished product batch size, $Q_F = mx$, is a function of delivery rate m and shipment order size x. The knowledge of the influence of batch size on total cost is essential, especially when batch size shifts significantly affect the overall cost. The batch size is influenced either, (i) by changing the both the component, m and x, simultaneously or, (ii) by changing m or x, one component at a time. Considering the above cases, a sensitivity analysis is performed to observe the effects of batch size on the total cost of the system.

5.1.1 Effect of Batch Size on Total Cost

In this model, shipment order size x is considered as a given parameter. Variation of batch sizes is primarily due to the variation of delivery rate m. Table 5.1 shows the effect of batch size on total cost when shipment order size is a fixed quantity. The Table indicates that minimum total cost corresponding to batch size of 768.6 units/cycle and the delivery rate of 25.62 delivery/cycle and constant shipment order size is 30 units/order.

Table 5.1 Effect of Batch Size on Total cost, *TC*

Shipping Order Size is Fixed				
Order size	*Delivery rate*	*Batch size, Q_F*	*Total cost,* $	*dTC/dQ_F*
	20	600	97,466.43	-19.25
	21	630	96,958.46	-14.72
	22	660	96,576.94	-10.80
	23	690	96,305.36	-7.38
	24	720	96,130.00	-4.38
	25	750	96,039.29	-1.73
30	**26***	**780***	**96,023.48**	0.63
	27	810	96,074.24	2.72
	28	840	96,184.43	4.59
	29	870	96,347.92	6.28
	30	900	96,559.37	7.79

To quantify the significant effect of finished product batch size on total cost, the rate and the direction of change of total cost, with respect to $Q_{F,}$ is considered. Mathematically, the expression is

$$\frac{\partial TC}{\partial Q_F} = -\frac{1}{{Q_F}^2}\left(An_a + Bn_b + Cn_c + f\right) + \frac{1}{x}\left(\frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y\right). \qquad (24)$$

Depending upon the parametric values given in Table 5.1, the term dTC/dQ_F holds positive value when batch size $Q_F \leq 770$ units. The effect of Q_F over the total cost for Q_F $\in$ [600, 900] following Equation (24) is shown in Figure 5.1. It is observed that the slope of the graph increases at the beginning, up to $Q_F = 750$ units and then the slope reduces. The result shows a negative value when the curve moves towards left, $Q_F < 770$ units. The batch size on the range [750,800] provides minimum total costs.

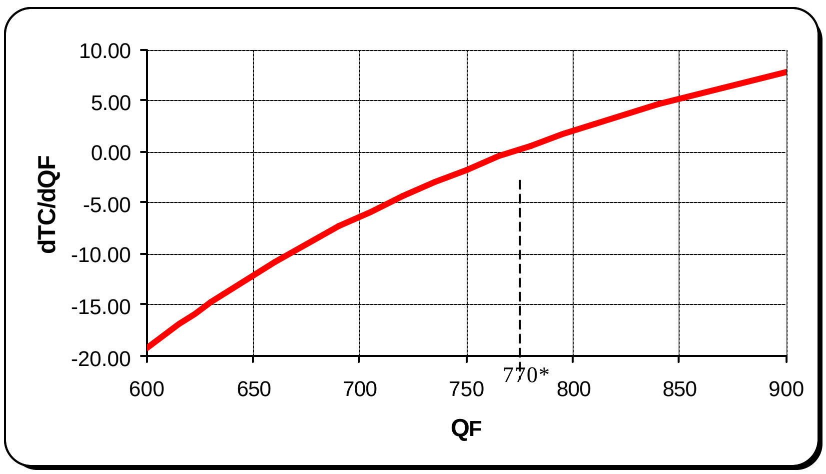

Figure 5.1. Effect of finished product batch size on total cost

5.1.2 Effect of Shipment Order Size on Total Cost

The total cost may vary due to the variation of shipment size. For any given batch size, if the shipment order size varies, delivery order rate fluctuates and total cost changes. Table 5.2 shows the effect of shipment order size on total cost. It is observed that total cost reduces when shipment size tends to be high and the corresponding delivery rate is low. If shipment size is equal to the batch size and delivered in a single shipment, total cost is likely to be minimum. The sensitivity of the shipment order size indicates that a higher shipment size with corresponding adjusted delivery rate may reduce total cost.

Table 5.2 Effect of shipment order size and shipping rate on total cost

Batch Size is a Fixed Quantity			
Order size	*Delivery rate*	*Batch size*	*Total cost,* $
25	31		105,797.14
26	30		103,509.44
27	29		101,392.69
28	28		99,428.56
29	27		97,601.27
30	**26**	**780***	**96,023.48**
31	25		94,304.23
32	24		92,812.14
33	23		91,411.68
34	23		90,094.78
35	22		88,854.28

5.2 Effect of Input Variables on Total Cost

The objective of this section is to identify the relation of end-product delivery rate and raw materials procurement rates on total cost function. In the assembly system, raw materials for each part type are procured separately from suppliers. Numerically, the optimum values of the delivery rate and procurement rates are already calculated based on the parametric values given in Table 4.1. The next two sub-sections will explore the sensitiveness of these parameters and show how they may affect the overall cost of the model in case of any changes.

5.2.1 Effect of Delivery Rate on Total Cost

In the present model, when shipment order size is a given parameter, total cost is affected by the change of delivery rate, m. From Table 5.1, it is observed that for a given shipment order size, there exists an optimum delivery rate and the corresponding total cost is the minimum value. For a fixed batch size, the total cost continuously decreases when the delivery rate increases and the corresponding order size decreases as shown in Table 5.2. To observe the impact of delivery rate on the total cost, the rate and the direction of change of total cost with respect to *m* is calculated shown as,

$$\frac{\partial TC}{\partial m} = -\frac{1}{m^2}\left(An_a + Bn_b + Cn_c + f\right) + \left(\frac{D}{n_a} + \frac{E}{n_b} + \frac{F}{n_c} + y\right). \tag{25}$$

Increasing the value of m from 10 to 40 and keeping the other parametric values unchanged, the effect of delivery rate on total cost is shown in Figure 5.2. The value of dTC/dm holds positive value at the delivery rate, $m \geq 22$. The intense sensitiveness of delivery rate on total cost is shown on the range [10,20]. Total cost variation is significantly lower when delivery rate approaches to higher value, ($m \geq 30$).

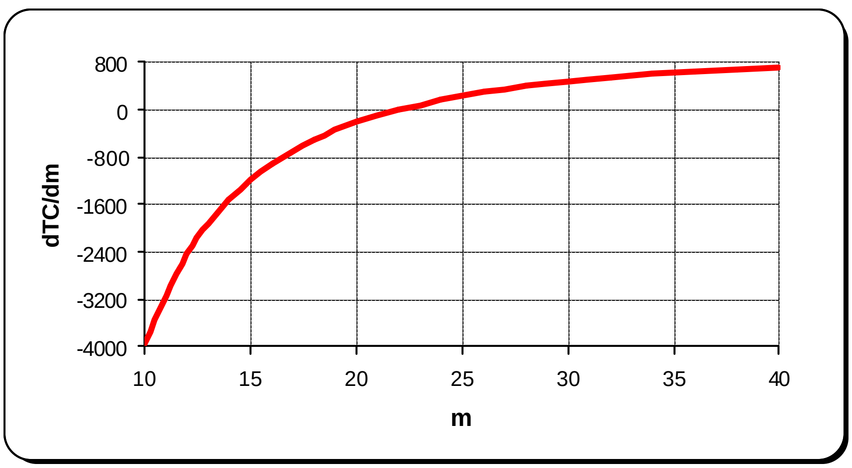

Figure 5.2. Effect of delivery rate on total cost

For a steady demand condition, the delivery rate is significantly dependant on the lot size of the shipment. The delivery rate is sensitive when shipment lot size is subject to change. For a given shipment order size, the change of delivery rate significantly effects the total cost until it reaches the optimal number of deliveries. When all the finished products are delivered (delivery rate reaches its optimal value), the rate of total cost change is significantly reduced.

5.2.2 Effect of Raw Materials Procurement Rate on Total Cost

The behavior of raw materials procurement rates is demonstrated in Figure 5.3. Implying the procurement rate variation of from 1 to 7, for all the raw materials, causes the total cost variation while the other parameters of the total cost function remain unchanged.

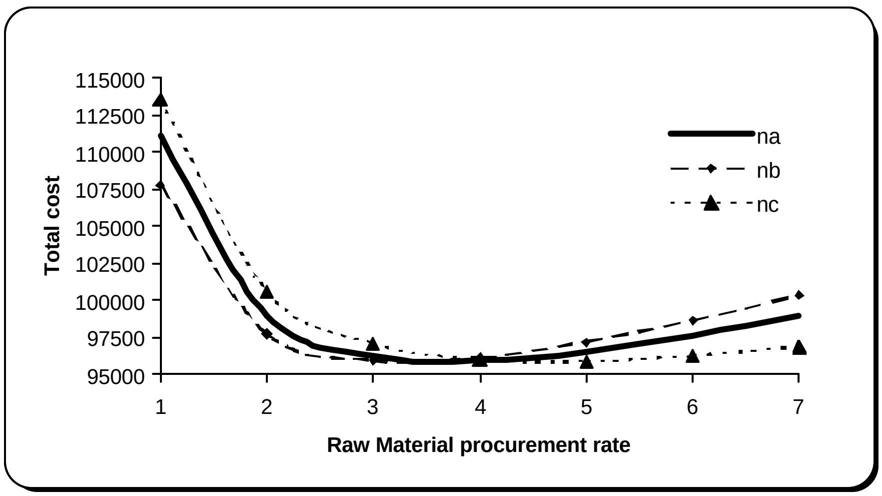

Figure 5.3. Change of total cost with procurement rates

It is observed that the cost remains stable in the range of 3 to 5 for each of the raw material. Total cost drastically increased when procurement rate falls below 2 and increases slowly beyond 5. The lower value of the procurement rates causes the higher raw materials order sizes, which results higher order costs and larger holding costs in the system. The previous analysis may be explicit when the outcomes are shown by using the rate of total cost with respect to procurement rates of each raw material. Mathematically, the expressions for each of the raw materials are as follows $\frac{\partial TC}{\partial n_a} = \frac{A}{m} - \frac{Dm}{n_a^{\ 2}}$,

$\frac{\partial TC}{\partial n_b} = \frac{B}{m} - \frac{Em}{n_b^{\ 2}}$ and $\frac{\partial TC}{\partial n_c} = \frac{C}{m} - \frac{Fm}{n_c^{\ 2}}$. Figure 5.4 shows the effect of procurement rate on total cost.

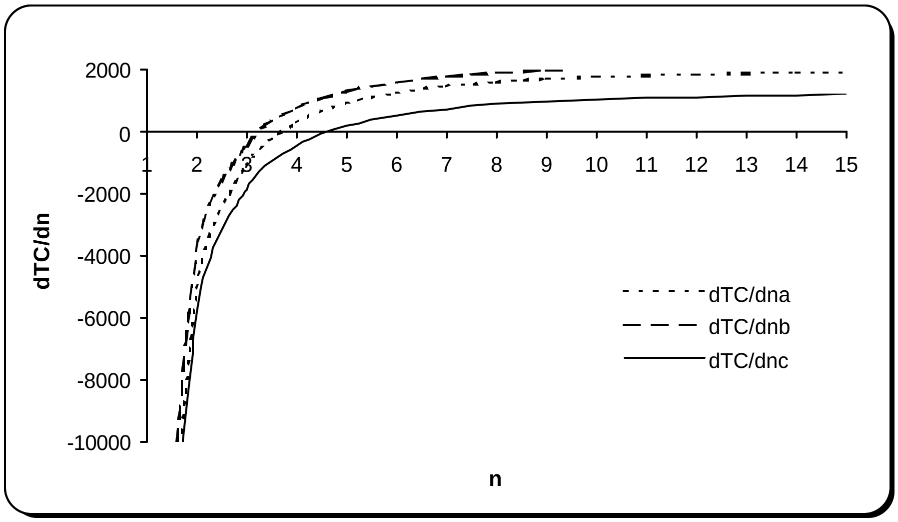

Figure 5.4. Effect of procurement rates on total cost

From the calculation, based on the parametric values given in Table 4.1, the optimum numbers of *unfinished, processed and ready raw materials* are obtained as 4, 3 and 5, respectively. The change of procurement rates drastically affect the total cost when the rates are lower than the optimal value, i.e. n_a<4, n_b<3 and n_c<5. Once procurement rates attain the optimal values, corresponding to the given parametric values of the cost function, the rate and direction change of total cost reduces as the shipment rates accomplishes the required quantities of the raw materials for the model.

5.3 Effect of Production Rate on Total Cost

The effect of production rate on total cost is investigated in this section. Total cost is calculated as a linear function of the production rate. Varying the production rates of a processing plant P_w and an assembly plant P_F from 5000 to 9000 units/year, the effect on total cost is shown in Figure 5.5. When production rates increases, the requirements of raw materials are also increased. Accordingly, the raw materials ordering cost, transportation cost, work-in-process inventory cost and the production of finished products are increased and ultimately the total cost is increased.

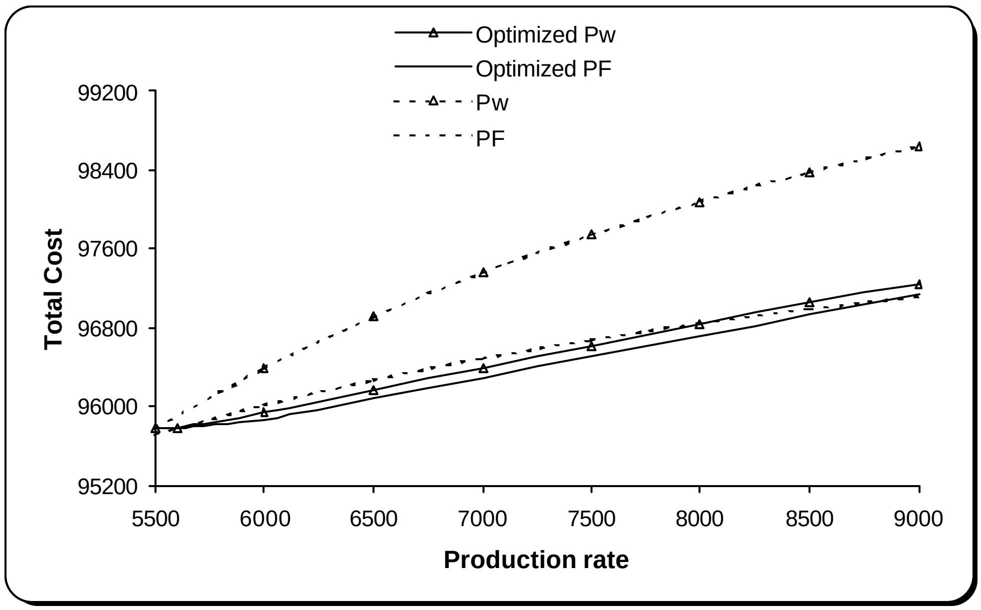

Figure 5.5. Effect of production rate on total cost

In the figure, the solid lines represent the change of total cost with the change of production rates when total cost is optimized. The dashed lines represent the variation of total costs when total cost is not optimized. When production rates are changed, the values of the variables are also changed. If the new values of the variables are adjusted in the total cost function, then the new cost is called adjusted or optimized total cost. Since the variation of adjusted total cost allows the adjustment of other parameters of the cost function, so the adjusted total cost gives the lower value than the total cost incurred by the unadjusted production rates variation.

The results also indicate that the sensitivity of production rate at processing stage is higher than that of the assembly production rate for both adjusted and unadjusted production rate changes. The production time in processing stage is higher than the production time in assembly stage. So, the change of production rate causes higher affect in processing plant than assembly plant.

5.4 Effect of Transportation Cost on Total Cost

The effect of transportation cost coefficients of raw materials S_a, S_b, S_c and finished product S_F, on total cost are studied. Figure 5.6 demonstrates the behavior of the transportation costs coefficient by arbitrarily changing the values from 150 to 350 dollars/shipment. It is observed that total costs increase linearly with the increase of transportation costs.

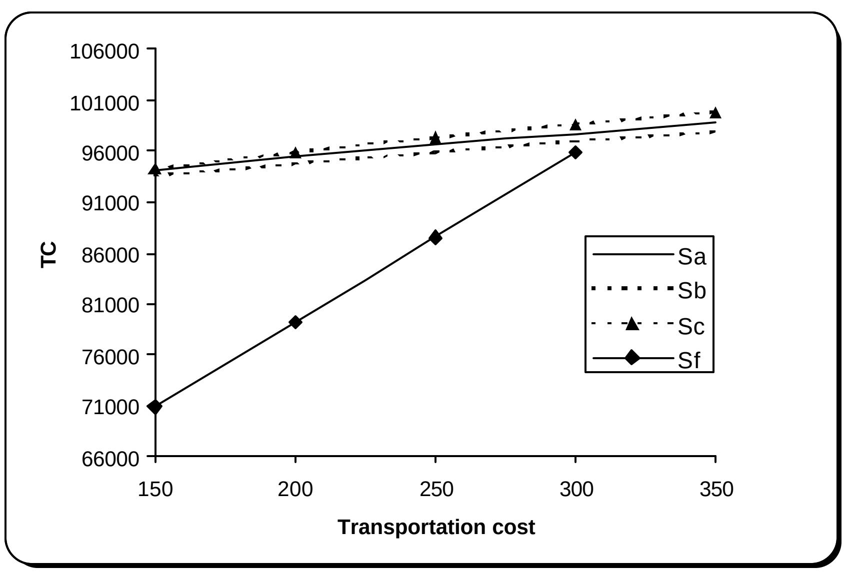

Figure 5.6. Effect of transportation cost on total cost

The raw materials transportation costs, S_a, S_b, S_c, are significantly less sensitive on total costs whereas finished product transportation cost S_F has significant impact on total cost. The explanation may be the raw materials procurements are less frequent (only 3-5 times per cycle) whereas the finished product delivery rate is considerably high (25-26 per cycle). So, a small change in transportation cost on high frequency element (i.e., delivery rate) causes significant effect on total cost. Consequently, the finished product transportation cost ingredient shows more sensitivity than the others.

5.5 Effect of Holding Cost on Total Incremental Cost

Assume the holding cost varies with the size of the batch, according to the following relationship:

$C_h = aQ_F$ unit/year

In the model, C_h is total holding costs (*THC*) ($C_h = H_A + H_B + H_C + H_w + H_F$); K is total ordering cost (*TOC*) ($K = K_A + K_B$); D_F is the annual demand and Q_F is and batch size of finished product. Total incremental cost (*TIC*) therefore is

$$TIC = THC + TOC = \frac{Q_F}{2} aQ_F + \frac{D_F}{Q_F} K \tag{26}$$

$$TIC^{/} = aQ_F - \frac{D_F}{{Q_F}^2} K = 0 \tag{27}$$

$$Q_F * = \sqrt[3]{\frac{D_F K}{a}} \tag{28}$$

$$TIC * = \frac{a{Q_F}^2}{2} + \frac{D_F}{Q_F} K \tag{29}$$

According to the parametric values given in Table 3.1, C_h = \$ 403 \$/unit/year, Q_F = 5000 unit/year, then $a = 403/5000 = 0.04$

$$TIC^* = \frac{aQ_F^{\ 2}}{2} + \frac{D_F}{Q_F}K = \frac{.04}{2}780^2 + \frac{5000}{780}185 = \$12{,}286.59 / year$$

Variation of a indicates the variation of unit holding cost when batch size remains unchanged. Figure 5.7 shows the effect of holding cost on total incremental cost at different batch sizes and different a-values (\$/unit/year). The total cost increment is linearly increasing with the variation of batch sizes.

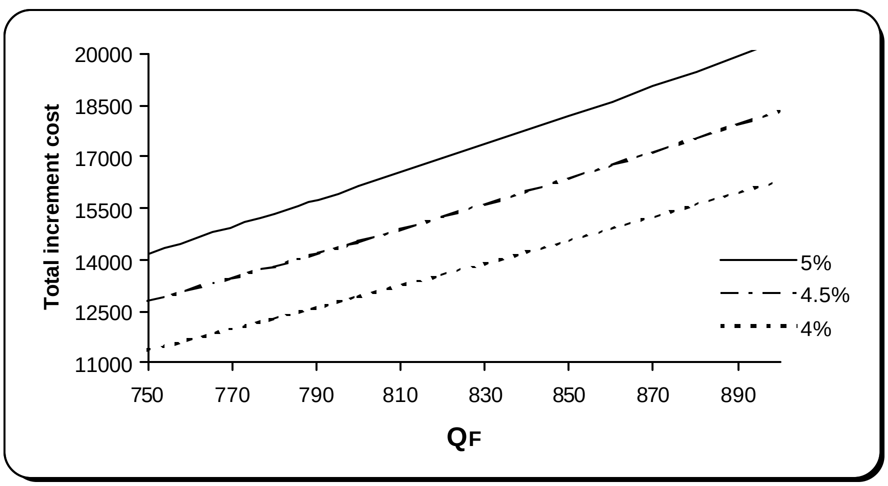

Figure 5.7. Effect of holding costs on total incremental cost

5.6 Effect of Setup (production and order) Cost on Total cost

In total cost function TC, raw materials ordering cost K_A, K_B and production setup cost A_w, A_F (processing and assembly plant) is the important factors. Assuming that K_A is varied to $K_A^{\ /}$, the changed total cost function, $TC(K_A)$, is written from Equation (14) as

$$TC(K_A) = n_a K_A^{\ /}\left(\frac{D_F}{Q_F}\right) + d_A. \quad (30)$$

where

$$d_A = \frac{D_F}{mx}\{S_A + n_b(K_B + S_B) + n_c S_C + A_w + A_F\} + \frac{mx}{2n_a}\left(\frac{H_A D_F}{f_w f_c^2 P_w}\right) + \frac{mx}{2n_b}\left(\frac{D_F H_B}{f_b P_F}\right)$$
$$+ \frac{mx}{2n_c f_c}\left(\frac{D_F H_C}{P_F} + H_w\right) + \frac{mx}{2}\left(1 - \frac{D_F}{P_F}\right)H_F + \frac{mx}{2f_c}\left(1 - \frac{D_F}{f_c P_w}\right)H_w + \left(\frac{D_F S_F}{x} + \frac{xH_F}{2}\right).$$

Similarly, K_B is varied to $K_B^{/}$, the changed total cost function, $TC(K_B)$, can be written from Equation (14) as

$$TC(K_B) = n_b K_B^{/}\left(\frac{D_F}{Q_F}\right) + d_B. \tag{31}$$

where,

$$d_B = \frac{D_F}{mx}\{n_a(K_A + S_A) + n_b S_B + n_c S_C + A_w + A_F\} + \frac{mx}{2n_a}\left(\frac{H_A D_F}{f_w f_c^2 P_w}\right) + \frac{mx}{2n_b}\left(\frac{D_F H_B}{f_b P_F}\right)$$
$$+ \frac{mx}{2n_c f_c}\left(\frac{D_F H_C}{P_F} + H_w\right) + \frac{mx}{2}\left(1 - \frac{D_F}{P_F}\right)H_F + \frac{mx}{2f_c}\left(1 - \frac{D_F}{f_c P_w}\right)H_w + \left(\frac{D_F S_F}{x} + \frac{xH_F}{2}\right).$$

A graphical representation of the variation of total cost, with respect to the variation of raw materials ordering cost, is shown in Figure 5.8. It is observed that total cost increases linearly with the increase of raw materials ordering cost and the slope of the curve on either side of the nodal point remains constant.

It is also important to find the impact of production setup cost on the total cost. Assuming production setup cost at processing plant A_w is varied to $A_w^{/}$ and corresponding total cost TC changes to $TC(A_w)$. The changed total cost with respect to $A_w^{/}$ is given as

$$TC(A_w) = A_w^{/}\left(\frac{D_F}{Q_F}\right) + q_w. \tag{32}$$

where

$$q_w = \frac{D_F}{mx}\{n_a(K_A+S_A)+n_bS_B+n_cS_C+A_F\}+\frac{mx}{2n_a}\left(\frac{H_AD_F}{f_wf_c^2P_w}\right)+\frac{mx}{2n_b}\left(\frac{D_FH_B}{f_bP_F}\right)$$
$$+\frac{mx}{2n_cf_c}\left(\frac{D_FH_C}{P_F}+H_w\right)+\frac{mx}{2}\left(1-\frac{D_F}{P_F}\right)H_F+\frac{mx}{2f_c}\left(1-\frac{D_F}{f_cP_w}\right)H_w+\left(\frac{D_FS_F}{x}+\frac{xH_F}{2}\right).$$

Similarly varying production setup cost at assembly stage from A_F to $A_F^{/}$, the total cost $TC(A_F)$ is obtained as

$$TC(A_F)=A_F^{/}\left(\frac{D_F}{Q_F}\right)+q_F. \tag{33}$$

where

$$q_F = \frac{D_F}{mx}\{n_a(K_A+S_A)+n_bS_B+n_cS_C+A_w\}+\frac{mx}{2n_a}\left(\frac{H_AD_F}{f_wf_c^2P_w}\right)+\frac{mx}{2n_b}\left(\frac{D_FH_B}{f_bP_F}\right)$$

$$+\frac{mx}{2n_cf_c}\left(\frac{D_FH_C}{P_F}+H_w\right)+\frac{mx}{2}\left(1-\frac{D_F}{P_F}\right)H_F+\frac{mx}{2f_c}\left(1-\frac{D_F}{f_cP_w}\right)H_w+\left(\frac{D_FS_F}{x}+\frac{xH_F}{2}\right).$$

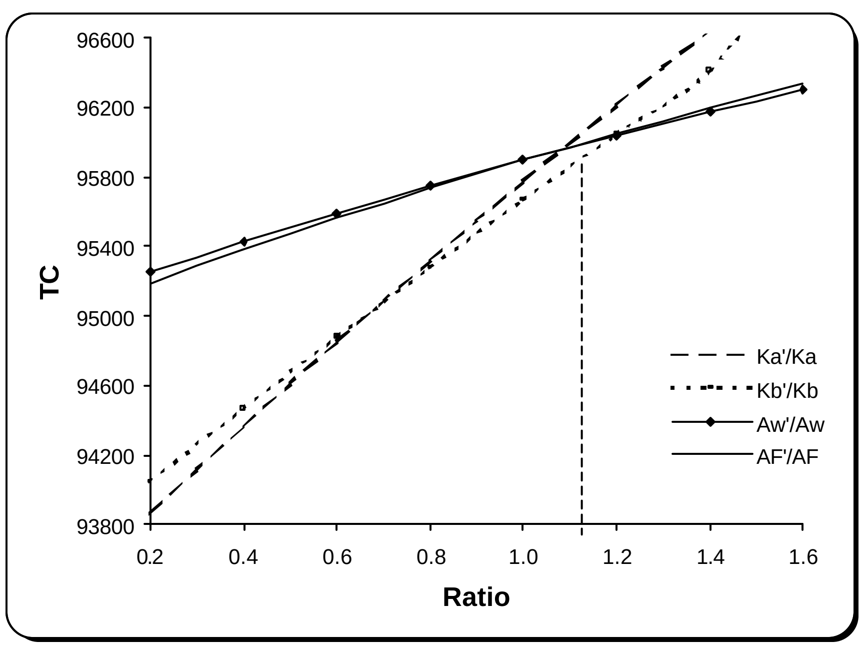

Figure 5.8. Effect of ordering and production setup cost on total cost

Comparing the two curves, it is observed that the slope of the production setup cost is steeper on the left of the nodal point ($K_A^{/} / K_A = A_w^{/} / A_w$ or $K_B^{/} / K_B = A_F^{/} / A_F$) than the raw materials ordering cost. The scenario is opposite on the other side of the nodal point. The reason may be the production setup cost is single for each cycle whereas multiple raw materials order costs is higher. As the production proceeds, the cumulative raw materials order costs exceed the production setup cost, so the ordering costs curve go above the production costs curve at higher production rate.

Ordering costs for unfinished raw materials is slightly lower than the ready raw materials order cost, but the number of shipment for *unfinished raw materials* is higher than the shipment number of *ready raw materials*. So the effect of *unfinished raw materials* is lesser at lower order cost, but rises sharply over *ready raw materials* when the order cost is higher.

CHAPTER 6

LOGISTIC OPERATIONAL SCHEDULE

This chapter considers the operational schedule of a single stage supply chain system model where the manufacturing system receives raw materials from the suppliers, processes them and delivers them to the finished goods buyer. The optimal parameters are the number of raw material orders and the number of finished products deliveries. An NLIP function has been derived to characterize the model. For a given set of parametric values, an optimal policy of the model has been obtained. Reviewing some special cases, the sensitivity of the system is analyzed. Some important issues, yet to be analyzed, are to schedule the order placement, deliveries and the configuration of the inventory system on a timescale.

6.1 Configuration of Single Stage Supply Chain Model on Timescale

A single-stage supply chain production system having two manufacturing plants, preprocessing and assembly plants, are considered. The optimal expressions for the number of orders, deliveries, finished product batch sizes have been derived. The *unfinished raw materials* are processed in the processing plant and are transported to the buffer inventory for the assembly operation. At this point, the *ready raw materials*, which do not need any preprocessing in the facilities, also arrive in buffer inventory for the assembly process. It is important to configure the order placement and the transportation of raw materials from suppliers to the workstations on a timescale. Figure 6.1 shows the configuration of raw materials shipments, workstation setups, and work-in-process inventory buildup on a timescale.

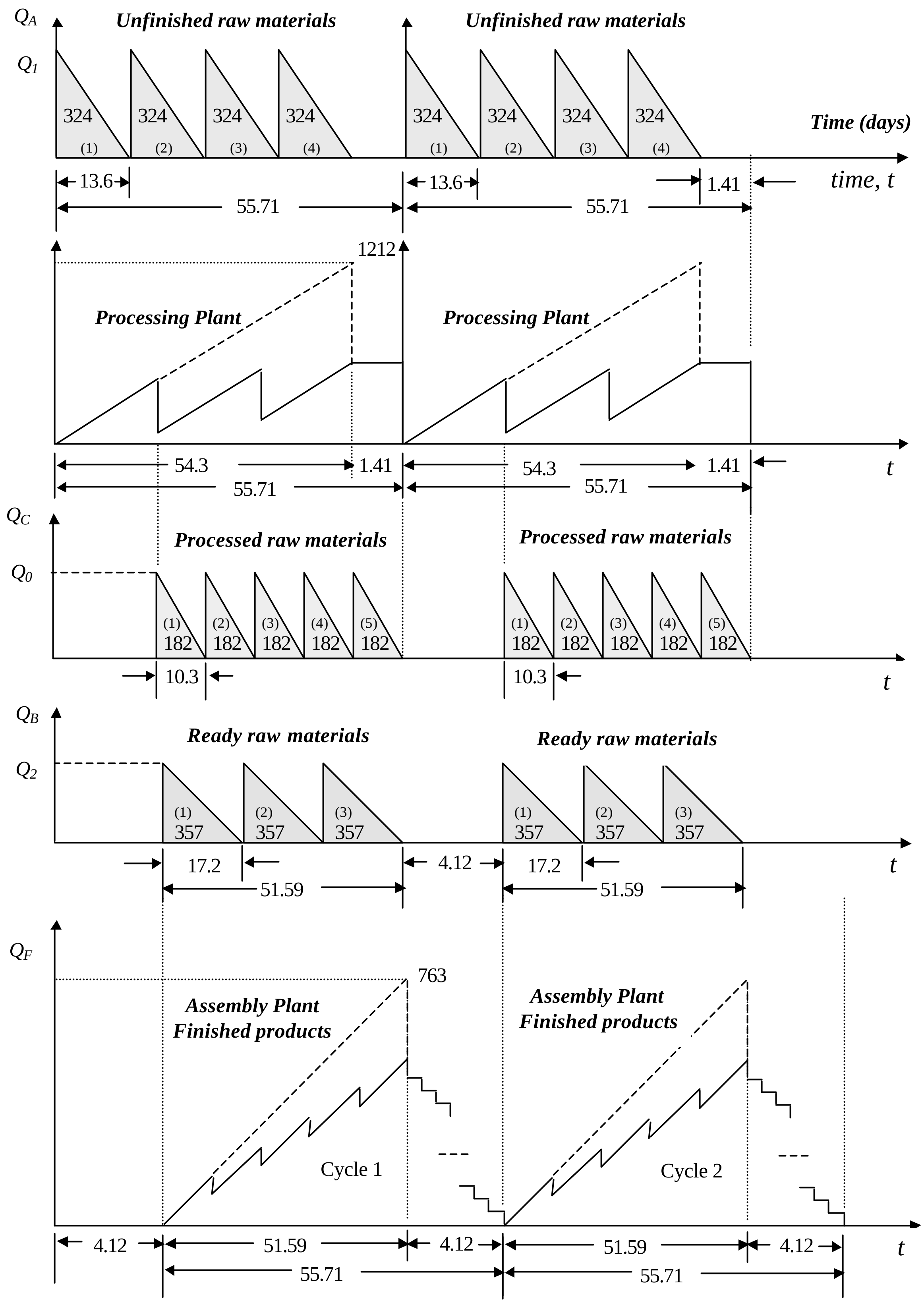

Figure 6.1. A single cycle schedule of SSSCS of (Example 3.1)

6.2 System Parameters

The demand of the finished product over a given a period of time, D_F, the production capacity of each production plant, P_w and P_F, are known. As the shortages are not allowed, the production rates are higher (or equal) than the demand rate. There are costs associated with the raw materials, such as the ordering cost, (K_A+K_B), shipping costs (S_A+S_B+S_C+S_F). The raw materials are carried in inventories and during work-in-process in processing plant and assembly plant which cause holding cost, (H_A+ H_B+ H_C+ H_w+ H_F). When unfinished raw materials are processed in the processing plants, are sent directly to the assembly plant buffer. Once finished products are manufactured, the goods are delivered when the quantity reaches order size. All of these parameters are given in Table 3.1. The results obtained from the numerical example are applied to operational scheduling.

6.3 Operations Planning

The output of the model are the optimal number of raw material orders and finished product deliveries that apply to the raw material suppliers and to the buyers to minimize cost. Since finished products are assembled from a group of raw materials with an assigned conversion factor, the batch size of the raw materials can be determined from finished product batch size and conversion factors. Also, the optimal number of orders, n^* can be used as to find the order size of the raw materials. The results of the sensitivity analysis show the variations in the total cost with appreciable variations in the input parameters. Depending on the variations of the total cost, the input parameters can be allowed to fluctuate in a specific range depending on the parametric values of the model.

Once the optimal parameters are determined, the quantity of the total raw materials that should be shipped in the production line in one cycle can be determined. Using these results, the amount of raw materials shipment that should be transported in processing plant and assembly plant can be determined. The manufacturing plants do not have to wait until it receives the complete shipment from suppliers or the previous workstation. The plant can start processing the products as soon as the first shipment arrives. This will help to reduce the cycle time of the production system. Thus, the productivity of the system can increase. Knowing the finished product demand, batch size and production rate of the assembly plant, the cycle time and the production time can be estimated. Once the production time is identified, the raw materials shipment arrival time can be determined. Similarly the cycle time and processing time of the processing plant and corresponding raw materials arrival time can be scheduled. Since there is down time in both processing plant and assembly plant in each cycle, the production setup for the next cycle can be done during this idle time. There are no shortages in the production system. To satisfy the annual demand, the production cycle runs throughout the year period.

6.4 Operational Schedule

The objective of this section is to compute the time intervals between each shipments and quantities for each type of transported raw materials and the time scale relation between the production of processing plant and assembly plant. It is also necessary to identify the arrival time of the raw materials at each of the manufacturing plant and flow time to transport components between the workstations. The demand of the each type of raw materials equals the sum of the quantities transported within the system.

6.4.1 Cycle Time

For a single-stage supply chain system that has two manufacturing plants, assembly plant and processing plant, the finished product demand and production rate and delivery size is given in Table 3.1. The delivery rate m is obtained from the solution. The batch size for the finished product $Q_F = mx$ is calculated as $25.44 \times 30 = 763.2$ units/cycle. The batch size for the *ready raw materials* and *unfinished raw materials* can be estimated from finished product batch size by using the conversion factor. The *ready raw materials* batch size is 763.2 / .65 = 1174.15 units/cycle. The batch size for unfinished raw materials is estimated as 763.2 / .9 x .7 = 1211.43 units/cycle. The calculation for cycle time and production time at processing plant and assembly plant single-period system is shown in Table 6.1.

Table 6.1. Cycle time and production time of the production plants

Assembly Plant	Cycle time	$T = \frac{Q_F}{D_F} \times 365$	$\frac{763.2}{5000} \times 365$	55.71 days/cycle
	Production time	$T_1 = \frac{Q_F}{P_F} \times 365$	$\frac{763.2}{5400} \times 365$	51.59 days/cycle
Processing Plant	Cycle time	$T_0 = \frac{Q_C}{D_C} \times 365$	$\frac{848}{5555.5} \times 365$	55.71 days/cycle
	Production time	$T_w = \frac{Q_C}{P_w} \times 365$	$\frac{848}{5700} \times 365$	54.3 days/cycle

6.4.2 Inventory Configuration

In the previous section, the cycle time and production time of manufacturing plants and the batch size for each type of raw materials were derived. The order quantities of each raw material can be determined by dividing the corresponding batch size with the number of orders needed to procure the entire batch from the suppliers. Once the production time for the manufacturing plants are known, the consumption time of ordered raw materials in each plant can be calculated. The calculation for yearly raw material inventory cost is shown in Table 6.2.

Table 6.2. Raw materials yearly inventory costs

Raw Materials	Batch Size (Q)	Order Number (n)	Order Size (q)	Prod. Time, *Days*	Cycle Time, *Days*	Holding Cost, $	$\frac{1}{2}QH\frac{T_1}{T}$, $
Unfinished	1211.43	3.74	323.9	54.3	55.71	48	8559.11
Processed	848.00	4.67	181.6	51.59	55.71	35	2946.21
Ready	1174.15	3.29	356.9	51.59	55.71	45	6749.00
Total raw materials yearly inventory costs							**$18,254.32**

As the parameters for ordering cost and shipping cost is given in Table 3.1, the total yearly ordering and shipping costs can be estimated. The expected yearly raw materials order and shipment costs are shown in Table 6.3.

Table 6.3. Raw materials yearly order and shipment costs

Raw Materials	Order/delivery Number (n)	Ordering Cost (K), $	Shipment Cost (S), $	No of Cycle/year (N)	(Shipment + Order) Cost/year $n(K+S)N$, $
Unfinished	3.74	90	220	6	6956.4
Processed	4.67	0	250	6	7005.0
Ready	3.29	95	200	6	5823.3
Finished Product	25.44	0	300	6	45792
Raw materials yearly (shipment + order) costs					**$65,576.7**

The work-in-process inventory costs for the processing plant and the assembly plant are calculated by summing up the average quantities of products are there during the operation. The summation of the total amount of products that accumulates during the production time of any plant and the same amount that carry during the down time, then deducing the amount of product that shipped during (n – 1) shipment gives the total work-in-process inventory for one cycle. Now, by dividing the total work-in-process inventory with the corresponding cycle time of any plant gives the average work-in-process inventory for that particular plant. The calculation for work-in-process inventory is shown in Table 6.4.

Table 6.4. Work-in-process (WIP) yearly inventory costs

Plants	(Q)	(N)	(T_1)	(T)	(H)	$\frac{1}{2}QT_1$ (a)	$\frac{Q(T-T_1)}{n}$ (b)	$\frac{Q(n-1)T}{n}$ (c)	WIP inventory $=\frac{(a+b-c)}{T}H$, $
Processing	848	4.67	54.3	55.7	33	23023	1195.7	7949.9	4,818.5
Assembly	763.2	25.44	51.6	55.7	38	19687	3144.4	1606.6	7,239.0
Total work-in-process yearly inventory costs									**$12,057.5**

Adding the costs obtained from (Table 6.1-6.3), the total yearly inventory cost equals = $18,254.32 + $65,576.7 + $12,057 = $95,888.52. The optimal total cost obtained is $95,896.34, which is $7.82 more than the calculated total cost.

CHAPTER 7

RESEARCH CONCLUSION

This chapter highlights the goals that were accomplished in the current research. A summary of the overall results is covered in the conclusion. Some suggestions for future research direction are recommended to develop more realistic model for the current production environment.

7.1 Conclusion

This paper addresses optimal order placement and delivery rate policies for a single-stage assembly type supply chain system. The system encompasses batch production process where finished product demand is approximately constant for an infinite planning horizon. Two distinct types of raw materials arrivals are in the system in which one group of raw materials requires preprocessing before emerging to the assembly line. The customer demand is assumed to be satisfied. In the total cost function, raw materials procurement rate and finished product delivery rate are the input variables. The objective of this model is to determine the integer solution of the variables so as to minimize the expected total costs incurred by inventory, ordering, shipping, (production) setup and purchasing costs. The system handles inventories for raw materials (unfinished, processed and ready raw materials), work-in-process inventory and finished products, separately.

Under certain assumptions, a (4-varible) nonlinear convex function was developed. Solving the original problem with relaxed integer constraints, a set of closed-form optimal solutions is obtained. The branch and bound algorithm is used to find the integer values of the variables. From the computational experiment results showed in Table 4.2, it is observed that the total cost found by B&B algorithm is within 99% of the optimal total cost. The sensitivity analysis has been focused on the dynamic nature of the system parameters and their influences on model costs. From the sensitivity investigation, it is observed that batch size, shipment order size and delivery rate have significant effect on total cost and are interactive in many cases. The sensitivity of procurement of raw materials is greater at lower rate (>3) and level at (3-5) and again increases slowly at the higher rate (<5). The production rate at processing stage and finished product transportation cost reveal significant effects on total cost. The analysis also identified the incremental total cost due change of holding costs and the interactive nature between raw materials order cost and production setup costs. Possible areas of modification are also suggested during the sensitivity analysis for effective improvement of the model.

This research reveals the insights into the order policies and inventory sequence of an assembly type problem when two components are required to produce the finished product. Using a rigorous mathematical approach, the optimal solution of the problem is accomplished. In the practical situation, the proposed model can be implicated in industries producing regularly consumable product where product supply is in the JIT environment and their demand pattern is approximately constant.

7.2 Scope of the Future Research

This research addressed raw materials procurement rate, different inventories, production of processing stage and assembly stage and finally, finished product delivery plan. The work may be extended in several directions.

- **Capacity constraints:** One direction for future research is the incorporation of capacity constraints in the model. In certain situations it might be more logistic to put the lower or upper bounds on production capacities. Budgetary constraints, storage space constraints can also be the extension of this research. The solution methodology of such type of problem would require the constrained optimization technique.

- **Demand pattern:** The second research direction may be the change of demand pattern of the products. The problem can be expanded to the stochastic demand pattern, which is suitable for short life products or high-tech products. Trend demand pattern with distinct phases (inception, market maturity and saturation) can be another possible extension, which is more appropriate for cutting-edge products (electronic product). For the stochastic model, the probabilistic nature of the demand pattern needs to be identified; whereas, aggregated rate (increment/decrement) of shipment with the initial delivery quantity needs to be developed for the model with a trend (linear increasing, maturity and decreasing) demand.

- **Multi-stage production** Consideration of multi-stage production process may be another possible extension of the present work. Several types of raw materials may be used and some of the raw materials may require a series of production system to convert into finished product.

- **Multiple finished products:** Another extension may include manufacturing multiple numbers of finished products. The nature of the raw materials may be different from each other and the quantity required of one raw material may be different than the other to convert into finished products.

- **Transporting system:** Integrating transporting system (Kanban mechanism) in the production process and their scheduling between workstations are the realistic issues in manufacturing systems. Incorporating the transportation facilities can be a possible extension of this research.

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APPENDIX A

DERIVATION OF WORK-IN-PROCESS INVENTORY

The Work-in-process inventory level at any time t, described in Figure 4, may be displayed, as the quantity of processed items by time t and the consumption of that inventory by the same time, in Figure A.1. Let the level of processed raw material inventory at any time t, $Q_C(t) = Q_P(t) - Q_w(t)$ be the excess amount of processed items, $Q_P(t)$, over the amount of transported item, $Q_w(t)$.

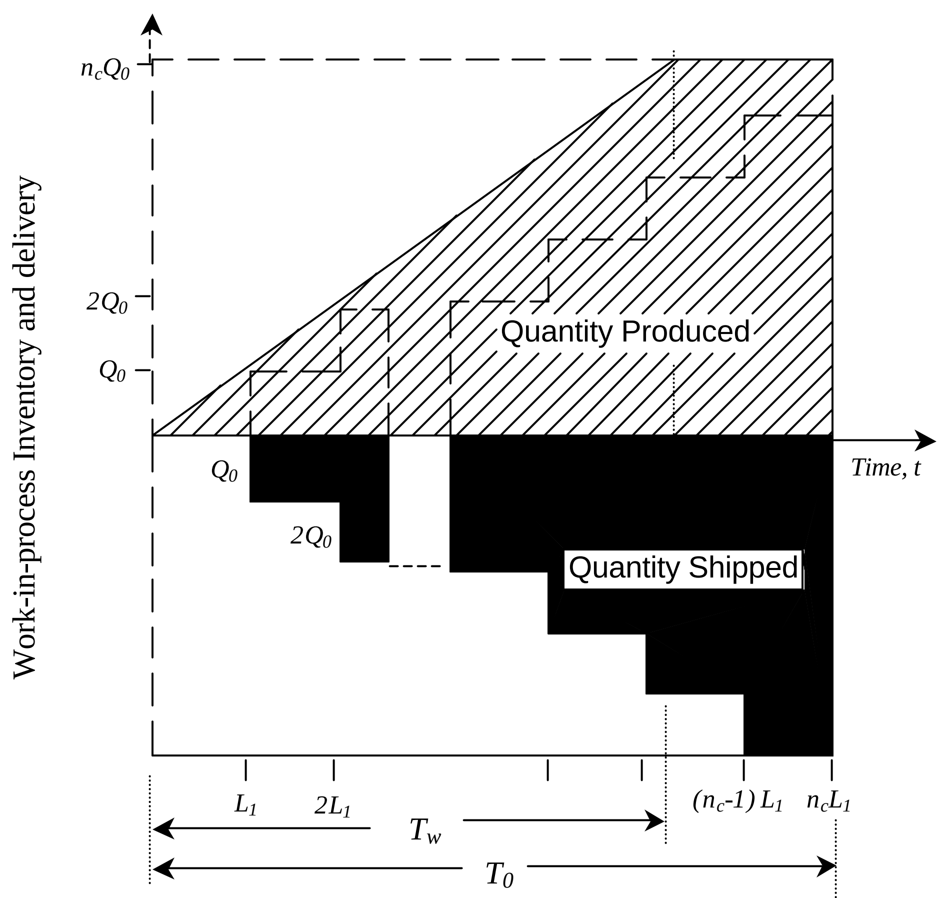

Figure A.1. Work-in-process inventory build-up at processing stage

The average work-in-process inventory at processing stage per cycle, Q_{avg}, is computed as

$$Q_{avg} = \frac{1}{T_0}\int_0^{T_0} Q_C(t)dt = \frac{1}{T_0}\left(\int_0^{T_{01}} Q_P(t)dt - \int_0^{T_0} Q_w(t)dt\right) \quad \text{(A.1)}$$

The function $Q_M(t)$ and $Q_s(t)$ are calculated as follows:

$$Q_P(t) = \begin{cases} P_w t & 0 \le t \le T_w \\ P_w T_w & T_w \le t \le T_0 \end{cases}$$

Thus,

$$\frac{1}{T_0}\int_0^{T_0} Q_P(t)dt = \frac{1}{T_0}\left(\int_0^{T_w} P_w t dt + \int_{T_w}^{T_0} P_w T_w dt\right) = \frac{1}{T_0}\left(\frac{P_w T_w{}^2}{2} + P_w T_w (T_0 - T_w)\right)$$

$$= P_w T_w\left(1 - \frac{T_w}{2T_0}\right) \quad \text{(A.2)}$$

At time $t = T_w$, the amount produced $Q_P(t) = P_w T_w$ and substituting $P_w T_w = Q_C$ in the Equation (A.2), it can be expressed as

$$\frac{1}{T_0}\int_0^{T_1} Q_P(t)dt = Q_C\left(1 - \frac{T_w}{2T_0}\right) \quad \text{(A.3)}$$

Since $T_0 = n_c L_1$, and the first Q_0 units of products are shipped out for $Q_0(n_c - 1)L_1$ unit-year, and then, the second shipment are made at $Q_0(n_c - 2)L_1$ unit-year and so on until the last shipments of Q_0 units are shipped for is $Q_0 L_1$ unit-year. Hence, the total shipment year is $Q_0 L_1[(n_c - 1) + (n_c - 2) + ... + 2 + 1]$. The average quantity of Work-in-process inventory during the cycle time T_1 is given by

$$\frac{1}{T_0}\int_0^{T_0} Q_w(t)dt = \frac{1}{T_0}\int_0^{n_c L} Q_S(t)dt = \frac{1}{T_0} Q_0 L[(n_c - 1) + (n_c - 2) + ... + 2 + 1]$$

$$= \frac{1}{T_0} Q_0 \frac{T_0}{n_c} \frac{(n_c - 1)(n_c - 1 + 1)}{2} = \frac{Q_0}{2}(n_c - 1) \tag{A.4}$$

in which the relation, $L_1 = \frac{T_0}{n_c}$ and substituting the value of Equations (A.3) and (A.4) in the Equation (A.1), the Work-in-process inventory, Q_{avg}, is calculated as

$$Q_{avg} = \frac{1}{T_0}\int_0^{T_0} Q_P(t)dt - \frac{1}{T_0}\int_0^{n_c L} Q_w(t)dt$$

$$= Q_C\left(1 - \frac{T_w}{2T_0}\right) - \frac{Q_0}{2}(n_c - 1) \tag{A.5}$$

The Work-in-process batch size Q_C is transferred by Q_0 amount per shipment by n_c number of shipments during the cycle time T_1, hence, $Q_0 = Q_c/n_c$, the Equation (A.5) can be simplified as

$$Q_{avg} = Q_C\left(1 - \frac{T_w}{2T_1}\right) - \frac{Q_C}{2n_c}(n_c - 1)\,. \tag{A.6}$$

The quantity of processed raw materials produced at the rate of P_w in time T_w satisfies the processed raw materials demanded amount D_C in the assembly line. Since the processed raw materials are carried during the processing stage cycle time T_0, the time T_w is adjusted by $T_w . P_w = T_0 . D_C$, which is rewritten as $T_w = T_0 \frac{D_C}{P_w}$. Arranging the Equation (A.6), the average finished product inventory, Q_{avg}, can be written as

$$Q_{avg} = \frac{Q_C}{2}\left(1 + \frac{1}{n_c} - \frac{D_C}{P_w}\right). \tag{A.7}$$

APPENDIX B

DERIVATION OF AVERAGE FINISHED PRODUCT INVENTORY

The inventory of the finished product at any time t is $Q_F(t) = Q_M(t) - Q_S(t)$ in which $Q_M(t)$ is the quantity of assembled items and $Q_S(t)$ the delivered quantity. The on-hand inventory and corresponding delivery of the assembled products are shown in Figure B.1.

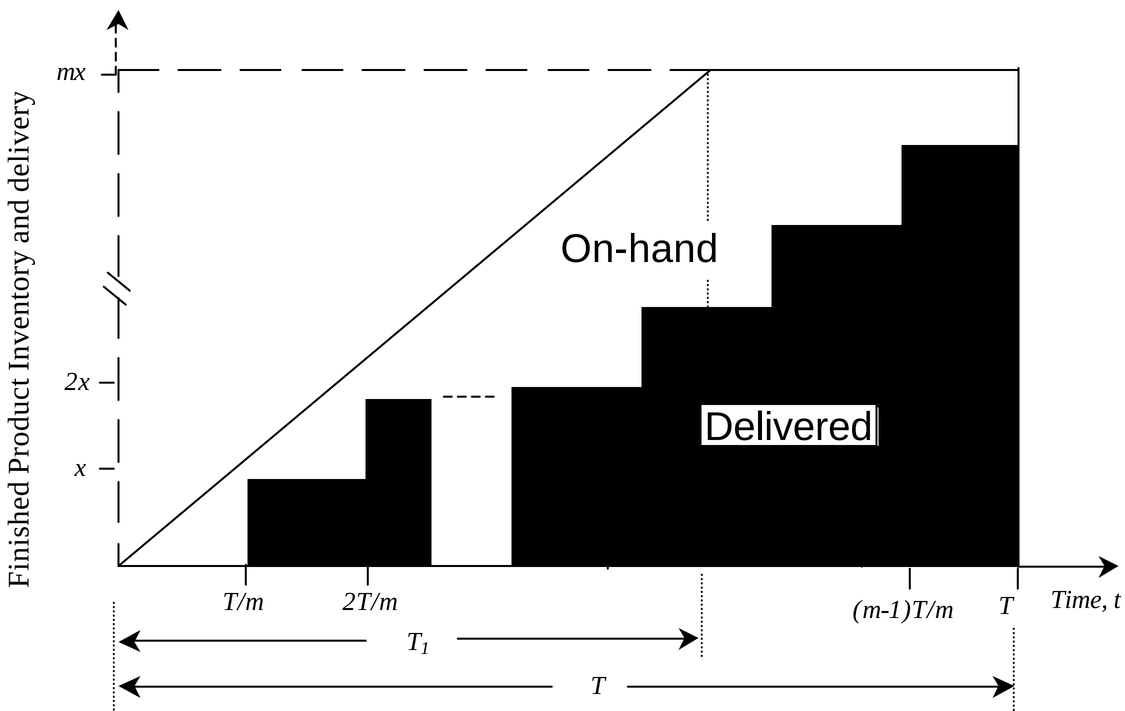

Figure B.1: Finished product inventory build-up

The average finished product inventory per cycle, I_{avg}, is computed as

$$I_{avg} = \frac{1}{T}\int_0^T Q_F(t)dt = \frac{1}{T}\left(\int_0^T Q_M(t)dt - \int_0^T Q_S(t)dt\right) \tag{B.1}$$

The calculation for the assembled items $Q_M(t)$ is as follows

$$Q_M(t) = \begin{cases} P_F t & 0 \le t \le T_1 \\ P_F T_1 & T_1 \le t \le T \end{cases}$$

Thus,

$$\frac{1}{T}\int_0^T Q_M(t)dt = \frac{1}{T}\left(\int_0^{T_1} P_F t dt + \int_{T_1}^{T} P_F T_1 dt\right)$$

$$= \frac{1}{T}\left(\frac{P_F T_1^{\ 2}}{2} + P_F T_1 (T - T_1)\right) = P_F T_1\left(1 - \frac{T_1}{2T}\right) \quad \text{(B.2)}$$

The amount of the assembled items produced at time $t = T_1$, is $P_F T_1 = Q_F$ and substituting $P_F T_1 = Q_F$ in the Equation (B.2)

$$\frac{1}{T}\int_0^T Q_M(t)dt = Q_F\left(1 - \frac{T_1}{2T}\right). \quad \text{(B.3)}$$

The calculation for the delivery of finished product quantity $Q_S(t)$ at any time t is stated as follows. The delivery quantity for the first shipment, x units, is shipped at $x(m-1)L_2$ unit-year, and then, the second shipment is ended by $x(m-2)L_2$ unit-year. The shipments continue until the last x units are delivered by xL_2 unit-year. Therefore, the total shipment year is $xL_2[(m-1)+(m-2)+...+2+1]$. Since $T = mL_2$, the average quantity of assembled finished product inventory is calculated as

$$\frac{1}{T}\int_0^T Q_S(t)dt = \frac{1}{T} xL_2[(m-1)+(m-2)+...+2+1]$$

$$= \frac{1}{T} x \frac{T}{m}\frac{(m-1)(m-1+1)}{2} = \frac{x}{2}(m-1) \quad \text{(B.4)}$$

Substituting the value of Equations (B.3) and (B.4) in the Equation (B.1), the finished products inventory, I_{avg}, is

$$I_{avg} = \frac{1}{T}\left(\int_0^T Q_M(t)dt - \int_0^T Q_S(t)dt\right) = Q_F\left(1 - \frac{T_1}{2T}\right) - \frac{x}{2}(m-1). \quad \text{(B.5)}$$

To complete the delivery of the finished product batch Q_F, it requires m number of shipment with a size of x units, therefore $Q_F = mx$, so the Equation (B.5) is stated as

$$I_{avg} = mx\left(1 - \frac{T_1}{2T}\right) - \frac{x}{2}(m-1)\,. \tag{B.6}$$

Since the quantity of assembled items is sufficient to meet the demand of the customers, so $\frac{T_1}{T} = \frac{D_F}{P_F}$, and rearranging the Equation (B.6), the average finished product inventory, I_{avg}, can be written as

$$I_{avg} = \frac{mx}{2}\left(1 + \frac{1}{m} - \frac{D_F}{P_F}\right). \tag{B.7}$$

APPENDIX C

PROOF OF CONVEXITY OF A NON-LINEAR FUNCTION

For a non-linear function, it is necessary to prove the convexity to assess that the stationary point x^* corresponds to a minimum. A function, *f(x)*, may be convex over some convex region *R*, if the Hessian matrix, *H(x)*, is positive definite or semi-definite for all value x in *R*.

The Hessian matrix associated with any function f(x_1, …, x_n) is an *n* x *n* symmetric matrix and is given by

$$H = \left[\frac{\partial^2 f}{\partial x_i \partial x_j} \right] = \nabla^2 f \qquad \text{(C.1)}$$

for Equation (17) the second derivatives are calculated as follows

$$\frac{\partial^2 Z}{\partial m^2} = \frac{2}{m^3}\left(An_a + Bn_b + Cn_c + f \right)$$

$$\frac{\partial^2 Z}{\partial m \partial n_a} = -\frac{A}{m^2} - \frac{D}{n_a{}^2} = -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2} \right)$$

$$\frac{\partial^2 Z}{\partial m \partial n_b} = -\frac{B}{m^2} - \frac{E}{n_b{}^2} = -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2} \right)$$

$$\frac{\partial^2 Z}{\partial m \partial n_c} = -\frac{C}{m^2} - \frac{F}{n_c{}^2} = -\left(\frac{C}{m^2} + \frac{F}{n_c{}^2} \right)$$

$$\frac{\partial^2 Z}{\partial n_a{}^2} = 2m\frac{D}{n_a{}^2}, \qquad \frac{\partial^2 Z}{\partial n_a \partial n_b} = 0, \qquad \frac{\partial^2 Z}{\partial n_a \partial n_c} = 0$$

$$\frac{\partial^2 Z}{\partial n_b{}^2} = 2m\frac{E}{n_b{}^2}, \qquad \frac{\partial^2 Z}{\partial n_a \partial n_c} = 0, \qquad \frac{\partial^2 Z}{\partial n_c{}^2} = 2m\frac{F}{n_c{}^2},$$

The Hessian matrix H is stated as

$$H = \begin{bmatrix} \frac{2}{m^3}(An_a + Bn_b + Cn_c + f) & -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) & -\left(\frac{C}{m^2} + \frac{F}{n_c{}^2}\right) \\ -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & 2m\frac{D}{n_a{}^2} & 0 & 0 \\ -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) & 0 & 2m\frac{E}{n_b{}^2} & 0 \\ -\left(\frac{C}{m^2} + \frac{F}{n_c{}^2}\right) & 0 & 0 & 2m\frac{F}{n_c{}^2} \end{bmatrix} \tag{C.2}$$

It is observed that all diagonal elements are positive, so the principle determinants $\boldsymbol{D_2}$, $\boldsymbol{D_3}$, and $\boldsymbol{D_4}$ are as follows:

$$D_2 = \begin{vmatrix} \frac{2}{m^3}(An_a + Bn_b + Cn_c + f) & -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) \\ -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & 2m\frac{D}{n_a{}^2} \end{vmatrix}$$

$$= \frac{4mD}{m^2 n_a{}^2}(An_a + Bn_b + Cn_c + f) - \left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right)^2$$

Let $a = \frac{A}{m^2}$, and $b = \frac{D}{n_a{}^2}$

$$D_2 = 4ab + (An_a + Bn_b + Cn_c + f) - (a + b)^2$$

$$D_2 = (An_a + Bn_b + Cn_c + f) - (a - b)^2$$

The expression is positive, if $D_2 \geq 0$. Therefore, it must be

$$(An_a + Bn_b + Cn_c + f) - (a - b)^2 \geq 0$$

$$(An_a + Bn_b + Cn_c + f) \geq (a - b)^2$$

or

$$(An_a + Bn_b + Cn_c + f) \geq \left(\frac{A}{m^2} - \frac{D}{n_a{}^2} \right)^2 \qquad \text{(C.3)}$$

$$D_3 = \begin{vmatrix} \frac{2}{m^3}(An_a + Bn_b + Cn_c + f) & -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) \\ -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & 2m\frac{D}{n_a{}^2} & 0 \\ -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) & 0 & 2m\frac{E}{n_b{}^2} \end{vmatrix}$$

$$D_3 = \frac{2}{m^3}(An_a + Bn_b + Cn_c + f)2m\frac{D}{n_a{}^2}2m\frac{E}{n_b{}^2} - \left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right)^2 2m\frac{E}{n_b{}^2}$$

$$+ \left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2 2m\frac{D}{n_a{}^2}$$

Proving the expression is positive, $D_3 \geq 0$:

$$\frac{8}{m}(An_a + Bn_b + Cn_c + f)\frac{D}{n_a{}^2}\frac{E}{n_b{}^2} + \left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2 \frac{2mD}{n_a{}^2} \geq \left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right)^2 2m\frac{E}{n_b{}^2}$$

or

$$\frac{4}{m^2}(An_a + Bn_b + Cn_c + f)\frac{D}{n_a{}^2}\frac{E}{n_b{}^2} + \left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2 \frac{D}{n_a{}^2} \geq \left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right)^2 \frac{E}{n_b{}^2} \qquad \text{(C.4)}$$

$$D_4 = \begin{bmatrix} \frac{2}{m^3}(An_a + Bn_b + Cn_c + f) & -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) & -\left(\frac{C}{m^2} + \frac{F}{n_c{}^2}\right) \\ -\left(\frac{A}{m^2} + \frac{D}{n_a{}^2}\right) & 2m\frac{D}{n_a{}^2} & 0 & 0 \\ -\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right) & 0 & 2m\frac{E}{n_b{}^2} & 0 \\ -\left(\frac{C}{m^2} + \frac{F}{n_c{}^2}\right) & 0 & 0 & 2m\frac{F}{n_c{}^2} \end{bmatrix}$$

$$D_4 = \frac{2}{m^3}(An_a + Bn_b + Cn_c + f)2m\frac{D}{n_a{}^2}2m\frac{E}{n_b{}^2}2m\frac{F}{n_c{}^2} - \left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2 2m\frac{D}{n_a{}^2}2m\frac{F}{n_c{}^2}$$

or

$$D_4 = 16(An_a + Bn_b + Cn_c + f)\frac{D}{n_a{}^2}\frac{E}{n_b{}^2}\frac{F}{n_c{}^2} - 4m^2\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2\frac{D}{n_a{}^2}\frac{F}{n_c{}^2}$$

For expression to be positive, $D_4 \geq 0$, thus

$$16(An_a + Bn_b + Cn_c + f)\frac{D}{n_a{}^2}\frac{E}{n_b{}^2}\frac{F}{n_c{}^2} - 4m^2\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2\frac{D}{n_a{}^2}\frac{F}{n_c{}^2} \geq 0$$

$$16(An_a + Bn_b + Cn_c + f)\frac{D}{n_a{}^2}\frac{E}{n_b{}^2}\frac{F}{n_c{}^2} \geq 4m^2\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2\frac{D}{n_a{}^2}\frac{F}{n_c{}^2}$$

or

$$4(An_a + Bn_b + Cn_c + f)\frac{E}{n_b{}^2} \geq m^2\left(\frac{B}{m^2} + \frac{E}{n_b{}^2}\right)^2 \qquad \text{(C.5)}$$

Consequently, if the equations (C.3), (C.4), (C.5), are satisfied for the given conditions, the principle determinant ***D₂***, ***D₃***, and ***D₄*** are all positive and H is proved to be a positive definite matrix. Therefore, the function of *Z* in Equation (18) is convex.

VITA

Mohammad Anwar Ashek Rahman was born in Bangladesh, on December 25, 1970. He was educated in Pabna Cadet College in Bangladesh. He received his bachelor of science degree in mechanical engineering and a master's degree in Industrial and Production Engineering from Bangladesh University of Engineering and Technology in 1995 and 1998, respectively.

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