

**OPTIMAL BATCH QUANTITY MODELS FOR A LEAN PRODUCTION
SYSTEM WITH REWORK AND SCRAP**

A Thesis

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ABSTRACT

In an imperfect manufacturing process, the defective items are produced with finished goods. Rework process is necessary to convert those defectives into finished goods. As the system is not perfect, some scrap is produced during this process of rework. In this research, inventory models for a single-stage production process are developed where defective items are produced and reworked, where scrap is produced, detected and discarded during the rework. Two policies of rework processes are considered (a) First policy: rework is done within the cycle, and (b) Second policy: rework is done after N cycles of normal production. Also, three types of scrap production and detection methods are considered for each policy, such as (i) scrap is detected before rework, (ii) scrap is detected during rework and (iii) scrap is detected after rework. Based on these inventory situations, the total cost functions for a single-stage imperfect manufacturing system are developed to find the optimum operational policy. Some numerical examples are provided to validate the model and a sensitivity analysis is carried out with respect to different parameters used to develop the model.

CHAPTER 1

INTRODUCTION

Manufacturing processes are sometime imperfect; for that reason their output may contain defective items. These defective items can be reworked, scrapped, subjected to other corrective processes or sold at reduced prices, but the result is increased extensive costs in every case. In recent decades, researchers tried to determine the optimal batch quantity of imperfect production system considering different operating conditions. Research is focused on the practical situations involved in a single or multi-stage production system.

To evaluate an inventory system policy properly requires determining the optimal batch-sizing model along with setup cost, inventory holding cost, processing cost and shortage cost. The demand and production pattern of a manufacturing facility affect the optimal batch quantity or the economic lot size. When raw materials are processed through a production process into finished goods, three types of finished products can be delivered due to various production qualities and material defects. These are (a) quality of finished products, (b) reworkable defective products, and (c) scrap. Additional resources with substantial costs are needed in cases where rework is involved.

In a multi-stage production process where products move from one stage to the next stage, the number of defectives may vary. Depending on the proportion of defectives, the optimal batch quantity also varies – these affect the costs of processing, setup, and holding of inventory. Whenever a production system has rework or repair facilities, some scrap could still be produced and, if no such facilities exists, all defective products go to scrap, incurring additional cost as well as the loss of goodwill if the

company fails to meet customers' demands. The flow diagram of a multi-stage, imperfect production system that produces both good product and defective items simultaneously, is shown in Figure 1.

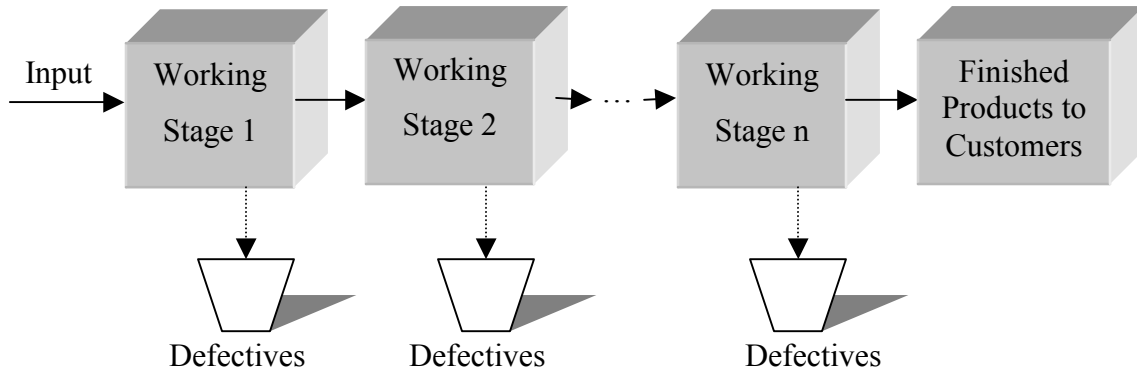


Figure 1.1: A multi-stage production system with defective items.

1.1 The Problem

Determining the optimal batch quantity of familiar inventory models has been a primary mission among the researchers for years. Most of the research is devoted to developing well-known inventory models with ideal conditions. In most manufacturing processes, with single or multiple stages, some defective items are produced. Even if the production process may have the rework capability, some percentage of scrap may not be avoided even after the rework process. For these reasons, some manufacturing systems experience a shortage of products. These result in customer dissatisfaction (arising from unfilled demand) and loss of goodwill.

This research focuses on reworking of defective items with less than 100% recovery, resulting in a certain percentage of scrap during or after the rework process. In an imperfect manufacturing process with reworking facility, the inventory of finished

goods can build up if the production rate is higher than the demand rate. The built-up of inventory contains both good and defective items. The defective items are reprocessed through the system's reworking facility. The inventory builds up again during the reworking process resulting in scrap being produced.

Depending on the scrap production, there might be a reduction in the inventory of finished goods. Hence, the total cost of the system, which is significantly dependent on the inventory of the finished goods, will be affected. To produce the required quantity, the system needs machinery setup, incurring a setup cost. Holding the inventory throughout the production period requires an inventory carrying cost. Processing the raw material for finished goods production and reworking the defectives involve processing cost. In this research the total cost of the production system will vary due to built-up of different inventories and processing cost of regular and rework processes. In order to discourage defective and scrap production, a penalty cost will be incorporated in the total cost function.

1.1.1 Rework Problem

The rework process is nothing but the correction process of the defective items produced during normal production. This research deals with two types of rework processes: (a) within-cycle rework, where the defectives are reworked within the same cycle, and (b) rework after N cycles, in which the defective items from each cycle are accumulated until completion of N cycles of normal production, after which the defective parts are reprocessed. For both policies the built-up inventory situations are different from the ideal ones. As a result, the modeling perspectives are also different.

1.1.2 Scrap Detection Problem

Some of the defectives might not be transformed into good items through the rework process described above. Hence, they are discarded as scrap and the inventory decreases. This scrap can be detected in three ways – before, during and after the rework process. In this research, the scrap production and detection techniques are involved for both operational policies: (a) within-cycle rework, and (b) rework after N cycles as described above. Thus the inventory of the system will form a different pattern from a traditional one due to rework process and scrap production. Figure 1.2 shows the problem structure and their relationship in a hierarchical order. The three scenarios of scrap detection are described below:

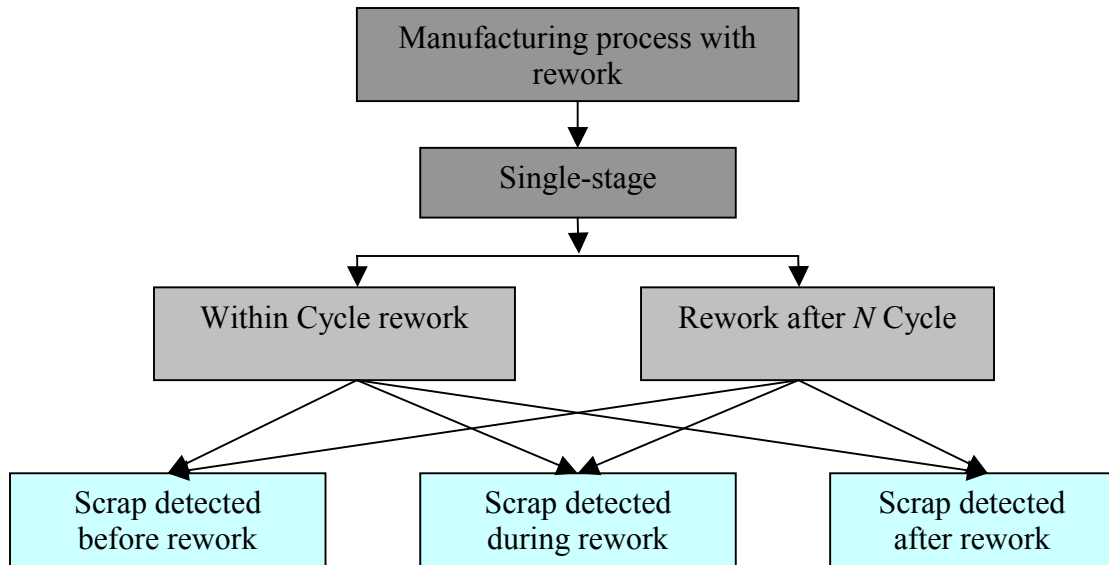


Figure 1.2: Problem structure and their relationship in hierarchical order.

(a) In an imperfect production system, the good and defective items produced together and the inventory builds up as the process continues. These defective items are reworked and the corrected items are added to the inventory. Some scrap produced

during the entire production process discarded. Defective items excluding the scrap identified at the beginning of rework are reprocessed.

(b) Sometimes scrap is produced and detected during the rework process. During rework scrap may take less time to produce than a good item. A reduction in the finished goods inventory occurs during the rework. For that reason, at the end of the production a shortage arises that results in unsatisfied customers' demand.

(c) Scrap may be detected at the end of the rework process of the defective items. After the end of that rework scrap is discarded which results in a further shortfall.

In this research, it is hypothesized that a large portion of the defective items can be transformed into good items through rework process. The optimal batch quantity might be obtained by optimizing the total cost of the production system with respect to the finished goods inventory of the system.

1.2 Applications

Glass and silicon wafer probes are illustrations of modern product. Every office or home needs glass for their doors, windows, etc., assembly industries need fasteners and cellular phone industries require wafer probes.

In a nut manufacturing industry the steel bar is the raw material for manufacturing nuts. At the beginning of the production these bars are sheared to length. Next one end of the bar is heated in the induction furnaces. After that head of the nut is forged by an upsetter and threads are either cut or rolled. At the end of the process the produced nuts are heat treated or galvanized. During the galvanization, some defects may occur and the nut turns into a defective product. Then, that item is fed again into the galvanizing stage for correction. Before the correction process scrap results due to improper thread cutting

and are discarded immediately. Thus, scrap is found before rework and good item inventory builds up.

The silicon wafer is an important part for the cellular phone manufacturing. The first step in the wafer manufacturing process is the formation of a large, silicon single crystal or ingot. This process begins with the melting of polysilicon, with minute amounts of electrically active elements such as arsenic, boron, phosphorous or antimony in a quartz crucible. Once the melt has reached the desired temperature, a silicon seed crystal is lower into the melt. The melt is slowly cooled to the required temperature, and crystal growth begins around the seed. As the growth continues, the seed is slowly extracted from the melt. The temperature of the melt and the speed of extraction govern the diameter of the ingot, and the concentration of an electrically active element in the melt governs the electrical properties of the silicon wafers to be made from the ingot. This is a complex and proprietary process requiring many control features on the crystal-growing equipment. After production of the ingots, they are extracted from the crystal pulling furnaces and allow them to cool. Then the ingots are grinded to the specified diameter. Next, the ingot is sliced into thin wafers using a 10-ton wire saw. The basic principle of wire sawing is to feed the ingot into a web of ultra-thin, fast moving wire. After the sawing process, the individual slices have sharp, fragile edges. These edges must be rounded in order to provide strength to the wafer. Profiling will ultimately prevent chipping or breakage in subsequent internal processing and during device fabrication. Lapping removes controlled amounts of silicon from a wafer using slurry. This process removes saw damage and final polishing and cleaning processes give the wafers the clean and super-flat mirror polished surfaces required for the fabrication of semiconductor

devices. Some of the products are further processed into epitaxial wafers. Thus, the wafer production is completed and during the process some of the wafers are turned into defectives due to extra silicon particles or breakage or chipping. They are processed again through the profiling or fabrication for correction. Due to various reasons, some of the defective wafers are turned into scrap during the rework process and are discarded. The corrected wafers are added to the inventory of good items and supplied to the customer. Thus, scrap can be produced and detected during the rework process.

Sometime the wafer scrap is detected after all rework processes are completed and at the end of the production the wafer scrap is discarded. This scrap results from various causes, such as imperfect polishing, sawing damages, etc. Thus, the total inventory of good wafers is reduced due to wafer scrap production. Glass manufacturing, bolt manufacturing also have the same type of rework with scrap problem.

Some examples of such industries are Cardinal Glass Industry, Eden Prairie, MN, Portland Bolt & Mfg. Co., Inc, and GGB Industries, Inc., Naples, FL which produce glass, nuts and fastners, wafer probe, respectively. They have a large inventory with single and multi-stage production system. These companies also produce defective items and reprocess them in their reworking facility. This research will significantly affect the inventory systems of these companies, which eventually might increase their profitability that they lose due to scrap production.

1.3 Research Goals

The intention of the research is to study and model the inventory system in an imperfect manufacturing facility where the rework option is available. In such a facility, defective items are produced with the finished goods, and these defectives are

reprocessed. Scrap is also produced and detected in different stages of production: before, during, and after the reprocessing of the defectives. This research also proposes a technique to satisfy the customers' demand, which may not be obtained due to scrap production. The principal motivation of this research is to minimize the total system cost of the inventory of an imperfect manufacturing process.

1.4 Research Objectives

The behavior of inventory patterns in production process with rework capabilities is different from the traditional inventory patterns for an ideal production process. Defective items and scrap is produced during the normal production due to imperfect system. To repair these defectives, a manufacturing process may incorporate various reprocessing techniques in rework facilities. Some scrap that is produced before, during and after the reprocess is discarded causing a reduction in the inventory. Due to the above reasons, the natures of the inventories of these systems are different from traditional ones. Hence, the primary objectives of this research are:

- (i) To study the behavior of the inventories in different reworking policies and scrap production.
- (ii) To find the optimal order policy for raw materials.
- (iii) To determine an optimal safety stock to meet the shortage of the inventory due to scrap (rejection).
- (iv) To set up the optimal batch size for production of the items.
- (v) To find the operational schedule (implementation) of the production process.

CHAPTER 2

LITERATURE REVIEW

Prior research having to do or repair option and scrap in the production systems are rare. Several researchers have developed economic lot size or optimal batch quantity model considering the perfect inventory model without any rework or scrap option. They have considered the perfect production process whereas most production processes are often imperfect. Kumar and Vrat (1979) tried to focus in this area and developed optimal batch size of finished goods inventory for a multi-stage production system. Goyal (1978) also mentioned the effect of amplified in-process inventory economic batch size model for a multi-stage production system. Neither of them has considered the defectives production. Chandra *et al.* (1997) have considered a model of batch quantity in a multi-stage production system with different proportion of defectives in every stage, but they ignored the rework option. Sarker *et al.* (2001b) have developed an optimum batch quantity considering rework but they also have ignored the scrap option during rework process.

2.1 Optimal Lot Sizing Problems

Goyal (1976) pointed out that, in a typical industrial purchasing situation, the buyer's order quantity is so small from a producer's perspective that the producer's setup cost per batch is usually larger than buyer's ordering cost. He suggested an integrated lot sizing approach that would minimize the joint total cost to both parties. Banerjee (1986) and Monahan (1989) developed lot-sizing models for the vendor, where the approach is to induce the customer to order in larger lots through offers of quantity price discount. These models did not consider the work-in-process, which occupy the significant amount

of total inventories. Banerjee (1992) considered periodic, discrete customer demand and assumed that the vendor's production rate was finite. Considering the effects of such a finite production rate on work-in-process inventories and consequently on the batching decision itself, they developed two models to determine the producer's optimal and independent course of action in terms of lot sizing, in response to customer's periodic ordering policy. In 1990, Banerjee and Burton made efforts to account for work-in-process inventories in their single and multi-stage batch sizing models under uniform demand, simultaneously. Clark and Armentano (1995) proposed a heuristic for the resource-limited multi-stage lot-sizing problem with general product structures, setup costs and resource usage, work-in-process inventory costs and lead times. Porteus (1986) derived a significant relationship between quality and lot size. He showed clearly that the improved output quality is achievable by reducing lot size. Porteus (1985) also proposed a model that can obtain the optimum setup cost and investment required in achieving this setup cost in discounted and undiscounted economic order quantity models.

Goyal and Gunashekharan (1990) developed a mathematical model, which showed how the total cost system could be affected by the investment in quality. They also considered the investment in quality and production batch size.

2.2 Optimal Batch Quantity with Rework and Scrap

In a manufacturing facility, production of defective items is a common. These defectives can reduce profitability. For a long time, researchers have avoided the defective items production problem. Gupta and Chakrabarty (1984) considered this problem and have brought it to researchers' attention. They dealt with the rework process in a multi-stage production system. They formed a model of a system where all defective

items are collected after producing the finished good, and those are fed into the first stage of production process for reworking. In their research, they developed a model for optimal production batch quantity and optimal recycling lot size to minimize the total operational cost of such circumstances. Chakrabarty and Rao (1988) introduced the rework process in a multi-stage production system incurring two operational policies for processing reworked lots. In one case the rework is done in the same stage where it occurs. For the first case they introduced a buffer to deliver the shortage occurred due to defective item production. The other case the rework is done immediately at the same stage from where they are produced before the whole lot is sent to the subsequent stages and subsequent batches are taken only after processing of reworked lots through all the stages is completed. They also developed the optimal batch quantity for both cases to minimize the total system cost.

Wein (1992) considered a yield problem within semiconductor fabrication and developed a mathematical model for rework and scrap decisions in a multi-stage production system to determine the effect of rework option using Markov decision model. Chandra *et al.* (1997) studied a problem that consists of optimum production batch size in a multi-stage production facility with scrap ignoring the rework option of the defectives. They also considered the optimal amount of investment in that manufacturing facility.

Recently, Sarker *et al.* (2001a) considered a single production system with rework options incorporating two cases of rework process. In first case they considered that the rework is done within the same cycle and the same stage where it produced. In the second case, the defectives items are accumulated up to N cycles and the accumulated items are

reworked in next cycle. They developed the economic batch quantity to minimize the total system cost and increase the profitability of the system. Sarker *et al.* (2001b) also developed the optimal batch quantity for a multi-stage production system to minimize the system cost under the same technique.

2.3 Imperfect Production Process

Lee and Rosenblatt (1986) considered a circumstance when manufacturing facility goes from 'in-control' to 'out-of-control' during production. They developed a model to determine the optimal order quantities with imperfect production process and established the relationship of quality, lot size, setup and holding cost and the deterioration of the production process. Teunter and Flapper (2000) considered a production line that produces a single item in multi-stage. The produced lots are non-defective, reworkable defective or non-reworkable defective. The rework process is done in the same production line. The authors developed the model for perishable items assuming that the rework time and the rework cost increase linearly with the time that a lot is held in stock and they derived an explicit expression for the average profit (sales revenue minus costs). Lee (1992) choose the lot-sizing problem containing the key characteristic of imperfection in a production process and developed a model which includes process shifting to *out-of-control* states, detection of the out-of-control production. Corrective actions follow the detections, and fixed setup and variable times of reworks. He found the problem from the wafer probe operation in semi conductor manufacturing.

2.4 Other Research with Rework and Inspection

Agnihotri and Kenett (1995) dealt with a production process which has an inspection process where the defectives items are sorted out and sent to the reworking stage. After reworking, the finished goods are delivered to the customers. They have developed a model, considering the number of defects is a random variable having geometric distribution, and investigated the impact of the defect distribution on system performance measures. Tay and Ballou (1988) considered that the product is produced in batches which are transported intact from stage to stage. The processing at one stage begins only after the completion of processing at previous stage. Each production stage has been considered as few defective items production, which are sorted out and sent for rework at one or more stages. They developed the model for any specified inspection configuration in sequential production process and have obtained a closed-form solution to determine the optimal lot size and rework batch size to minimize the total system cost.

So and Tang (1995a) presented a model of a bottleneck system that performs two distinct types of operations such as regular production and rework process. In their research, each job is passed through an inspection, and the job that passes the inspection is fed to the downstream of the production process, otherwise it is fed to that stage for rework. They formulated the problem as a semi Markov decision process. They also developed a simple procedure to compute the critical value that identifies the optimal threshold policy and evaluated the impact of batch sizes, yield, and switchover time on an optimal threshold policy. So and Tang (1995b) presented another model of a bottleneck facility which performs two separate types of operations such as 'regular' and 'repair.' They considered two policies, 'repair none' and 'repair all,' and found the optimality

conditions for both policies. Hong *et al.* (1998) developed an economic design of inspection procedure when the scrap items are reworked. Their cost model consists of the cost incurred by imperfect quality, reprocessing cost and inspection cost and they developed a probabilistic method for solving that problem.

2.5. Drawbacks of Previous Research

The above literature study indicates that determination of optimal batch quantities of production processes was the principal search for most of the researchers. Research has focused on developing *optimal lot size* of traditional inventory models, *economic batch size* for imperfect production process, *comparison* for different production policies and *inspection policies* for various inventory models with rework or reprocessing options. This research may not be sufficient to solve the problem adequately. From the above survey, some drawbacks are found that exist in previous solution methods. The drawbacks are in the following areas:

- (a) As described in the previous section, researchers [(Goyal, 1976, 1978), (Monahan, 1989), (Kumar and Vrat, 1978), (Banerjee, 1986, 1992), (Clark and Armentano, 1995)] tried to develop optimal batch quantity for perfect production process in ideal conditions. Practically, the production facility involves a lot of imperfection, which results defective items.
- (b) Many researchers [(Gupta and Chakrabarty, 1984), (Chakrabarty and Rao, 1988), (Chandra *et al.*, 1997)] dealt with defective production processes, and tried to develop optimal batch sizes to improve the quality and minimizing the total cost of the system. Some of them did not consider rework which result in material wastage. Some of them consider rework,

but during the rework process, the production equipment is considered to be a perfect reworking facility.

- (c) As defective items are produced and rework in an imperfect production system, some of the defective items cannot be reworked and those go to scrap. For that reason some shortages may occur in customers' demand satisfaction. Few researchers [(Sarker *et al.*, 2001), (Tay and Ballou, 1988), (Lee, 1992)] focused on developing optimal batch quantities in this type of production system, but they did not consider about the shortages that may occur due to scrap produced during rework process.

Table 2.1 shows some comparison between some previous research which considered the rework facility in production systems.

In this research, optimal batch quantity models are to be developed considering rework process and scrap production during or before the rework, which will overcome the drawbacks of the previous research. It is anticipated that this research will provide better results to the real problems. Table 2.2 shows the comparison of the model features between this research and other research.

Table 2.1: Comparison between some research with rework option.

#	Title	Summary	Attributes
1	Gupta and Chakrabarty (IJPR, 22(2), 1984, 299-311): Looping in a multi-stage production system	In a multi-stage production system, a model for optimum production batch quantity as well as optimal recycling lot size has been developed with respect to minimize the total cost function of the situation where the defective items are produced during normal production of finished goods.	<ol style="list-style-type: none"> 1. Multi-stage system 2. Uniform demand 3. Deterministic processing time 4. Deterministic setup time 5. Constant defectives 6. Negligible waiting time 7. No scrap 8. Balanced work loads
2	Chakrabarty and Rao (Opsearch, 1988, 25(2), 75-88): EBQ for a multi-stage production system considering rework	Models are developed to determine optimum number of cycles for rework and optimum batch quantity in a multi-stage production system considering two different policies of processing reworked lots.	<ol style="list-style-type: none"> 1. Multi-stage system 2. Uniform demand 3. Variable defectives 4. No scrap 5. No defectives during rework 6. Balanced workloads.
3	Tay and Ballou (IJPR, 1988, 26(8), 1299-1315): An integrated production-inventory model with reprocessing and inspection	The cost and quality levels of production system have examined through a model, and obtained a closed form solution for optimal lot size and reprocessing batch size using Markovian process.	<ol style="list-style-type: none"> 1. Multi-stage system 2. Probabilistic demand 3. Batch production 4. Defectives are produced every stage 5. Rework is considered 6. No scrap
4	Agnihotri and Kenett (EJOR, 1995, 80(2), 308-327): The impact of defects on a process with rework	To quantify the impact of defects on various system performance measures for a production system with 100% inspection followed by rework. The model has developed assuming the number of defects to be random variable with geometric distribution and investigation has done for the impact of the defect distribution on system performance measures.	<ol style="list-style-type: none"> 1. Three stage manufacturing system 2. Probabilistic demand 3. Defects are detected one at a time 4. Number of defects have a discrete probability distribution 5. FCFS discipline applied 6. Unlimited buffer 7. Perfect rework conditions 8. Steady state system 9. Multiple parallel servers
5	Chandra <i>et al.</i> (PPC, 1997, 8(6), 586-596): Optimal batch size and investment in multi-stage production systems with scrap.	The problems are selecting the optimum production batch size in a multi-stage manufacturing system with scrap and determining the optimal amount of investment. The effects of investment for quality improvement on the system parameters have also analyzed.	<ol style="list-style-type: none"> 1. Multi-stage system 2. Constant and uniform demand 3. Constant price per unit of product 4. Constant setup costs 5. All defectives are scrapped 6. Production is greater than demand 7. Each stage defectives production 8. Normally distributed product quality
6	Sarker <i>et al.</i> (Working paper, 2001a): Manufacturing batch sizing for rework process in a single-stage production system.	To determine the optimal batch quantity of a single-stage production system with rework facility considering two policies of rework process to minimize the total cost of the system.	<ol style="list-style-type: none"> 1. Single-stage system 2. Constant demand 3. Constant production rate 4. No scrap 5. No defectives during rework <p>Inspection cost is ignored</p>
7	Sarker <i>et al.</i> (Working paper, 2001b): Manufacturing batch sizing for rework process in a multi-stage production system.	The optimal batch quantity of a multi-stage production system has determined with rework facility considering two policies of rework process to minimize the total cost of the system.	<ol style="list-style-type: none"> 1. Multi-stage system 2. Constant demand 3. Constant production rate 4. No scrap 5. No defectives during rework 6. Inspection cost is ignored

Table 2.2: Comparison of different features between current research and other research.

<i>Properties considered</i>	<i>Gupta and Chakrabarty (1984)</i>	<i>Chakrabarty and Rao (1988)</i>	<i>Tay and Ballou (1988)</i>	<i>Agnihotri and Kenett (1995)</i>	<i>Chandra et al. (1997)</i>	<i>Sarker et al. (2001a)</i>	<i>Sarker et al. (2001b)</i>	<i>Current Research</i>
Products	Single	Single	Single	Single	Single	Single	Single	Single
Stages	N	N	N	N	N	1	N	1
Demand rate	Uniform	Uniform	Prob.	Prob.	Constant	Const.	Const.	Constant
Production rate	Constant	Constant	Const.	Constant	Constant	Const.	Const.	Constant
Rework option	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes
Scrap considered	No	No	No	No	Yes	No	No	Yes
Rework policy	R	T & U	V	W	-	Y & Z	Y & Z	Y & Z
Ways of Scrap detection	-	-	-	-	A	-	-	B, C & D
Inspection cost	No	No	Yes	Yes	Yes	No	No	No
Buffer considered	No	Yes	No	No	No	No	No	Yes
Shortage cost	No	No	No	No	No	Yes	Yes	Yes

Prob. = probabilistic, Const. = constant.

A = Scrap detection during the production,

B = Scrap detection before the rework,

C = Scrap detection during the rework process, and

D = Scrap detection after the rework process.

N = Multi-stage

R = Rework option: after defectives production, they fed in to the first stage,

T = Rework is done in the same stage where defective produced

U = Rework is done immediately after a defective produced at same stage,

V = Rework is done after the end of production,

W = Rework is done after production in a different stage,

Y = Rework is done immediately within the cycle where the defectives produced, and

Z = Rework is done after N cycles of production in a different stage.

CHAPTER 3

MODEL DEVELOPMENT

This section of the research contains the formulation of the inventory model as described previously for different policies with rework and identification of different cases for scrap in a single-stage production system. The formulations of the models depend on some assumptions and notation. They are described at the beginning after which the average inventories for different cases and the total cost functions of the inventories are derived.

3.1 Assumptions

The following assumptions are made to develop the model:

- (a) A single type of product in a single-stage production system is considered,
- (b) Production rate is constant and greater than demand rate,
- (c) Proportion of defective is constant in each cycle,
- (d) Only one type of defective is produced in each cycle,
- (e) Scrap is produced and detected in different ways,
- (f) The defectives are reprocessed once, after which they are discarded as scrap, if not corrected properly,
- (g) Proportion of scrap is less than the proportion of defectives,
- (f) Inspection cost is ignored since it is negligible with respect to other costs.

3.2 Notation

The following notation will be necessary to explain and formulate the problem.

- α Proportion of scrap during rework with respect to total produced defective items.

β	Proportion of defectives in each cycle with respect to total produced items.
δ	Scrap production factor, where $0 \leq \delta \leq 1$.
C	Processing cost in first operational policy, dollars/unit.
H_I	Inventory carrying cost for within cycle rework, dollars/unit/year.
H_n	Unit inventory carrying cost for unit finished product per unit time for rework after N cycles, dollars/unit/year.
C_p	Unit penalty cost, dollars/unit/year.
C_s	Setup cost, dollars/minute.
C_d	Setup cost for defective items, dollars/minute.
K_S	Setup cost, dollars/year,
K_{SD}	Setup cost for defectives, dollars/year,
K_T	Scrap handling cost, dollars/year,
C_w	Unit in-process inventory carrying cost, dollars/unit/year,
C_t	Unit scrap handling cost, dollars/unit,
D	Demand rate, units/year.
N	Number of production cycles after which the defective items are reworked.
P	Production rate, units per planning period, units/year.
Q	Batch quantity per cycle, units/batch.
t_1	Time of normal production, year.
t_2	Time of rework process, year.
t_3	Time of consumption after production stops for the Case I, and scrap production time for Case II, year.

- t_4 Consumption time after production stops for Case II, year.
- Q_1 The quantity of good items remaining after consumption at the end of time t_1 .
- Q_2 The quantity of good items that should remain after consumption at the end of time t_1 .
- Q_3 The quantity remaining after consumption at the end of time t_2 , when rework is completed without the scrap.
- Q_4 The quantity that is produced at the end of rework.
- Q_5 The quantity remaining after the consumption during the production, detection and separation of scrap.
- S Setup cost in within cycle rework policy, dollars/batch.
- t_s Setup time, minute/batch.
- t_d Setup time for defectives, minute/batch.
- T_1 Total cycle time for Case I, years.
- T_2 Total cycle time for Case II, years.

3.3 The Models

In this research, the inventory models are developed for two operational policies. The *first policy* covers rework is done whenever defectives are produced. Therefore, the finished products are delivered to the customers at the end of the cycle and scrap is discarded. The *second policy* encompasses defective items produced in each cycle and accumulated until N cycles of production are completed, and then the rework is performed. As a result, the reworking cycle may be different from the normal cycle. The scrap is also accumulated during reprocessing. When good items are produced they are

delivered to the customer directly, for scrap production shortage occurs and penalty cost is imposed on both scrap items and buffer inventory.

3.4 First Policy: Within Cycle Rework Process

As described above, the rework in the first policy is done within the same cycle in which the defectives are produced, and some scrap is produced and detected before, during and after the rework process. Figure 3.1 shows the block diagram of the production process for the first policy.

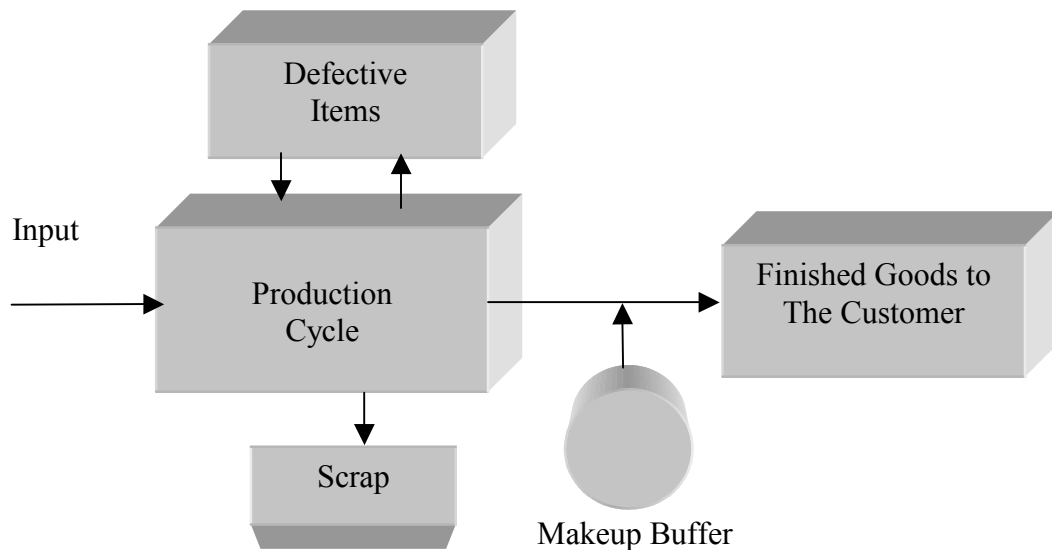


Figure 3.1: Block diagram of the process with rework and scrap for within cycle rework.

Scrap is divided into three different cases: (a) scrap detected before rework, (b) scrap detected during rework and (c) scrap detected after rework. All cases are considered below.

3.4.1 Case I: Scrap Detected before Rework

In this case, scrap can be detected before the rework process starts to produce good items from the defectives. Figure 3.2 shows inventory when defective items are reworked within the cycle and scrap is detected before rework starts. Accordingly, Figure 3.2, t_1 , t_2 , and t_3 , are the time segments, which represent the processing time (uptime), rework time without scrap and downtime or consumption time, respectively.

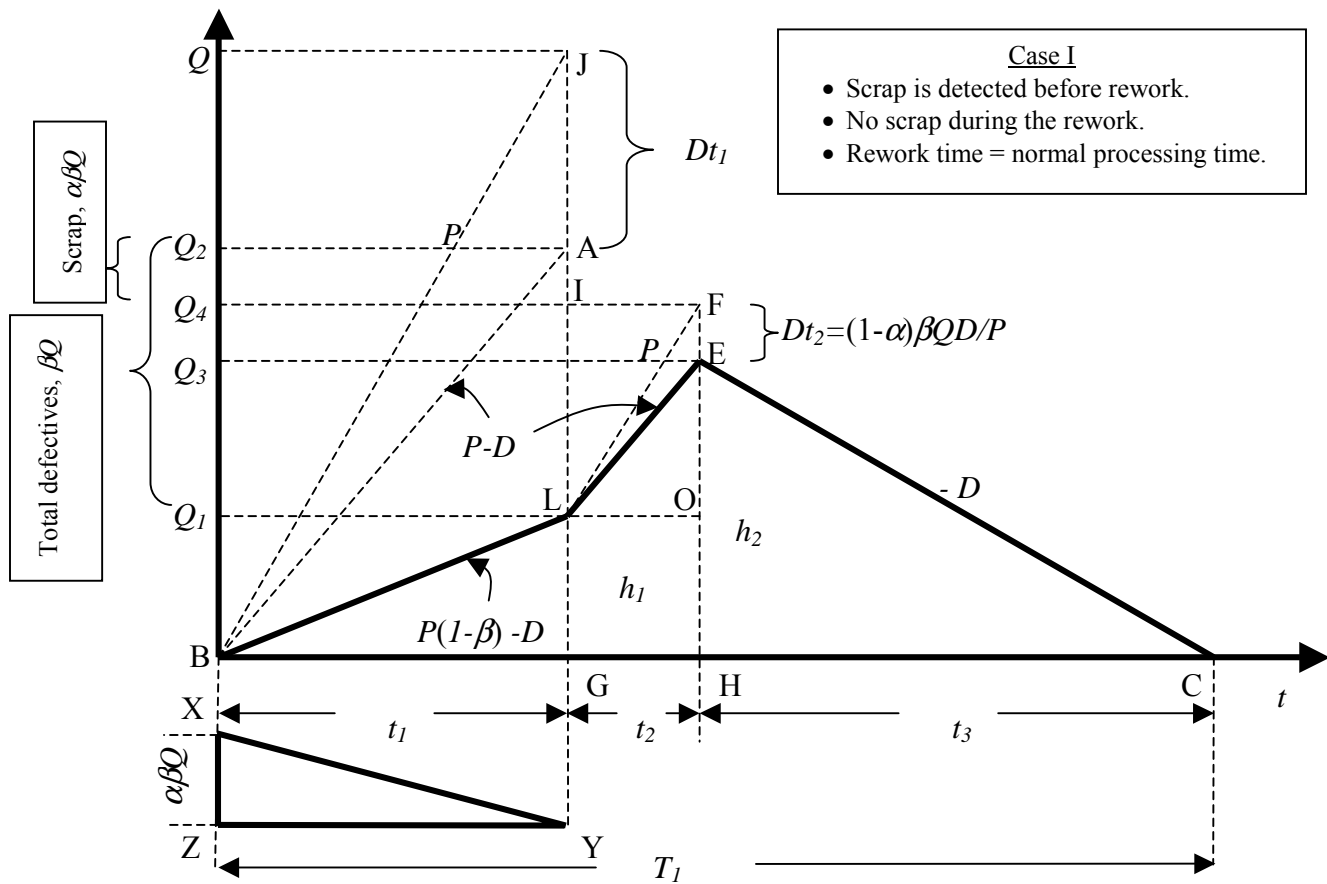


Figure 3.2: Inventory built-up when scrap is found before rework process.

The inventory level represented by the triangle (BAG) with dashed line indicates the ideal case of inventory when no defective or scrap items are produced in the production cycle. From the beginning to the end of the production process both good and

defective items are produced together. When a defective item is produced, the item is immediately reworked. In Figure 3.2, it is shown that the defective items are produced at a rate of β during time t_1 . The triangle BLG represents inventory when the defective items are produced during the uptime, t_1 . The line BL indicates the slope of $P(1-\beta)-D$, i.e., the net replenishment rate when the defective items are produced during time t_1 at a defective proportion of β . The net amount of defectives produced during time t_1 is βQ .

It is assumed that α % of defectives is scrap. Hence, at the end of time t_1 , the scrap units $\alpha\beta Q$ are identified and separated from the main inventory, and say, line AI indicates that amount. The remaining defective $(1-\alpha)\beta Q$ units represented by LI are reworked at the rate of P units/year, as the rework rate is assumed as the same as production rate. Therefore, inventory builds up again as the rework process continues from point L to point F during time t_2 . The triangle ECH indicates the pure consumption occurred after the production stops at the end of time t_2 , and only pure consumption continues during time t_3 .

As some scrap is detected during the time of t_1 , therefore, to maintain the goodwill of the company, an equivalent quantity of buffer, $\alpha\beta Q$ represented by the triangle XYZ at the bottom of the inventory diagram is maintained during the time period t_1 .

3.4.1.1 Average Inventory Calculation

According to the definition, $DT = Q$. Again $Q = Pt_1$, which leads to $t_1 = Q/P$. In Figure 3.2, Q_1 represents the quantity of good items remaining after consumption at the end of time t_1 ; Q_2 represents the quantity of items that should remain after consumption, if no defective item is produced at the end of time t_1 ; and Q_3 indicates the quantity

remaining after consumption at the end of time t_2 , when rework is completed without the scrap. Hence, it can be shown that $Q_2 = (1 - D/P)Q$ and $Q_1 = (1 - \beta - D/P)Q$, and number of defective items produced is $AL = Q_2 - Q_1 = \beta Q$. Here, line AJ represents the consumption during uptime t_1 , so $AJ = Dt_1$, and during time $t_2 = (1-\alpha)\beta Q/P$, the inventory used is $Dt_2 = EF = (1-\alpha)\beta DQ/P$. Therefore, $Q_3 = EH = Q_2 - AI - EF = (1 - D/P)Q - \alpha\beta Q - (1-\alpha)\beta DQ/P = [1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q$. Now t_3 can be found as follows:

$$EH = Dt_3 \Rightarrow t_3 = \frac{[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q}{D}. \quad (3.1)$$

Hence, the total cycle time $T_1 = t_1 + t_2 + t_3$ can be calculated as

$$T_1 = t_1 + t_2 + t_3 = \frac{Q}{P} + \frac{(1-\alpha)\beta Q}{P} + \frac{[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q}{D} = \frac{(1-\alpha\beta)Q}{D}. \quad (3.1a)$$

If $\alpha = \beta = 0$, then $T_1 = Q/D$, which is the standard inventory model.

Therefore, according to Figure 3.2, the average inventory, \bar{I} , can be calculated as follows:

$$\begin{aligned} \bar{I}_I &= \frac{h_1 t_1}{2T_1} + \frac{h_1 t_2}{T_1} + \frac{(h_2 - h_1)t_2}{2T_1} + \frac{h_2}{2T_1}(T_1 - t_1 - t_2) = \frac{h_1}{2T_1}(t_1 + t_2) + \frac{h_2}{2T_1}(T_1 - t_1) \\ &= \frac{D[P(1-\beta) - D]}{2(1-\alpha\beta)P} \left[\frac{Q}{P} + \frac{(1-\alpha)\beta Q}{P} \right] + \frac{D}{2(1-\alpha\beta)Q} \left[\left(\frac{[P(1-\beta) - D]Q}{P} \right. \right. \\ &\quad \left. \left. + \frac{(P-D)(1-\alpha)\beta Q}{P} \right) \left(\frac{(1-\alpha\beta)Q}{D} - \frac{Q}{P} \right) \right], \end{aligned}$$

which, upon simplification, yields

$$\bar{I}_I = \frac{Q}{2(1-\alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \alpha\beta P + \alpha\beta D)]. \quad (3.2)$$

When $\alpha = \beta = 0$, equation (3.2) reduces to $\bar{I} = \frac{Q}{2} \left(1 - \frac{D}{P}\right)$, which indicates the standard finite production inventory model.

If $\alpha = 0$, equation (3.2) reduces to

$$\bar{I} = \frac{Q}{2P} [P - D(1 + \beta + \beta^2)]. \quad (3.3)$$

which indicates the finite production model with rework option and no scrap.

3.4.1.2 Makeup Buffer Inventory Calculation

A makeup buffer XYZ is maintained during time t_1 due to scrap production to satisfy customers' demand. Hence, the average inventory of area XYZ can be calculated as

$$\bar{I}_{IB} = \frac{1}{2} \alpha \beta Q t_1 = \frac{\alpha \beta D Q}{2(1 - \alpha \beta) P}. \quad (3.4)$$

3.4.2 Case II: Scrap Detected during Rework

In this case, scrap is detected during the rework process. It is assumed that the time to qualify a reworking item as scrap is less than the time to produce a good item. Figure 3.3 shows inventory during the entire cycle when scrap is detected during rework process.

In Figure 3.3, the processing time (uptime), rework time, scrap production time and pure consumption time are represented by the time segments t_1 , t_2 , t_3 and t_4 , respectively. The processes of production of good and defective items and scrap declaration have already been described in previous sections. Here, it is assumed that the reworked good items are produced at the rate of P units/year, and scrap is produced at the rate of P/δ units/year, where δ is the scrap production factor, ($0 \leq \delta \leq 1$).

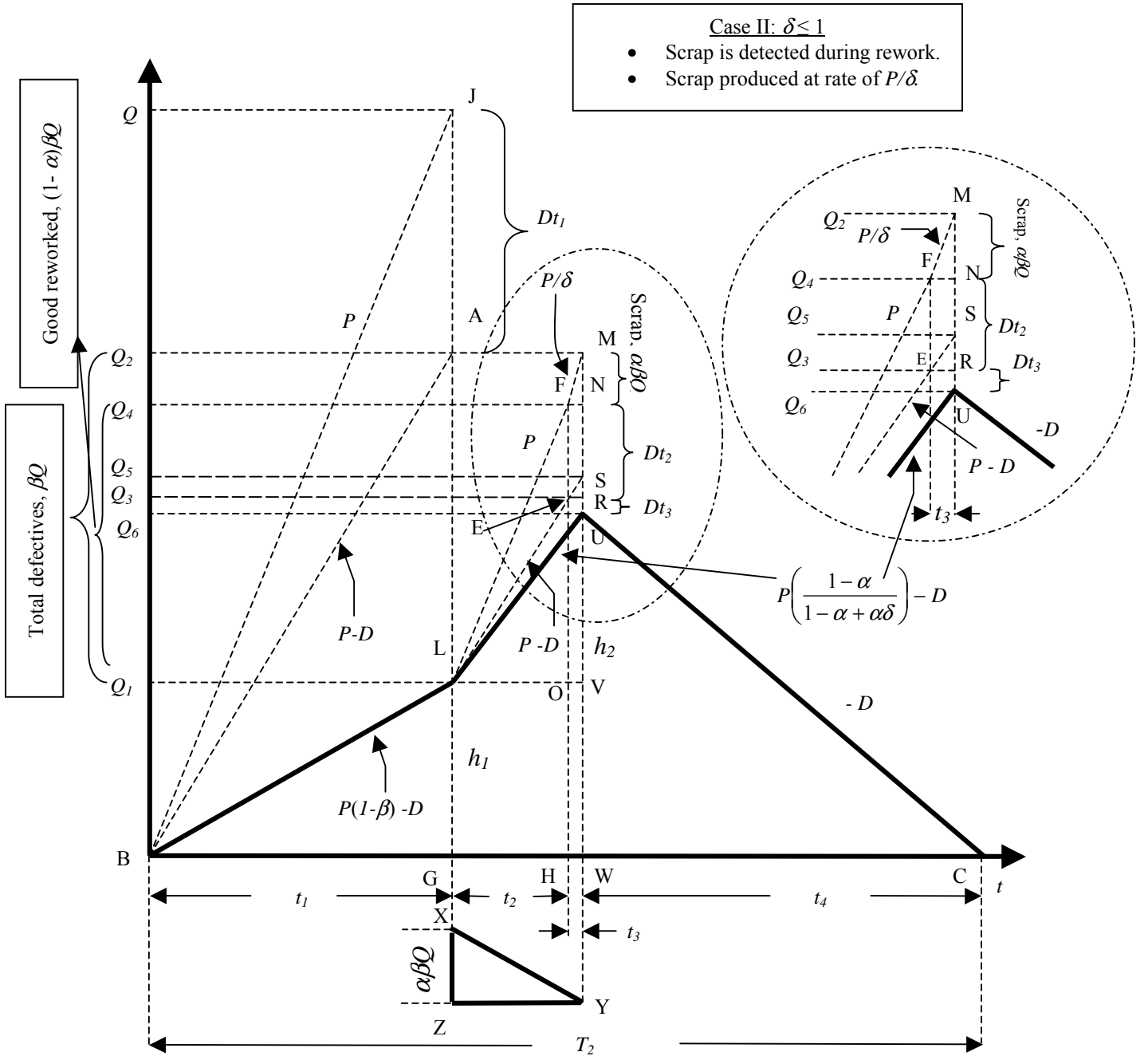


Figure 3.3: Inventory when scrap is detected during rework process.

According to the assumptions, the scrap items are α % of the total defective items. The total *good* items produced during rework time are $(1-\alpha)\beta Q$. Thus, the

inventory continues to build as the rework process proceeds from point L to point E during time t_2 , and after the production, $\alpha\beta Q$ units of scrap is detected. Though scrap is produced more or less uniformly during the rework process, in order to isolate the scrap production, it is shown separately from point F to M in the Figure 3.3, where in FM indicates the slope P/δ . As scrap is separated during the rework process, the time to produce them is added to the actual inventory production time.

At the end of time t_3 , production stops and consumption represented by the line UC occurs through the time period t_4 . The triangle UCW shows the inventory consumption during time t_4 . As scrap is found during that time periods t_2 and t_3 a compensating buffer is maintained during the time of rework to meet the demand.

In Figure 3.3, the triangle XYZ represents the compensating buffer inventory maintained from the beginning to the end of rework process.

3.4.2.1 Average Inventory Calculation

The representation and values of Q_1 , Q_2 , Q_3 , AJ, AL and EF have been described in previous sections. In this case, Q_4 is the quantity that is produced at the end of rework, Q_6 is the quantity remaining after the consumption during the production detection and separation of scrap (at the end of time t_3). Hence, $t_2 = (1-\alpha)\beta Q/P$, and the inventory consumed during t_2 time is $Dt_2 = EF = (1-\alpha)\beta DQ/P$. Therefore, $Q_4 = FH = Q_1 + EO + EF = (1 - \beta - D/P)Q + (P-D)(1-\alpha)\beta DQ/P + (1-\alpha)\beta DQ/P = [1-(1-\beta)D/P]Q$, so the actual inventory without the scrap is Q_3 , and $Q_3 = EH = Q_1 + EM = (1 - \beta - D/P)Q + (P - D)(1-\alpha)\beta Q/P = [1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q$. After the production, $\alpha\%$ scrap is found, which is $\alpha\beta Q$. These items are produced during the rework, but in Figure 3.3 it is shown separately from point F to M. As these are separated during the rework process, the time

to produce scrap is added to the actual inventory production time. Hence, t_3 can be calculated as $t_3 = \alpha\beta\delta Q/P$. During the time t_3 , the inventory is consumption represented by line RU can be calculated as $RU = Dt_3 = \alpha\beta\delta DQ/P$. After the end of the production, the actual inventory of good items remaining is Q_6 units:

$$\begin{aligned} Q_6 = UW = Q_3 - RU &= \left(1 - \alpha\beta - \frac{(1 + \beta - \alpha\beta)D}{P}\right)Q - \frac{\alpha\beta\delta DQ}{P} \\ &= Q[1 - \alpha\beta - (1 + \beta - \alpha\beta + \alpha\beta\delta)D/P]. \end{aligned} \quad (3.5)$$

Now t_4 can be found as follows:

$$UW = Dt_4 \Rightarrow t_4 = \frac{Q}{D}[1 - \alpha\beta - (1 + \beta - \alpha\beta + \alpha\beta\delta)D/P]. \quad (3.6)$$

Hence, the total cycle time $T_2 = t_1 + t_2 + t_3 + t_4$ can be calculated as

$$\begin{aligned} T_2 = t_1 + t_2 + t_3 + t_4 &= \frac{Q}{P} + \frac{(1 - \alpha)\beta Q}{P} + \frac{\alpha\beta\delta Q}{P} + \frac{Q_6}{D} \\ &= \frac{Q}{P}(1 + \beta - \alpha\beta + \alpha\beta\delta) + \frac{Q}{D}\left[1 - \alpha\beta - \frac{D(1 + \beta - \alpha\beta + \alpha\beta\delta)}{P}\right] = Q\left[\frac{1 - \alpha\beta}{D}\right]. \end{aligned} \quad (3.7)$$

If $\alpha = \beta = 0$, then $T_2 = Q/D$, which is the standard inventory model. A comparison between different inventory level of Case I and Case II is shown in Table 3.1.

Table 3.1: A comparison between Case I and Case II.

Parameters	Scrap	
	Before Rework	During Rework ($0 \leq \delta \leq 1$)
Q	Pt_1	Pt_1
t_2	$(1 - \alpha)\beta Q/P$	$(1 - \alpha)\beta Q/P$
t_3	$[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q/D$	$\alpha\beta\delta Q/P$
t_4	–	$Q(1 - \alpha\beta - (1 + \beta - \alpha\beta + \alpha\beta\delta)D/P)/D$
T	$t_1 + t_2 + t_3$	$t_1 + t_2 + t_3 + t_4$
Q_1	$(1 - \beta - D/P)Q$	$(1 - \beta - D/P)Q$
Q_2	$(1 - D/P)Q$	$(1 - D/P)Q$
Q_3	$[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q$	$[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q$
Q_4	$[1 - \alpha\beta - D/P]Q$	$[1 - (1 - \beta)D/P]Q$
Q_6	–	$Q(1 - \alpha\beta - (1 + \beta - \alpha\beta + \alpha\beta\delta)D/P)$

According to Figure 3.3, the average inventory, \bar{I} , can be evaluated in this fashion below,

$$\begin{aligned}\bar{I}_{II} &= \frac{h_1 t_1}{2T_2} + \frac{h_1(t_2 + t_3)}{T_2} + \frac{(h_2 - h_1)(t_2 + t_3)}{2T_2} + \frac{h_2(T_2 - t_1 - t_2 - t_3)}{2T_2} = \frac{1}{2T_2} [h_1(t_1 + t_2 + t_3) + h_2(T_2 - t_1)] \\ &= \frac{D}{2(1 - \alpha\beta)Q} \left[(P(1 - \beta) - D) \frac{Q}{P} \left(\frac{Q}{P} + \frac{(1 - \alpha)\beta Q}{P} + \frac{\alpha\beta\delta Q}{P} \right) \right. \\ &\quad \left. + Q \left(1 - \beta\alpha - \frac{D(1 + \beta - \alpha\beta + \alpha\beta\delta)}{P} \right) \left(\frac{(1 - \alpha\beta)Q}{D} - \frac{Q}{P} \right) \right].\end{aligned}$$

Simplifying the above equation, the average inventory can be evaluated as

$$\begin{aligned}\bar{I}_{II} &= \frac{Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) \\ &\quad - \alpha\beta(2P - 2D - 2\beta D - \beta\delta D - \alpha\beta\delta D + \alpha\beta D - \alpha\beta P)].\end{aligned}\quad (3.8)$$

When $\alpha = \beta = 0$ and $\delta = 1$, equation (3.8) reduces to $\bar{I} = \frac{Q}{2} \left(1 - \frac{D}{P} \right)$, which is the same as the standard finite production inventory model and when $\alpha = 0$, that means when no scrap is producing during rework, equation (3.8) reduces to equation (3.3).

3.4.2.2 Makeup Buffer Inventory Calculation

In this case, as scrap is produced during the rework a makeup buffer XYZ is maintained from the beginning of time t_2 to the end of time t_3 . Therefore, the average inventory of the buffer can be found from the area XYZ as follows:

$$\bar{I}_{IB} = \frac{1}{2} \alpha\beta Q(t_2 + t_3) = \frac{\alpha\beta(\beta - \alpha\beta + \alpha\beta\delta)Q^2}{2P}.\quad (3.9)$$

3.4.2.3 Case III: Scrap Detected after Rework

This section deals with the detection of scrap after the rework is completed. This is a special case of Case II, described in the previous section. In this case, δ , the scrap production factor, is assumed to be 1, which means scrap is producing at the same rate P . Hence, the equation (3.8) becomes

$$\bar{I}_{III} = \frac{Q}{2(1-\alpha\beta)P} [P - D - \beta D - \beta^2 D - \alpha\beta(2P - 2D - \beta D - \alpha\beta P)], \quad (3.10)$$

and the average inventory for the makeup buffer reduces to

$$\bar{I}_{III} = \frac{1}{2} \alpha\beta Q(t_2 + t_3) = \frac{\alpha\beta^2 Q^2}{2P}. \quad (3.11)$$

3.4.2.4 Special Case of Case II, $\delta = 0$

This is another special case, where the scrap production factor, $\delta = 0$, which means scrap is detected at the beginning of production. Hence, equation (3.8) reduces to

$$\bar{I}_{SP} = \frac{Q}{2(1-\alpha\beta)P} [P - D - \beta D - \beta^2 D - \alpha\beta(2P - 2D - 2\beta D + \alpha\beta D - \alpha\beta P)], \quad (3.11a)$$

and the buffer inventory becomes

$$\bar{I}_{SPB} = \frac{1}{2} \alpha\beta Q(t_2 + t_3) = \frac{\alpha\beta(\beta - \alpha\beta)Q^2}{2P}. \quad (3.11b)$$

3.5 Total Cost for First Policy

In the current and previous sections, the configurations of inventory are described under the assumption of the first policy. Generally, the total cost of a production system consists of three major costs: such as (a) setup cost, (b) inventory carrying cost, and (c) processing cost. As a makeup buffer is maintained to overcome the shortage of the customers' demand due to scrap production, a buffer maintenance cost is also included in the total cost. In this case, the total cost function consisting setup cost, inventory carrying

cost for all cases, and buffer maintenance and processing cost can be calculated as follows:

3.5.1 Setup Cost

Each and every production process needs a setup for processing the raw materials to manufacture finished product. Hence, setup cost (K_s) can be calculated as

$$K_s = \frac{D}{Q} S, \quad (3.12)$$

where D is demand rate (units/year), Q is the batch quantity (units/year) and S is setup cost per batch.

3.5.2 Inventory Carrying Cost

Inventory builds up during the uptime because of higher production rate than the demand rate. Thus, the production facility incurs inventory-carrying cost due to the accumulated inventory in each cycle. According to this policy, the inventory carrying costs are calculated as follows:

3.5.2.1 Inventory Carrying Cost for All Cases

Usually inventory-carrying cost of finished products is proportional to the average inventory of the product in a cycle. Hence, inventory-carrying cost can be calculated as average inventory of the produced items in the cycle multiplied by unit inventory carrying cost of the product. Using equations (3.2), (3.8), (3.10) and (3.11a) inventory carrying cost is calculated for Case I, Case II, Case III, and Special Case are respectively as follows:

$$IC_I = \frac{H_1 Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \alpha\beta P + \alpha\beta D)], \quad (3.13)$$

$$IC_{II} = \frac{H_1 Q}{2(1-\alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \beta\delta D - \alpha\beta\delta D + \alpha\beta D - \alpha\beta P)], \quad (3.14)$$

$$IC_{III} = \frac{H_1 Q}{2(1-\alpha\beta)P} [P - D - \beta D - \beta^2 P - \alpha\beta(2P - 2D - \beta D - \alpha\beta P)], \text{ and} \quad (3.15)$$

$$IC_{SP} = \frac{H_1 Q}{2(1-\alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D + \alpha\beta D - \alpha\beta P)]. \quad (3.15a)$$

3.5.2.2 Makeup Buffer Inventory Carrying Cost for All Cases

Due to scrap a shortage occurs. To compensate for these shortages, in every case a makeup buffer is maintained so that the carrying cost for the makeup buffer plays a role in the total cost function. The buffer carrying cost can be evaluated by multiplying inventory carrying cost H_I and average inventories of the makeup buffers. Using the equations (3.4), (3.9), (3.11) and (3.11b) the makeup buffer carrying cost can be evaluated as

$$B_I = \frac{\alpha\beta H_1 D Q}{2(1-\alpha\beta)P}, \quad (3.16)$$

$$B_{II} = \frac{\alpha\beta(\beta - \alpha\beta + \alpha\beta\delta) H_1 Q}{2(1-\alpha\beta)P}, \quad (3.17)$$

$$B_{III} = \frac{\alpha\beta^2 H_1 Q}{2(1-\alpha\beta)P}, \text{ and} \quad (3.18)$$

$$B_{SP} = \frac{\alpha\beta(\beta - \alpha\beta) H_1 Q}{2(1-\alpha\beta)P}. \quad (3.18a)$$

3.5.3 Processing Cost

Here, in each cycle, the batch quantity Q is processed and the defective items are reworked, incurring processing cost. Hence, the total processing cost of the system is the

accumulation of the processing costs of batch quantity Q , and defective items βQ found during production of total batch quantity.

If the processing cost per batch is C , then the processing cost of batch quantity Q is CQ . Hence, the total processing cost for the whole planning period, K_p can be calculated as

$$K_p = CQ \times D / Q = CD. \quad (3.19)$$

3.5.3.1 Processing Cost for Rework

In this model the defective items are produced in every cycle at β proportion of total batch quantity Q , so the total quantity that will be defective is βQ . According to the first policy, the defective items are processed within the same cycle for correction, and there is no setup cost involved for them. Only the processing cost plays the role in the total cost function. Again, the inventory cost for the defective items is also taken into consideration and it is already added to the inventory carrying cost, so the processing cost of the rework of defective items βQ over the whole planning period can be calculated as

$$K_{RP} = C\beta Q \times D / Q = \beta CD. \quad (3.20)$$

Here, a processing cost for the buffer quantity $\alpha\beta Q$ is also assessed during the production period to makeup the shortage due to scrap. Therefore, the processing cost of the buffer quantity $\alpha\beta Q$ is

$$K_{BP} = C\alpha\beta Q \times D / Q = \alpha\beta CD. \quad (3.20a)$$

Hence, the total processing cost for rework and buffer maintenance can be evaluated by adding equations (3.20a) and (3.20b) as follows:

$$K_{RBP} = \beta CD + \alpha\beta CD = (\beta + \alpha\beta)CD. \quad (3.20b)$$

3.5.4 Scrap Handling Cost

To handle scrap it is necessary to add a cost for the amount of $\alpha\beta Q$ unit scrap. It can be evaluated by multiplying the unit scrap handling cost, C_r and the amount of scrap produced in entire period of time as

$$K_T = C_r \alpha \beta Q = C_r \alpha \beta Q . \quad (3.21)$$

3.5.5 Total System Cost for All Cases

The total cost of the system $TC(Q)$, is the accumulation of the setup cost, inventory-carrying cost, makeup buffer carrying cost, processing cost and processing cost due to reworking and buffer maintenance. Hence, the total cost of the system for Case I can be calculated by adding equations (3.12), (3.13), (3.16), (3.19), (3.20b) and (3.21) as follows:

$$TC_I(Q) = \frac{DS}{Q} + CD(1 + \beta + \alpha\beta) + C_r \alpha \beta Q + \frac{H_1 Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \alpha\beta P + \alpha\beta D)] + \frac{\alpha\beta Q D H_1}{2(1 - \alpha\beta)P} . \quad (3.22)$$

The total cost of the system for Case II can be calculated by adding equations (3.12), (3.14), (3.17), (3.19), (3.20b) and (3.21), as

$$TC_{II}(Q) = \frac{DS}{Q} + CD(1 + \beta + \alpha\beta) + C_r \alpha \beta Q + \frac{H_1 Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \beta\delta D - \alpha\beta\delta D + \alpha\beta D - \alpha\beta P)] + \frac{\alpha\beta(\beta - \alpha\beta + \alpha\beta\delta)DQH_1}{2(1 - \alpha\beta)P} . \quad (3.23)$$

The total cost of the system Case III can be calculated by adding equations (3.12), (3.15), (3.18), (3.19), (3.20b) and (3.21) as follows:

$$TC_{III}(Q) = \frac{DS}{Q} + CD(1 + \beta + \alpha\beta) + C_r \alpha \beta Q + \frac{H_1 Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2)$$

$$-\alpha\beta(2P - 2D - 3\beta D - \alpha\beta P)] + \frac{\alpha\beta^2 Q D H_1}{2(1 - \alpha\beta)P}. \quad (3.24)$$

The total cost of the system for Special Case of Case II can be calculated by adding equations (3.12), (3.15a), (3.18a), (3.19), (3.20b) and (3.21) as follows:

$$TC_{SP}(Q) = \frac{DS}{Q} + CD(1 + \beta + \alpha\beta) + C_i\alpha\beta Q + \frac{H_1 Q}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D + \alpha\beta D - \alpha\beta P)] + \frac{\alpha\beta(\beta - \alpha\beta)DQH_1}{2(1 - \alpha\beta)P}. \quad (3.25)$$

At this stage, it is necessary to find the nature of the above functions for optimization purpose. If the functions are convex, then the partial differentiation with respect to batch quantity can be set to zero; that means, $\frac{\partial TC(Q)}{\partial Q} = 0$ and the optimal batch quantity Q^* can be evaluated. If the functions are non-convex, a single-variable direct search method such as (a) random search method, (b) univariate method, (c) pattern search method, and (d) Rosenbrock's method of rotating coordinates method can be applied to find the optimum order quantity.

3.6 Optimality

It can be easily shown that $TC(Q)$ is a convex function in Q (see Appendix C). Hence, an optimum batch quantity Q^* , can be calculated from $\partial TC(Q)/\partial Q = 0$, which yields

$$\frac{\partial TC_i(Q)}{\partial Q} = -\frac{DS}{Q^2} + C_i\alpha\beta + \frac{H_1}{2(1 - \alpha\beta)P} [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 2\beta D - \alpha\beta P + \alpha\beta D)] + \frac{\alpha\beta D H_1}{2(1 - \alpha\beta)P} = 0, \quad (3.26)$$

$$Q_I^* = \sqrt{\frac{2DS}{\frac{H_1}{(1-\alpha\beta)P} \gamma + 2C_i \alpha\beta}}. \quad (3.27)$$

where $\gamma = [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 3D - 2\beta D - \alpha\beta P + \alpha\beta D)]$,

Similarly, from equations (3.23), (3.24), and (3.25) the optimal batch quantity Q^* , can be evaluated, respectively, as

$$Q_{II}^* = \sqrt{\frac{2DS}{\frac{H_1}{(1-\alpha\beta)P} \pi + 2C_i \alpha\beta}}, \quad (3.28)$$

where $\pi = [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 3\beta D - \beta\delta D - 2\alpha\beta\delta D + 2\alpha\beta D - \alpha\beta P)]$,

$$Q_{III}^* = \sqrt{\frac{2DS}{\frac{H_1}{(1-\alpha\beta)P} \lambda + 2C_i \alpha\beta}}, \text{ and} \quad (3.29)$$

where $\lambda = [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 4\beta D - \alpha\beta P)]$,

$$Q_{SP}^* = \sqrt{\frac{2DS}{\frac{H_1}{(1-\alpha\beta)P} \omega + 2C_i \alpha\beta}}, \quad (3.30)$$

where $\omega = [P - D(1 + \beta + \beta^2) - \alpha\beta(2P - 2D - 3\beta D + 2\alpha\beta D - \alpha\beta P)]$.

3.6.1 Special Cases

If the production system is considered to be ideal, i.e., no defective items and scrap are produced, means the value of β and α is set to zero. In that case, equations (3.27), (3.28), (3.29) and (3.30) reduce to the classical economic batch quantity model as follows:

$$Q^* = \sqrt{\frac{2SD}{H_1(1-D/P)}}. \quad (3.31)$$

When the defective items are produced and scrap is not, the equations (3.27), (3.28), (3.29) and (3.30) reduce to the economic batch quantity model with defectives as follows:

$$Q^* = \sqrt{\frac{2DS}{\frac{H_1}{P}[P - D(1 + \beta + \beta^2)]}}. \quad (3.32)$$

The solutions above are validated through numerical examples in the following section.

3.7 Numerical Example

The optimum value of Q^* for all cases can be obtained by substituting the parameter values in equations (3.27), (3.28), (3.29) and (3.30). Assume, $D = 300$ units/year, $P = 550$ units/year, $C = \$7$ /units, $S = \$50$ /batch, $H_1 = \$50$ /units/year, $C_t = \$5$ /units, $\beta = 0.05$ and $\alpha = 0.20$ and $\delta = 0.07$ (meaning β is assumed as 0.05 i.e., 5% defects, and α is assumed as 0.20 i.e., 20% scrap from the defectives). The optimum batch quantities Q^* and the total costs with respect to Q^* are obtained for all cases and are calculated by using above parametric values and equations (3.22), (3.27), (3.23), (3.28), (3.24), (3.29), (3.25), (3.30), and (3.31), respectively. All results are represented in tabular form below:

Table 3.2: Results of the numerical example for first policy.

Cases	Case I	Case II	Case III	Special Case	Ideal Case
Parameters					
Q^* , units	37.39	37.62	37.61	37.62	36.33
$TC(Q^*)$, \$	3028.31	3023.13	3023.73	3023.37	2925.72

According to the above table, it can be observed that the optimal total cost and optimum batch sizes of this research are greater than the ideal inventory model.

CHAPTER 4

REWORK AFTER N CYCLES

In this section after- N -cycles policy of reworking process is described. Here, the reworking cycle occurs after the completion of N cycles of regular production. The inventory for this policy is described, average inventory is calculated, and the total cost function based on these inventories is formulated.

4.1 Second Policy: Rework after N Cycles

Under this policy, the defective items are accumulated up to N cycles of normal production, and after which they are reworked. As the production in a cycle continues, the finished goods are supplied to the customer.

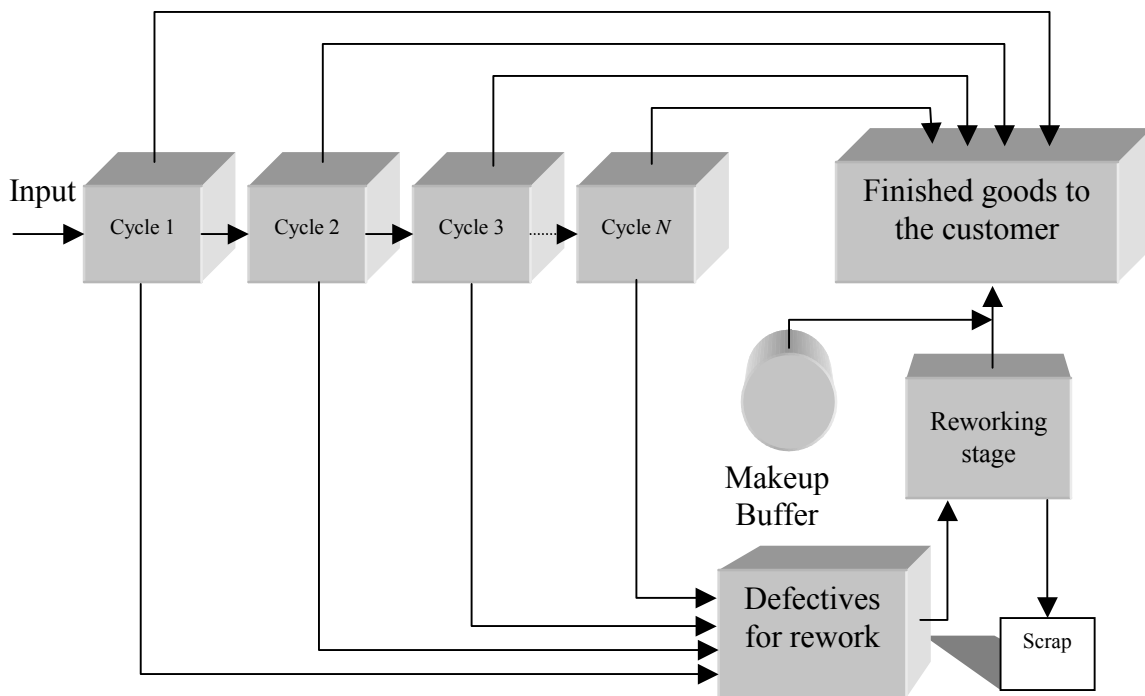


Figure 4.1: Block diagram of the production process with rework and scrap for rework after N cycles policy.

The system is assessed with a penalty cost for shortages as each cycle contains some of the defective items, till the rework is accomplished. Figure 4.1 shows the block diagram of the entire production process with rework. Under this policy, the defective items from each cycle are accumulated until completion of N cycles of production, after which the defective parts are reprocessed. The scrap is detected during the time of rework and the makeup buffer is used.

4.2 Inventory of Finished Product

Figure 4.2 shows the inventory built-up for one cycle during the production, as the defective items are separated.

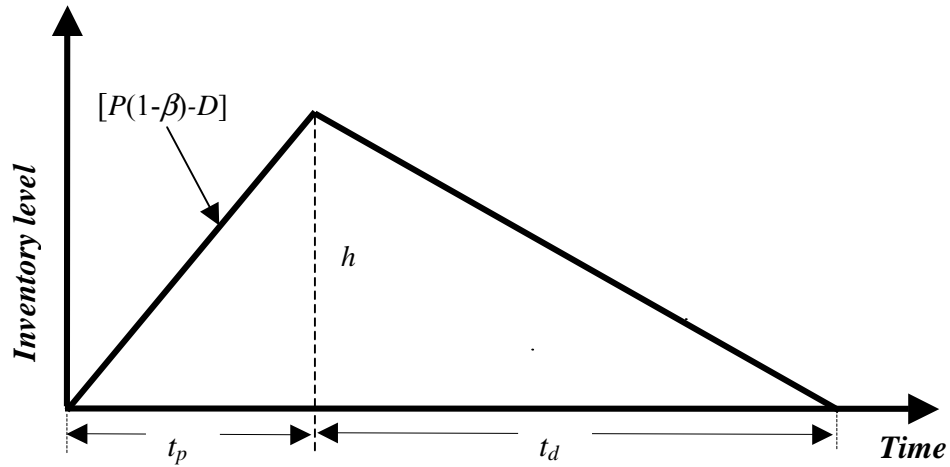


Figure 4.2. Inventory of finished goods in one cycle

According to Figure 4.2, the inventory of one cycle can be calculated as

$$\bar{I} = \frac{1}{2} h(t_p + t_d). \quad (4.1)$$

Since, $h = [P(1 - \beta) - D]Q/P$, the production uptime $t_p = Q/P$, downtime $t_d = h/D$, and the total time of the cycle is $T = (1 - \beta)Q/D$.

Hence, the average inventory \bar{I} , in equation (4.1) can be written as

$$\bar{I} = \frac{1}{2} [P(1-\beta) - D] \frac{Q}{P} \frac{Q(1-\beta)}{D} = \frac{1}{2} \left[1 - \beta - \frac{D}{P} \right] \frac{Q^2(1-\beta)}{D},$$

so the average inventory for the entire period is given by

$$\bar{I} = \frac{1}{2} \left[1 - \beta - \frac{D}{P} \right] \frac{Q^2(1-\beta)}{D} \frac{D}{Q} = \frac{Q(1-\beta)}{2} \left[1 - \beta - \frac{D}{P} \right]. \quad (4.2)$$

4.3 Inventory for Reworked Items

Under this policy the rework is done after accumulation of the defectives through N cycles of production and some portion of defectives scrapped. As described in Chapter 3, scrap detection and production takes place in three ways; they are described below and the inventories of the reworked items are calculated. Also a buffer is maintained in each case to makeup the shortages due to the scrap production at the time of rework, and both normal inventory during rework and buffer inventory are calculated together.

4.3.1 Case I: Scrap Detected before Rework

In this case the scrap is detected at the beginning of the rework process. The inventory for reworked good items is shown in the Figure 4.3. The total scrap for one cycle found is $\alpha\beta Q$, as the proportion of scrap is assumed 100 α % of total defectives produced. That is why, the total reworked items is evaluated as $(1-\alpha)\beta Q$.

From the Figure 4.3 the average inventory can be calculated as follows:

Here, t_p production uptime for a reworked lot = $(1-\alpha)\beta Q/P$, down time $t_d = h/D$, where $h = (P-D)(1-\alpha)\beta Q/P$. Therefore, the average inventory can be calculated as

$$\bar{I}_{IR} = \frac{h}{2} (t_{pr} + t_{dr}) = \frac{(P-D)(1-\alpha)\beta Q}{2P} \left[\frac{(1-\alpha)\beta Q}{P} + \frac{(P-D)(1-\alpha)\beta Q}{DP} \right]$$

$$= \frac{(1-\alpha)^2 \beta^2 Q^2}{2D} \left[1 - \frac{D}{P} \right]. \quad (4.3)$$

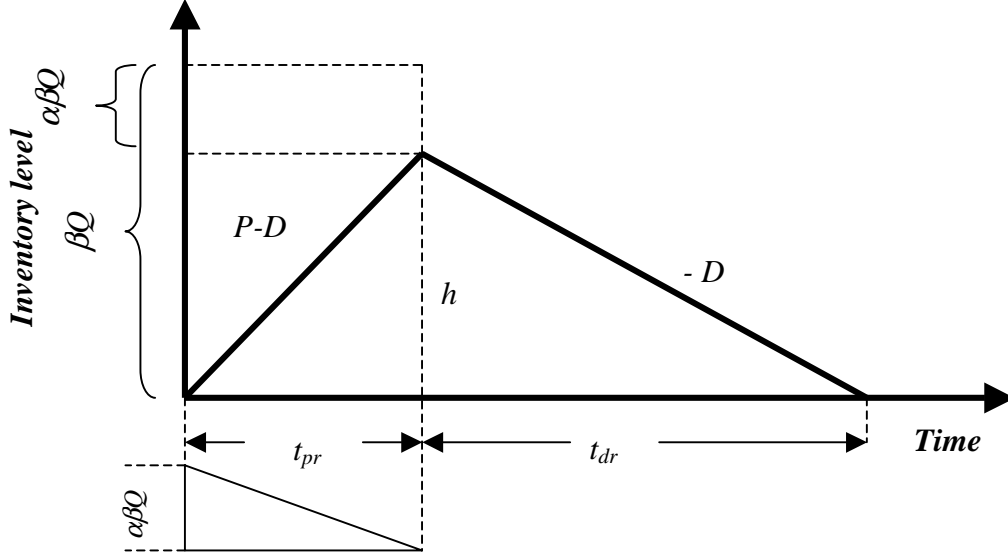


Figure 4.3: Inventory of finished goods during rework for Case I.

At the bottom of the Figure 4.3, it is shown that the inventory for the makeup buffer of $\alpha\beta Q$ quantity is maintained until production of good items is completed from the defectives. The shortage might occur at that time due to the detection of scrap before rework, so the makeup buffer inventory is

$$\bar{I}_{IB} = \frac{1}{2} \alpha\beta Q t_{pr} = \frac{\alpha(1-\alpha)\beta^2 Q^2}{2P}. \quad (4.4)$$

Therefore, the total aggregated inventory during the rework process for Case I can be evaluated by adding equations (4.3) and (4.4) as

$$\bar{I}_{IF} = \bar{I}_{IR} + \bar{I}_{IB} = \frac{(1-\alpha)^2 \beta^2 Q^2}{2D} \left[1 - \frac{D}{P} \right] + \frac{\alpha(1-\alpha)\beta^2 Q^2}{2P}$$

$$= \frac{(1-\alpha)\beta^2 Q^2}{2D} \left[1-\alpha - \frac{(1-2\alpha)D}{P} \right]. \quad (4.5)$$

Hence, the average inventory for the entire period is

$$\bar{I}_{ITF} = \frac{(1-\alpha)\beta^2 Q^2}{2D} \left[1-\alpha - \frac{(1-2\alpha)D}{P} \right] \frac{D}{Q} = \frac{(1-\alpha)\beta^2 Q}{2} \left[1-\alpha - \frac{(1-2\alpha)D}{P} \right]. \quad (4.6)$$

4.3.2 Case II: Scrap Detected during Rework

In this case scrap is detected when rework process continues and scrap production rate is assumed as P/δ , where δ is the scrap production factor ($0 \leq \delta \leq 1$). The total scrap produced for one cycle is $\alpha\beta Q$, as the proportion of scrap is assumed as $100\alpha\%$, of the total defectives produced. That is why, the total reworked items are calculated as $(1-\alpha)\beta Q$. The inventory built-up for reworked good items is shown in Figure 4.4. The average inventory can be calculated as follows:

here, the production uptime for the reworked lot is

$$t_p = t_{pr} + t_{ps} = (1-\alpha)\beta Q/P + \alpha\beta\delta Q/P = (\beta - \alpha\beta + \alpha\beta\delta)Q/P, \text{ down time } t_d = h/D,$$

$$\text{where } h = \left[\frac{P(1-\alpha)}{1-\alpha+\alpha\delta} - D \right] t_p = Q \left[\beta - \alpha\beta - \frac{(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right]. \quad (4.7)$$

Therefore, the average inventory can be calculated as

$$\begin{aligned} \bar{I}_{IRR} &= \frac{h}{2}(t_p + t_{dr}) = \frac{Q}{2} \left[\beta - \alpha\beta - \frac{(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right] \left[\frac{(\beta - \alpha\beta + \alpha\beta\delta)Q}{P} \right. \\ &\quad \left. + \frac{Q}{D} \left(\beta - \alpha\beta - \frac{(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right) \right] = \frac{(\beta - \alpha\beta)Q^2}{2D} \\ &\quad \times \left(\beta - \alpha\beta - \frac{(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right). \end{aligned} \quad (4.8)$$

As shown in Figure 4.4, the makeup buffer inventory of $\alpha\beta Q$ units is maintained until the completion of good items and scrap production from the defectives, so the makeup buffer inventory is given by

$$\bar{I}_{IB} = \frac{1}{2} \alpha\beta Q t_p = \frac{\alpha\beta(\beta - \alpha\beta + \alpha\beta\delta)Q^2}{2P}. \quad (4.9)$$

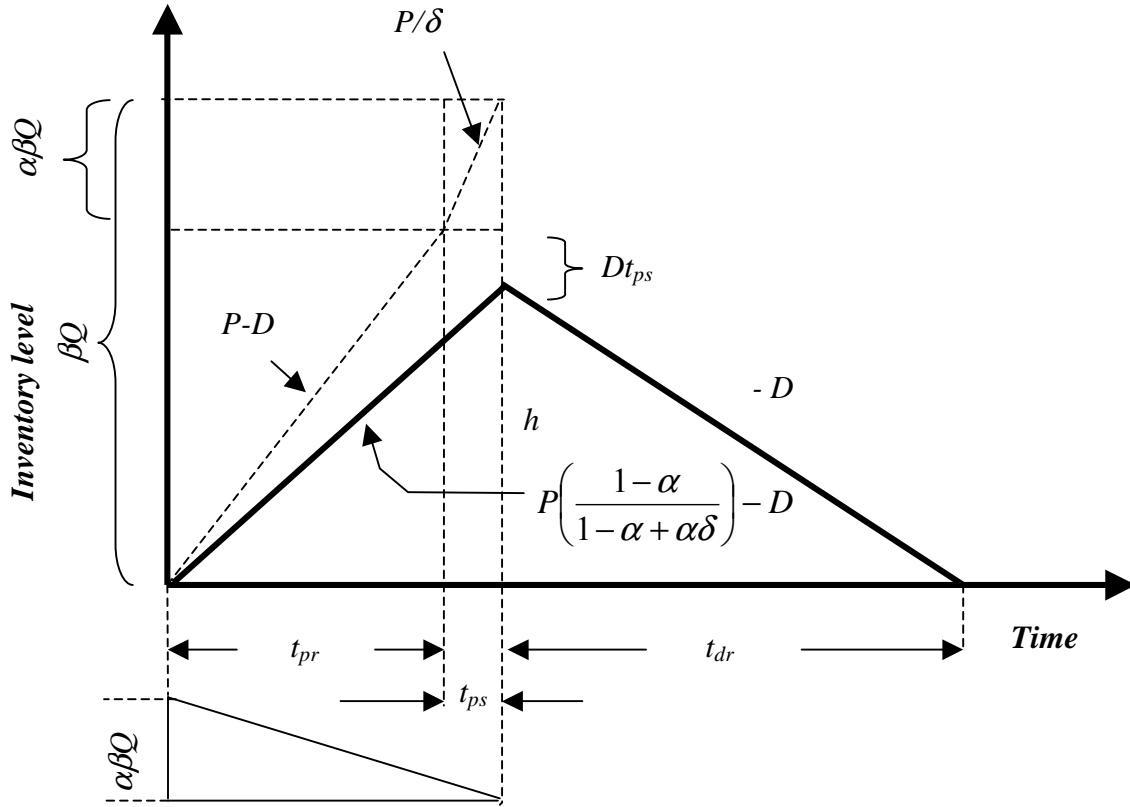


Figure 4.4: Inventory of finished goods during rework for Case II.

Therefore, the total aggregated inventory during the rework process for Case II is found from equation (4.8) and (4.9) as

$$\bar{I}_{IF} = \bar{I}_{IR} + \bar{I}_{IB} = \frac{(\beta - \alpha\beta)Q^2}{2D} \left(\beta - \alpha\beta - \frac{(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right) + \frac{\alpha\beta(\beta - \alpha\beta + \alpha\beta\delta)Q^2}{2P}$$

$$= \frac{\beta Q^2}{2D} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(1 + \alpha\delta)(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right]. \quad (4.10)$$

Hence, the average inventory for the entire period is

$$\begin{aligned} \bar{I}_{ITF} &= \frac{\beta Q^2}{2D} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(1 + \alpha\delta)(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right] \frac{D}{Q} \\ &= \frac{\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(1 + \alpha\delta)(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right]. \end{aligned} \quad (4.11)$$

4.3.3 Case III: Scrap Detected after Rework, $\delta = 1$

Case III is a special case of Case II, where $\delta = 1$. As described in Section 3.5.3.1, equation (4.11) reduces to

$$\bar{I}_{ITF} = \frac{\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(\beta - 2\alpha\beta + \alpha^2\beta)D}{P} \right]. \quad (4.12)$$

4.3.4 Special Case of Case II: $\delta = 0$

This is another special case of Case II, where $\delta = 0$. Equation (4.11) reduces to

$$\bar{I}_{SPTF} = \frac{\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(\beta - \alpha\beta)D}{P} \right]. \quad (4.12a)$$

4.4 In-Process Inventory of Rejected Materials

In the rework after N cycles policy, some defective items are produced in each cycle and are accumulated until the end of N cycles of production. Figure 4.5 shows the in-process inventory of the rejected material for N cycles.

The in-process inventory of rejected materials depends on the waiting time for the entire batch of quantity Q during the setup and processing time for one component at N^{th} cycle. The setup waiting time can be given as

$$T_{ws} = (N-1)t_s + (N-2)t_s + (N-3)t_s + \dots + 2t_s + t_s = \frac{N(N-1)t_s}{2}. \quad (4.13)$$

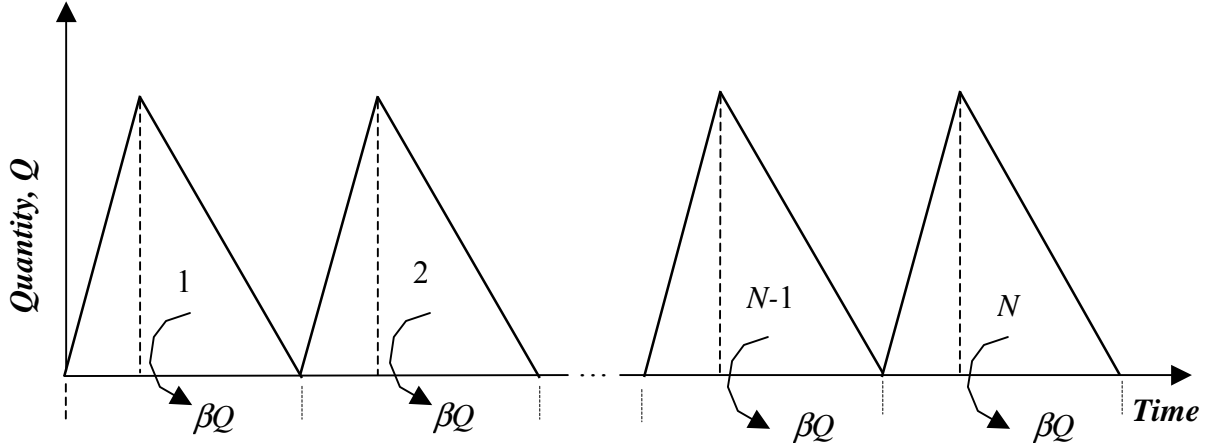


Figure 4.5. In-process inventory of the rejected material over N cycles for Second Policy.

The waiting time for processing the batches can be calculated as

$$\begin{aligned}
 T_{wp} &= \left(\frac{Q}{2P} + \frac{Q(N-1)}{P} \right) + \left(\frac{Q}{2P} + \frac{Q(N-2)}{P} \right) + \left(\frac{Q}{2P} + \frac{Q(N-3)}{P} \right) + \dots + \frac{Q}{2P} \\
 &= \frac{NQ}{2P} + \frac{Q}{P} [(N-1) + (N-2) + (N-3) + \dots + 2 + 1] = \frac{NQ}{2P} + \frac{N(N-1)Q}{2P} = \frac{N^2Q}{2P}. \quad (4.14)
 \end{aligned}$$

Hence, the total waiting time is found by adding equations (4.13) and (4.14):

$$T_w = T_{ws} + T_{wp} = \frac{N(N-1)t_s}{2} + \frac{N^2Q}{2P}. \quad (4.15)$$

The total in-process inventory of the rejected material for the entire period can be evaluated by accumulating the in-process inventories for all lots:

$$\bar{I}_{wip} = \left[\frac{N(N-1)t_s}{2} + \frac{N^2Q}{2P} \right] \frac{\beta Q}{1-\beta} = \frac{\beta}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right]. \quad (4.16)$$

4.5 Total Cost for Second Policy

In the rework after N cycles policy, the rework process occurs in the $(N+1)^{\text{th}}$ cycle after completion of N cycles of production. To formulate the total cost function for this

model, it is necessary to calculate the setup cost for this process, in-process inventory carrying cost, reworked and buffer inventory carrying cost, penalty cost due to shortage created by the defective items taken out during normal production, and processing cost. These costs are described and calculated in this section.

4.5.1 Setup Cost

Each production facility needs a setup for producing finished goods, which incurs a cost for setup. Hence, the setup cost is found for the whole batch quantity Q as

$$K_s = C_s t_s. \quad (4.17)$$

In rework after N cycles policy, another setup cost is needed for rework as the rework is done after completion of normal production process. Hence, the setup cost for rework process for entire batch of defective quantity βQ is

$$K_{SD} = C_d t_d. \quad (4.17a)$$

4.5.2 Inventory Carrying Cost

Four types of inventories are found in this policy – they are finished goods inventory for N cycles, reworked finished good inventory, inventory of rejected items during regular process, and makeup buffer inventory. These inventories are calculated in previous sections, so the inventory carrying cost can be evaluated by multiplying the unit carrying cost and the average inventory of the cycle. By accumulating different inventories and multiplying them by unit inventory carrying cost, the total average inventory carrying cost can be calculated.

The inventory carrying cost can be found for Case I by using the equations (4.2), (4.6) and (4.16), for Case II by using the equations (4.2), (4.11) and (4.16), for Case III

and Special case of Case II by using the equations (4.2), (4.12), (4.16) and (4.12a) respectively, as follows:

$$IC_I = \frac{H_n(1-\beta)Q}{2} \left[1 - \beta - \frac{D}{P} \right] + \frac{H_n(1-\alpha)\beta^2Q}{2} \left[1 - \alpha - \frac{(1-2\alpha)D}{P} \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right], \quad (4.18)$$

$$IC_{II} = \frac{H_n(1-\beta)Q}{2} \left[1 - \beta - \frac{D}{P} \right] + \frac{H_n\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(1+\alpha\delta)(\beta - \alpha\beta + \alpha\beta\delta)D}{P} \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right], \quad (4.19)$$

$$IC_{III} = \frac{H_n(1-\beta)Q}{2} \left[1 - \beta - \frac{D}{P} \right] + \frac{H_n\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(\beta - 2\alpha\beta + \alpha^2\beta)D}{P} \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right], \text{ and} \quad (4.20)$$

$$IC_{SP} = \frac{H_n(1-\beta)Q}{2} \left[1 - \beta - \frac{D}{P} \right] + \frac{H_n\beta Q}{2} \left[\beta - 2\alpha\beta + \alpha^2\beta - \frac{(\beta - \alpha\beta)D}{P} \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right]. \quad (4.20a)$$

4.5.3 Penalty Cost

The production of defective items in every cycle results in shortages by βQ items, and these shortages are fulfilled after the rework is completed at the end of $(N+1)^{th}$ cycle. For that reason, a penalty cost is assessed in the total cost function. To calculate the penalty cost, it is necessary to calculate the total time elapsed in shortage. The shortage time consists of production runtime and downtime, which is the same for N cycles. At the end of N^{th} cycle the rework is performed i.e., in $(N+1)^{th}$ cycle. Hence, the total shortage

time is different in this cycle as the accumulated defective items up to N cycles are reworked here. Shortage times for these two cases are calculated below.

The shortage time up to N cycles is calculated from Figure 4.2 by adding the production uptime and down time for each cycle, $(t_p + t_d) = t_p + ht_p / D = (1 - \beta)Q / D$.

Therefore, the total shortage time up to N cycles, T_s^n , is given by

$$T_s^n = [(N - 1) + (N - 2) + \dots + 2 + 1] \left[\frac{Q(1 - \beta)}{D} \right] = \frac{(1 - \beta)(N - 1)}{2}, \quad (4.21)$$

where $N = D/Q$.

The shortage time for $(N+1)^{th}$ cycle can be calculated in three ways, as rework time varies due to scrap production. Also they consist of rework production time and consumption time, so they are calculated for different cases, respectively, as follows:

$$T_{IS}^{(n+1)th} = \frac{(1 - \alpha)\beta QN}{P} + \frac{(P - D)(1 - \alpha)\beta QN}{DP} = \frac{(1 - \alpha)\beta QN}{D} = (1 - \alpha)\beta, \quad (4.22)$$

$$\begin{aligned} T_{IIS}^{(n+1)th} &= \frac{(\beta - \alpha\beta + \alpha\beta\delta)QN}{P} + \left[\frac{P(1 - \alpha)}{1 - \alpha + \alpha\delta} - D \right] \frac{(\beta - \alpha\beta + \alpha\beta\delta)QN}{DP}, \\ &= \frac{(1 - \alpha)\beta QN}{D} = (1 - \alpha)\beta, \text{ and} \end{aligned} \quad (4.23)$$

$$T_{IIS}^{(n+1)th} = T_{SPS}^{(n+1)th} = \frac{\beta(1 - \alpha)QN}{D} = (1 - \alpha)\beta. \quad (4.24)$$

Hence, the total shortage times can be evaluated for different cases by using equations (4.21), (4.22), (4.23), and (4.24) as

$$\begin{aligned} T_{IS} = T_{IIS} = T_{IIS} = T_{SPS} = T_s^n + T_{IS}^{(n+1)th} &= (N - 1)(1 - \beta) / 2 + (1 - \alpha)\beta \\ &= [N - 1 - \beta N + 3\beta - 2\alpha\beta] / 2, \end{aligned} \quad (4.25)$$

The total penalty cost over a planning period for different cases can be obtained as

$$\begin{aligned}
PC_I = PC_{II} = PC_{III} = PC_{SP} &= C_p \frac{\beta Q}{2(1-\beta)} [N(1-\beta) - 1 + 3\beta - 2\alpha\beta] \\
&= \frac{C_p \beta D}{2(1-\beta)} \left[1 - \beta - \frac{1 - 3\beta + 2\alpha\beta}{N} \right]. \tag{4.26}
\end{aligned}$$

4.5.4 Scrap Handling Cost

Scrap needs a cost for handling the amount of $\alpha\beta Q$ unit scrap during production. It can be evaluated by multiplying the unit scrap handling cost, C_i and the amount of scrap produced in entire period as

$$K_T = C_i \alpha \beta Q. \tag{4.27}$$

4.5.5 Total System Cost

The production of total batch quantity, Q is divided into N cycles to minimize the inventory carrying cost, so it is necessary to find the total cost for the entire production period. Hence, it is required to form the total cost function with respect to number of cycles, $N (= D/Q)$. Thus, the total system cost of the production and rework process over N cycles for the second policy with different cases of scrap production can be evaluated from equations (4.17), (4.18), (4.19), (4.20), (4.20a), (4.26), and (4.27). Since the total cost is a function of $N (= D/Q)$, it can be written as

$$\begin{aligned}
TC_I(N) &= NC_s t_s + C_d t_d + \frac{C_i \alpha \beta D}{N} + \frac{H_n D}{2N} \left[(1-\beta) \left(1 - \beta - \frac{D}{P} \right) + (1-\alpha) \beta^2 \right. \\
&\quad \left. \times \left(1 - \alpha - \frac{(1-2\alpha)D}{P} \right) \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right] \\
&\quad + \frac{C_p \beta D}{2(1-\beta)} \left[1 - \beta - \frac{1 - 3\beta + 2\alpha\beta}{N} \right], \tag{4.28}
\end{aligned}$$

$$TC_{II}(N) = NC_s t_s + C_d t_d + \frac{C_i \alpha \beta D}{N} + \frac{H_n D}{2N} \left[(1-\beta) \left(1 - \beta - \frac{D}{P} \right) \right]$$

$$\begin{aligned}
& + \beta^2 \left((1-\alpha)^2 - \frac{(1+\alpha\delta)(1-\alpha+\alpha\delta)D}{P} \right) \\
& + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right] + \frac{C_p \beta D}{2(1-\beta)} \left[1-\beta - \frac{1-3\beta+2\alpha\beta}{N} \right], \tag{4.29}
\end{aligned}$$

$$\begin{aligned}
TC_{III}(N) = & NC_s t_s + C_d t_d + \frac{C_i \alpha \beta D}{N} + \frac{H_n D}{2N} \left[(1-\beta) \left(1-\beta - \frac{D}{P} \right) \right. \\
& \left. + \left(\beta^2 - 2\alpha\beta^2 + \alpha^2 \beta^2 - \frac{(\beta^2 + \alpha\beta^2)D}{P} \right) \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right] \\
& + \frac{C_p \beta D}{2(1-\beta)} \left[1-\beta - \frac{1-3\beta+2\alpha\beta}{N} \right], \text{ and} \tag{4.30}
\end{aligned}$$

$$\begin{aligned}
TC_{SP}(N) = & NC_s t_s + C_d t_d + \frac{C_i \alpha \beta D}{N} + \frac{H_n D}{2N} \left[(1-\beta) \left(1-\beta - \frac{D}{P} \right) \right. \\
& \left. + \left(\beta^2 - 2\alpha\beta^2 + \alpha^2 \beta^2 - \frac{\beta(\beta - \alpha\beta)D}{P} \right) \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(N-1)t_s}{2} + \frac{D^2}{2P} \right] \\
& + \frac{C_p \beta D}{2(1-\beta)} \left[1-\beta - \frac{1-3\beta+2\alpha\beta}{N} \right]. \tag{4.31}
\end{aligned}$$

which need to be minimized. For the minimization purpose, it is necessary to find the nature of these functions and it is found that the above functions are convex function [see Appendix D].

4.6 Optimality

It can be easily shown that $TC(N)$ is a convex function in N (see Appendix D). Hence, an optimum number of cycles N^* , can be calculated from $\partial TC(N)/\partial N = 0$, which yields

$$\frac{\partial TC_I(N)}{\partial N} = C_s t_s - \frac{C_i \alpha \beta D}{N^2} - \frac{H_n D}{2N^2} \left[(1-\beta) \left(1-\beta - \frac{D}{P} \right) + (1-\alpha)\beta^2 \right]$$

$$\times \left(1 - \alpha - \frac{(1 - 2\alpha)D}{P} \right) \Big] + \frac{\beta C_w D t_s}{2(1 - \beta)} + \frac{C_p \beta (1 - 3\beta + 2\alpha\beta) D}{2(1 - \beta) N^2} = 0,$$

which yields

$$\frac{1}{N^2} \left[C_i \alpha \beta D + \frac{H_n D}{2} \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + (1 - \alpha) \beta^2 \left(1 - \alpha - \frac{(1 - 2\alpha)D}{P} \right) \right] - \frac{C_p \beta (1 - 3\beta + 2\alpha\beta) D}{2(1 - \beta)} \right] = C_s t_s + \frac{\beta C_w D t_s}{2(1 - \beta)},$$

from which

$$N_I^* = \sqrt{\frac{2(1 - \beta) C_i \alpha \beta D + (1 - \beta) H_n D \theta - \beta (1 - 3\beta + 2\alpha\beta) D C_p}{2(1 - \beta) C_s t_s + \beta C_w D t_s}} \quad (4.32)$$

$$\text{where } \theta = \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + (1 - \alpha) \beta^2 \left(1 - \alpha - \frac{(1 - 2\alpha)D}{P} \right) \right].$$

Similarly, from equations (4.29), (3.30), and (3.31) the optimum number of cycles N^* , can be evaluated, respectively, as

$$N_{II}^* = \sqrt{\frac{2(1 - \beta) C_i \alpha \beta D + (1 - \beta) H_n D \phi - \beta (1 - 3\beta + 2\alpha\beta) D C_p}{2(1 - \beta) C_s t_s + \beta C_w D t_s}}, \quad (4.33)$$

$$\text{where } \phi = \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + \beta^2 \left((1 - \alpha)^2 - \frac{(1 + \alpha\delta)(1 - \alpha + \alpha\delta)D}{P} \right) \right],$$

$$N_{III}^* = \sqrt{\frac{2(1 - \beta) C_i \alpha \beta D + (1 - \beta) H_n D \psi - \beta (1 - 3\beta + 2\alpha\beta) D C_p}{2(1 - \beta) C_s t_s + \beta C_w D t_s}}, \quad (4.34)$$

$$\text{where } \psi = \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + \beta^2 \left((1 - \alpha)^2 - \frac{(1 + \alpha)D}{P} \right) \right], \text{ and}$$

$$N_{SP}^* = \sqrt{\frac{2(1 - \beta) C_i \alpha \beta D + (1 - \beta) H_n D \phi - \beta (1 - 3\beta + 2\alpha\beta) D C_p}{2(1 - \beta) C_s t_s + \beta C_w D t_s}}, \quad (4.35)$$

$$\text{where } \phi = \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + \left(\beta^2 - 2\alpha\beta^2 + \alpha^2\beta^2 - \frac{\beta^2(1 - \alpha)D}{P} \right) \right].$$

From the above equations, the optimum batch quantity for all cases can be evaluated as

$$Q_i^* = D/N_i^* \quad (\text{where } i = I, II, III \text{ and } SP). \quad (4.36)$$

4.6.1 Special Cases

If no defective and scrap are produced during the production, i.e., $\alpha = \beta = 0$, then equations (4.32), (4.33), (4.34), and (4.35) reduce to

$$N^* = \sqrt{\frac{H_n D(1 - D/P)}{2C_s t_s}}. \quad (4.37)$$

Since $Q^* = D/N^*$, the optimal batch quantity reduces to the classical model for the optimum batch quantity which is

$$Q^* = \sqrt{\frac{2C_s t_s D}{H_n(1 - D/P)}}. \quad (4.38)$$

When no scrap is produced during the rework, i.e., $\alpha = 0$, equations (4.32), (4.33), (4.34), and (4.35) reduce to

$$N^* = \sqrt{\frac{(1 - \beta)H_n D \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + \beta^2 \left(1 - \frac{D}{P} \right) \right] - \beta(1 - 3\beta)DC_p}{2(1 - \beta)C_s t_s + \beta C_w D t_s}}. \quad (4.39)$$

Hence,

$$Q^* = \sqrt{\frac{D[2(1 - \beta)C_s t_s + \beta C_w D t_s]}{(1 - \beta)H_n \left[(1 - \beta) \left(1 - \beta - \frac{D}{P} \right) + \beta^2 \left(1 - \frac{D}{P} \right) \right] - \beta(1 - 3\beta)C_p}}. \quad (4.40)$$

The derived solutions above are validated through numerical examples in the following section.

4.7 Numerical Example

Using the same $\beta = 0.05$, $\alpha = 0.2$, and $\delta = 0.07$ N is obtained using equations (4.36), (4.37), (4.38), and (4.39) and shown in a tabular form. For $C_s = \$1.00/\text{min}$ (equivalently $\$2,628,000/\text{year}$), $t_s = 50 \text{ min}/\text{setup}$ (equivalently $0.000095129 \text{ year}/\text{setup}$), $C_d = \$1.00/\text{min}$, $t_d = 50 \text{ min}$, $D = 300 \text{ units}/\text{year}$, $P = 550 \text{ units}/\text{year}$, $H_n = \$118/\text{units}/\text{year}$, $C_p = \$177/\text{units}/\text{year}$, and $C_w = \$88.5/\text{units}/\text{year}$, the following values can be obtained by using equations (4.26), (4.27), (4.28), (4.29), (4.32), (4.33), (4.34), (4.35), (4.36), (4.37), and (4.38), respectively.

Table 4.1: Results of the numerical example for second policy.

Cases	Case I	Case II	Case III	Special Case	Ideal Case
Parameters					
N^*	10.59	10.58	10.58	10.58	12.68
Q^* , units	28.32	28.34	28.34	28.34	23.65
$TC(N^*)$, \$	2819.28	2818.49	2817.64	2818.55	1318.50

According to Table 4.1 it can be concluded that the values of Q^* , and $TC(N^*)$ for ideal inventory model is less than for the evaluated inventory models.

CHAPTER 5

OPERATIONAL SCHEDULE

It is necessary to implement evaluated models with numerical data for practical situations of a production process. These operational schedules determine the optimum order quantity, time of production and rework, quantity of produced items, quantity of scrap and buffer, etc., in a production process with rework and scrap. This chapter deals with the operational schedules of the models of this research for entire production process.

5.1 Operational Schedule for First Policy: Within Cycle Rework

Under this policy, the rework is done within the same cycle in which the defectives are produced, together with defective items produced and detected before, during and after the rework process. Depending on the severity of defects, they are reworked or scrapped. To evaluate the operational schedule, it is necessary to calculate the time of production and rework, quantity of produced good items, defective items and scrap, optimum order quantity, etc. The following values of parameters are used to calculate the operational schedule:

$D = 300$ units/year, $P = 550$ units/year, $C = \$7$ /units, $S = \$50$ /batch, $H_1 = \$50$ /units/year, $C_r = \$5$ /units, $\beta = 0.05$ and $\alpha = 0.20$ and $\delta = 0.07$ (meaning β is assumed as 0.05, i.e., 5% defects, and α is assumed as 0.20, i.e., 20% scrap from the defectives. The operational schedule for different cases is calculated as follows:

5.1.1 Operational Schedule for Case I of First Policy

Case I of the first policy states that, the scrap can be detected before the rework process starts to produce good items from the defectives. Using the above parametric

$$t_3 = \frac{[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q}{D} = \frac{[1 - 0.20 \times 0.05 - (1 + 0.05 - 0.20 \times 0.05)300/550]37}{300}$$

= 0.052 years. Hence, the total cycle time is $T_I = t_1 + t_2 + t_3 = 0.067 + 0.0027 + 0.052 = 0.122$ years. Using the above values, the operational schedule for Case I of first policy is graphically represented in Figure 5.1.

The related costs involved in operational schedule of Case I are calculated using the above values and equations (3.12), (3.13), (3.16), (3.19), (3.20b), (3.21) and (3.22) as follows:

Setup cost = \$401.18/setup, inventory carrying cost = \$394.11/year, makeup buffer inventory carrying cost = \$5.16/year, processing cost = \$2100, processing cost for rework = \$126, scrap handling cost = \$1.87 and the optimum total cost = \$3028.32. The optimum total cost found here is same as the optimum total cost evaluated in numerical example for Case I of within cycle rework policy.

5.1.2 Operational Schedule for Case II of First Policy

In Case II of first policy, scrap is detected during the rework process, and it is assumed that the time to produce a reworking item as scrap is less than the time to produce a good item. Hence, another parameter is considered in this case which is known as scrap production factor, $\delta = 0.07$. Using equation (3.28) it is found that $Q_{II}^* = 38$ units/year. Hence, $t_1 = Q_{II}^*/P = 38/550 = 0.068$ year, $Q_1 = 15$ units/year and $Q_2 = 17$ units/year, number of defectives produced is $\beta Q_{II}^* = 2$ units and number of scraps is $\alpha\beta Q_{II}^* = 0.4$ are calculated as for Case I. The rework time is $t_2 = (1 - \alpha)\beta Q_{II}^*/P = (1 - 0.2) \times 0.05 \times 38/550 = 0.0027$ years, $t_3 = \alpha\beta\delta Q_{II}^*/P = 5 \times 10^{-5}$ and using equation (3.6) t_4 is found as

$$t_4 = \frac{38}{300} \left(1 - 0.20 \times 0.05 - \frac{(1 + 0.05 - 0.20 \times 0.05 + 0.20 \times 0.05 \times 0.07) 300}{550} \right) = 0.0524 \text{ years.}$$

Hence, $T_2 = t_1 + t_2 + t_3 + t_4 = 0.068 + 0.0027 + 0.0524 + 5 \times 10^{-5} = 0.123$ years.

The related costs of operational schedule for Case II are calculated using the above values and equations (3.12), (3.14), (3.17), (3.19), (3.20b), (3.21) and (3.23) as follows:

Setup cost = \$398.72/setup, inventory carrying cost = \$396.54/year, makeup buffer inventory carrying cost = \$0.0007/year, processing cost = \$2100, processing cost for rework = \$126, scrap handling cost = \$1.87 and the optimum total cost = \$3023.13. The optimum total cost is matched with the optimum total cost calculated in numerical example for Case II of within cycle rework policy.

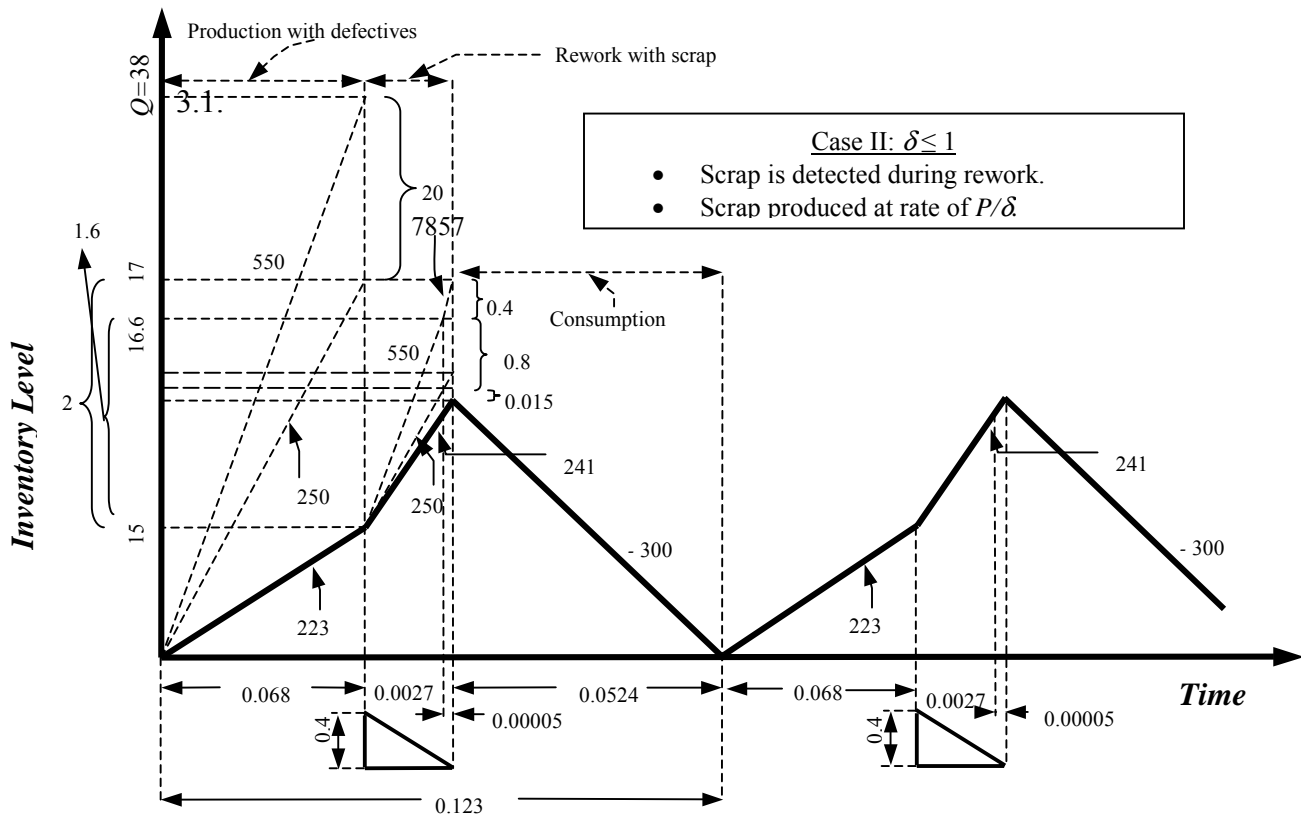


Figure 5.2: Operational schedule for Case II of First Policy.

Figure 5.2 represents the operation schedule for Case II of first policy and the calculated parameters of operational schedule for first policy are shown in Table 5.1.

Table 5.1: Calculation of operational schedule for within cycle rework policy.

Parameters	Case I	Case II	Units
Q^*	37.39	37.62	units/year
Q_1^*	15.00	15.00	units/year
Q^*	17.00	17.00	units/year
βQ^*	2.00	2.00	units
$\alpha\beta Q^*$	0.40	0.40	units
t_1	0.067	0.068	years
t_2	0.0027	0.027	years
t_3	0.052	5×10^{-5}	years
t_4	–	0.0524	years
T	0.122	0.123	years
K_s	401.18	398.72	\$/setup
IC	394.11	396.54	\$
B	5.16	0.0007	\$
K_P	2100.00	2100.00	\$
K_{RBP}	126.00	126.00	\$
K_T	1.87	1.87	\$
$TC(Q^*)$	3028.32	3023.13	\$

5.2 Operational Schedule for Second Policy: Rework after N Cycles

In this policy, the defective items are accumulated up to N cycles of normal production, after which they are reworked. As the production in a cycle continues, the finished goods are supplied to the customer. For the second policy, the operational schedule is computed using the following values:

$D = 300$ units/year, $P = 550$ units/year, $\beta = 0.05$, $\alpha = 0.2$, $\delta = 0.07$, $C_s = \$1.00/\text{min}$ (equivalently $\$2,628,000/\text{year}$), $t_s = 50$ min/setup (equivalently 0.000095129 year/setup), $C_d = \$1.00/\text{min}$, $t_d = 50$ min, $H_n = \$118/\text{units/year}$, $C_p = \$177/\text{units/year}$, and $C_w = \$88.5/\text{units/year}$. The operational schedule for different cases is calculated below.

5.2.1 Operational Schedule for Case I of Second Policy

To calculate the operational schedule of Case I (scrap detected before rework) of the second policy, it is first required to calculate the optimum number of cycles, N_I^* . Applying above values in equation (4.32), the optimum number of cycles is found as $N_I^* = 10.59 \approx 11$, and from equation (4.36), the optimum quantity of item produced in a cycle is $Q_I^* = 28.32 \approx 28$ units. Hence, the uptime $t_p = Q_I^*/PN_I^* = 28/(550 \times 11) = 0.005$, quantity produced in a cycle $= Q_I^*/N_I^* = 3$ units and downtime is $t_d = [P(1-\beta)-D] Q_I^*/PDN_I^* = [550(1-0.05)-300]28/(550 \times 300 \times 11) = 0.004$ years. The quantity is remained after the end of the production and consumption of the first cycle is $Q = [550(1-0.05) - 300] 3/550 = 1$ units. The number of defectives produced in this cycle is 0.15 units. The total number of defectives items produced in 11 cycles is $0.15 \times 10.59 = 2 = \beta Q$. According to second policy, the rework is completed in $(N + 1)^{th}$ cycle, here it is $(11+1)^{th} = 12^{th}$ cycle. Scrap produced is $\alpha\beta Q = 0.20 \times 2 \approx 0.4$ units and number of defective item for rework remains $2 - 0.4 = 1.6$ units. According to Figure 4.3 the rework time can be calculated as $t_{pr} = 1.6/[P - D] = 0.0072$ years and consumption time for rework is $t_{dr} = 1.6/D = 0.0053$ years. Hence, the total time of the entire cycle is $T = N_I^* (t_p + t_d) + (t_{pr} + t_{dr}) = 0.108$ years. Figure 5.3 represents the operational schedule for Case I of second policy.

The costs related to this operational schedule are calculated using the equations (4.17), (4.17a), (4.18), (4.26), (4.27), and (4.28), respectively, as follows:

Setup cost for entire batch, $K_S = \$529.50/\text{setup}$, setup cost for rework, $K_{SD} = \$50/\text{setup}$, inventory carrying cost, $IC_I = \$1025.66/\text{year}$, penalty cost, $PC_I = \$1212.71$, scrap handling cost, $K_T = \$1.41$ and the optimum total cost, $TC(N_I^*) = \$2819.28$. The

optimum total cost found in this section is equal to the optimum total cost evaluated in numerical example for Case I of rework after N cycles policy, because the same values of the parameters are used to calculate.

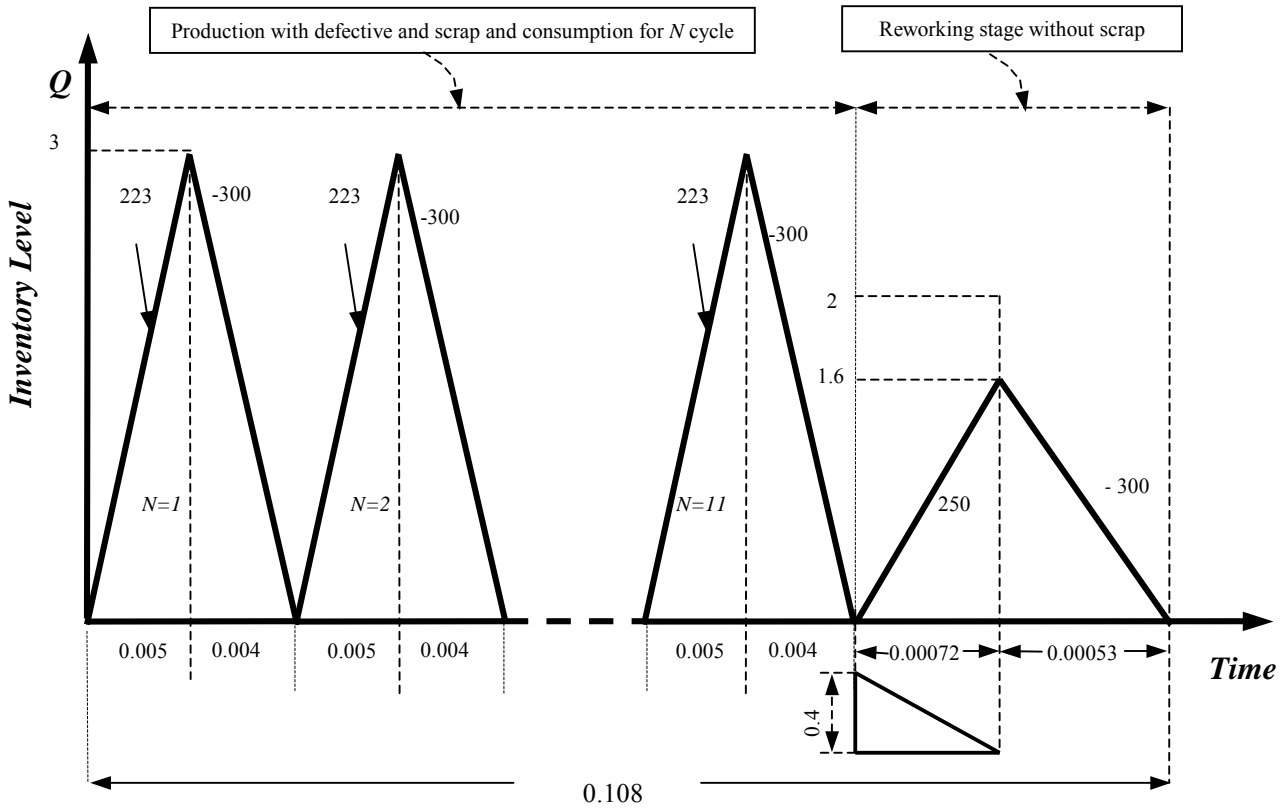


Figure 5.3: Operational schedule for Case I of Second Policy

5.2.2 Operational Schedule for Case II of Second Policy

In this section the operational schedule for Case II (scrap detected during rework) of second policy is calculated by using the data described before. Using equation (4.33), the optimum number of cycles calculated is $N_{II}^* = 10.58 \approx 11$, and from equation (4.36) the optimum batch quantity is calculated as $Q_{II}^* = 28$ units/year. Some values such as the quantity of item produced in a cycle = 3 units, $t_p = 0.005$, $t_d = 0.004$ years, number of

defectives produced in this cycle = 0.15 units and defectives items produced in 11 cycles = 2 = βQ_{II}^* , number of scrap produced $\alpha\beta Q_{II}^* = 0.4$ units and number of defective item remained for rework = 1.6 units as calculated previously. According to Figure 4.4 the rework time can be calculated as $t = t_{pr} + t_{ps} = (\beta - \alpha\beta)Q_{II}^*/PN_{II}^* + \alpha\beta\delta Q_{II}^*/PN_{II}^* = 0.0029 + 3 \times 10^{-6} = 0.003$ years and consumption time for rework is $t_{dr} = (\beta Q_{II}^* - \alpha\beta Q_{II}^* - Dt_{ps})/D = 0.0053$ years. Hence, the total time of the entire cycle is $T = N_{II}^* (t_p + t_d) + (t_{pr} + t_{sr} + t_{dr}) = 11.58(0.005 + 0.004) + (0.0029 + 3 \times 10^{-6} + 0.0053) = 0.104$ years. The graphical representation of the operational schedule for Case II of second policy is shown in Figure 5.4.

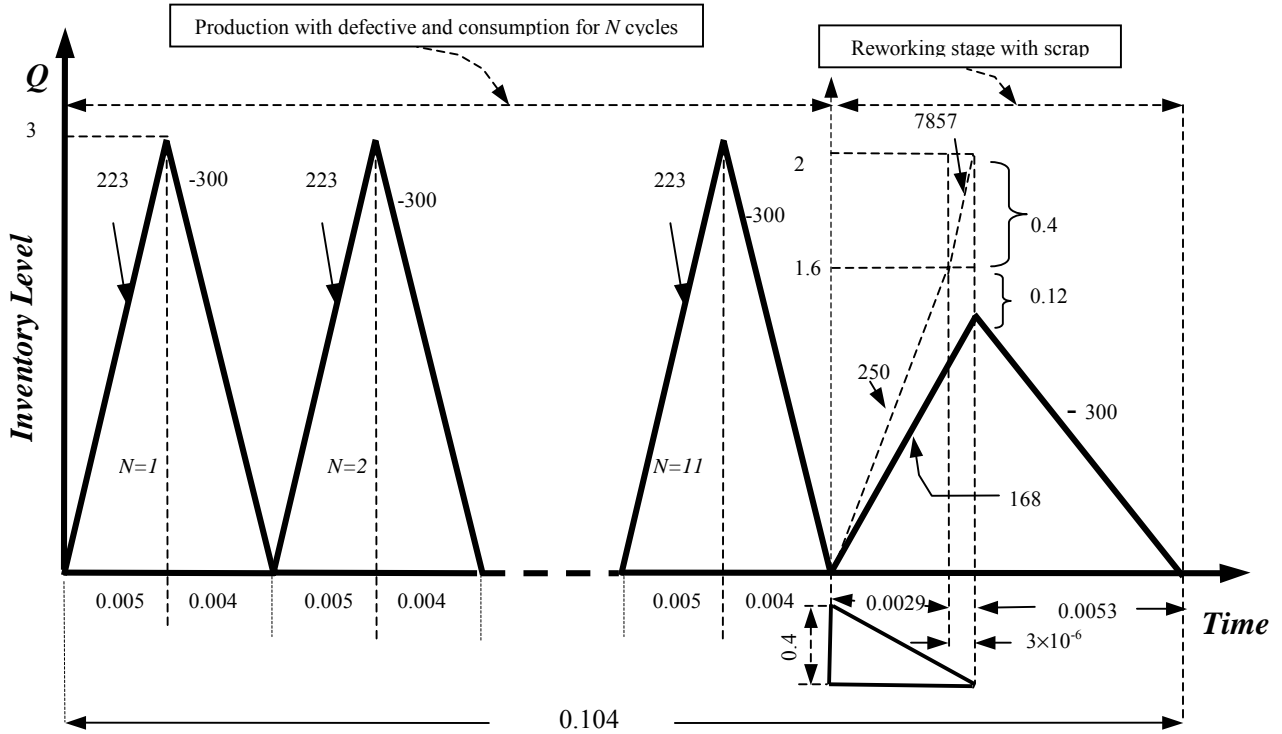


Figure 5.4: Operational schedule for Case II of Second Policy.

The costs involved in this operational schedule for Case II are calculated using the equations (4.17), (4.17a), (4.19), (4.26), (4.27), and (4.29), respectively, below:

Setup cost for entire batch, $K_S = \$529.00/\text{setup}$, setup cost for rework, $K_{SD} = \$50/\text{setup}$, inventory carrying cost, $IC_{II} = \$1025.48/\text{year}$, penalty cost, $PC_{II} = \$1212.59$, scrap handling cost, $K_T = \$1.42$ and the optimum total cost, $TC(N_{II}^*) = \$2818.49$. The optimum total cost found in this section is equal to the optimum total cost evaluated in numerical example for Case II of rework after N cycles policy where the same parametric values are used for numerical computation of the equations. The calculated values of the operational schedule for second policy are shown in Table 5.2.

Table 5.2: Calculation of operational schedule for rework after N cycles policy.

Parameters	Case I	Case II	Units
Q^*	28.34	28.34	units/year
N^*	10.59	10.58	–
βQ^*	2.00	2.00	units
$\alpha \beta Q^*$	0.40	0.40	units
t_p	0.005	0.068	years
t_d	0.004	0.027	years
t_{pr}	0.0072	0.002903	years
t_{dr}	0.0053	0.0053	years
T	0.108	0.104	years
K_S	529.50	529.00	\$/setup
K_{SD}	50.00	50.00	\$
IC	1025.66	1025.48	\$
PC	1212.71	1212.59	\$
K_T	1.41	1.42	\$
$TC(N^*)$	2819.28	2818.49	\$

In the operational schedule, Case III and Special Case are not shown as they are the special case of Case II.

CHAPTER 6

SENSITIVITY ANALYSIS

The total cost functions are the real solution in which the model parameters (batch quantity, proportion of defectives, proportion of scrap) are assumed to be static values. It is reasonable to study the sensitivity, i.e., the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analyses of various system parameters for the models of this research are required to observe whether,

- (a) The current solutions remain unchanged,
- (b) The current solutions become sub-optimal,
- (c) The current solutions become infeasible, etc.

In this research, two alternative models with three different cases are developed for the optimal production lot size with allowance for rework of defective items and scrap. A sensitivity analysis is carried out for both policies to determine how the total cost of the system and the optimum batch quantity are affected due to the changes of defective rates β , percentage of scrap α , both α and β , scrap production factor δ for Case II, setup cost S , and scrap handling cost C_r .

6.1 Effect of β on Q^* and $TC(Q^*)$

The proportion of the defectives is a major parameter in developing the model. Both the batch quantity and the total cost are affected due to variation of the proportion of the defectives. Mathematically, for Case I of the first policy,

$$\frac{dQ^*}{d\beta} = \left[\frac{\sqrt{PDS} (H_1 \xi - 2\alpha(1-\alpha\beta)^2 C_i P)}{\sqrt{2(1-\alpha\beta)} v^{3/2}} \right], \quad (6.1)$$

where $\xi = \alpha P(1-\alpha\beta)^2 + D(1+2\beta-2\alpha-4\alpha\beta+2\alpha^2\beta-\alpha\beta^2+2\alpha^2\beta^2-\alpha^3\beta^2)$, and

$$v = [H_1((P-D(1+\beta+\beta^2))-\alpha\beta(2P-3D-2\beta D-\alpha\beta P+\alpha\beta D))+2C_i\alpha\beta(1-\alpha\beta)P].$$

The rate and the direction of change of Q with respect to β depend upon the parametric values used in numerical examples. Hence, $dQ^*/d\beta > 0$ holds on the real values of β . According to equation (6.1), $dQ^*/d\beta \in \Omega$ if $\beta \in [0.01, 0.5931]$ and $dQ^*/d\beta \notin \Omega$ if $\beta \in [0.5931, 1]$ and the effect of Q over the defective proportion β is shown in Figure 6.1. From the Figure 6.1, it can be observed that the change over Q^* is occurring slowly up to point [0.55, 1700] due to change of β , and after that Q^* increases with an increase of β .

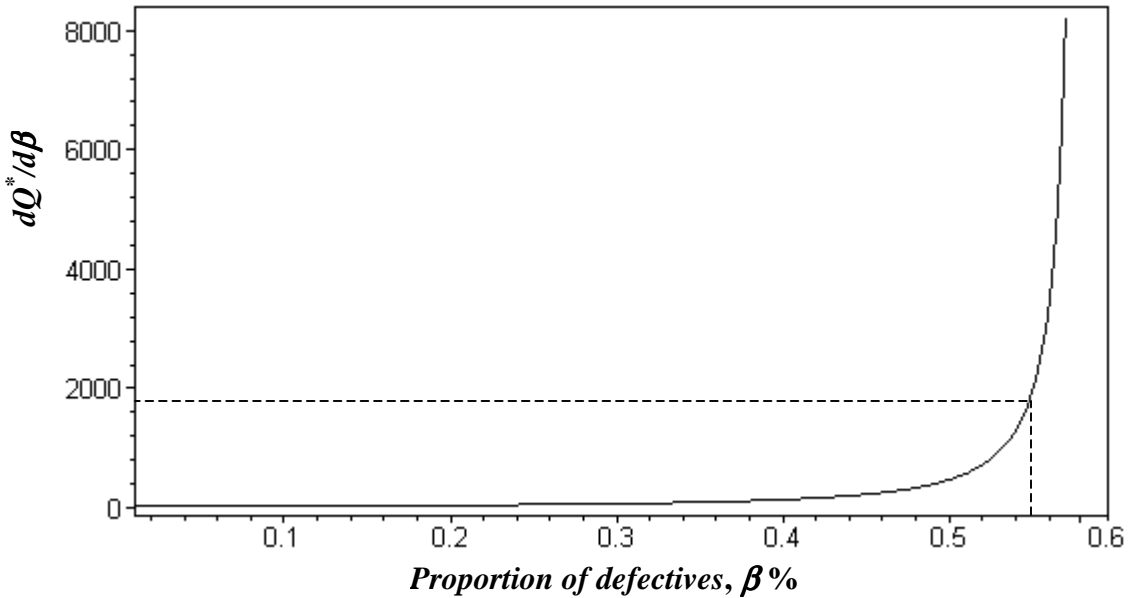


Figure 6.1: Effect of proportion of defective on batch size.

Again, the effect of proportion of the defective items over the $TC(Q^*)$ can be shown mathematically for Case I of first policy by equation (6.2) as follows:

$$\frac{\partial TC(Q^*)}{\partial \beta} = CD(1 + \alpha) + C_i \alpha Q^* - \frac{H_1 Q^*}{2(1 - \alpha\beta)^2 P} [(1 - \alpha\beta)^2 P + D(1 + 2\beta - 2\alpha - 4\alpha\beta + 2\alpha^2 \beta - \alpha\beta^2 + 2\alpha^2 \beta^2 - \alpha^3 \beta^2)] \quad (6.2)$$

Using equation (6.2), it can be found that $dTC(Q^*)/d\beta \in \Phi$, if $\beta \in [0.01, 0.81]$ and $dTC(Q^*)/d\beta \notin \Phi$ if $\beta \in [0.81, 1]$ the effect of Q over the defective proportion β is shown in Figure 6.2.

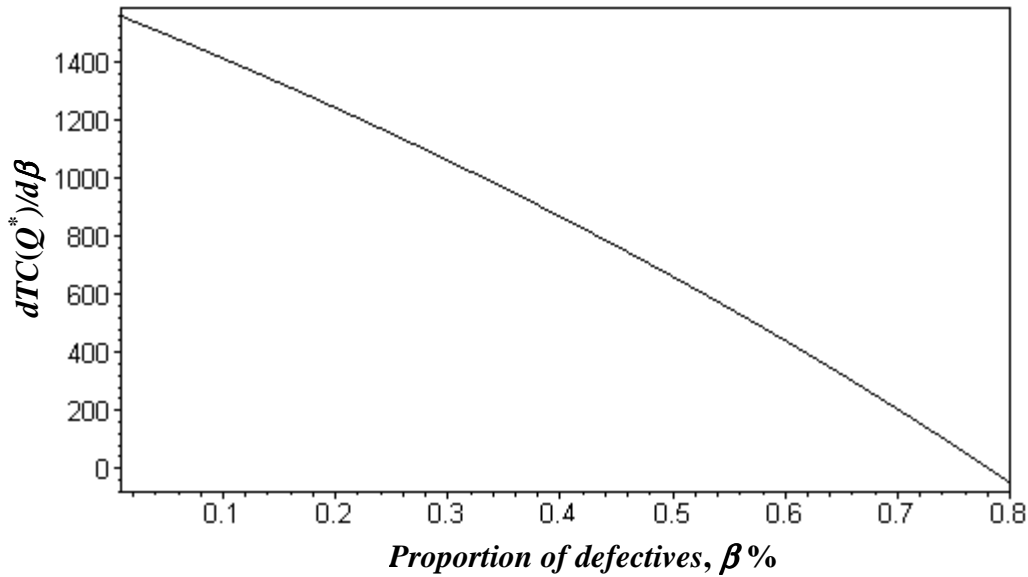


Figure 6.2: Effect of proportion of defective on total cost.

The effect of proportion of defectives is studied by changing the value of β from 0.05 to 0.23. It is observed that the total cost, $TC(Q^*)$ and optimal batch quantity, Q^* are directly related with defective rates and their values increase as β increases. This study is shown in Table 6.1 and 6.2 for all cases for both policies.

It is observed that, in second policy, the optimum batch size increases up to a certain level with the increase of the proportion of defectives. After that it starts to decrease, which indicates that second policy is more sensitive than first policy.

Table 6.1: Effect of β over $TC(Q^*)$ and Q^* for Case I and Case II.

β	CASE I				CASE II			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.05	37.39	3028.31	28.32	2819.28	37.62	3023.39	28.34	2818.49
0.08	38.12	3088.56	32.25	3735.95	38.51	3080.71	32.34	3733.65
0.10	38.65	3128.13	35.59	4354.61	39.15	3118.34	35.79	4350.64
0.15	40.17	3224.83	48.53	5931.06	40.98	3210.17	49.65	5918.80
0.20	42.02	3317.95	77.91	7557.42	43.22	3298.36	87.39	7521.13
0.23	43.34	3371.86	123.90	8562.96	44.82	3349.25	221.12	8475.02

Table 6.2: Effect of β over $TC(Q^*)$ and Q^* for Case III and Special Case.

β	CASE III				SPECIALCASE			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.05	37.61	3023.73	28.34	2817.64	37.62	3023.37	28.34	2818.55
0.08	38.46	3081.59	30.85	3425.05	38.51	3080.65	32.34	3733.82
0.10	39.08	3119.74	32.05	3736.56	39.16	3118.24	35.79	4350.94
0.15	40.79	3213.48	34.76	4363.59	41.00	3209.92	49.65	5919.71
0.20	42.83	3304.58	45.00	5938.57	43.25	3297.90	87.34	7523.98
0.23	44.25	3357.79	46.75	6617.35	44.87	3348.61	219.86	8484.42

6.2 Effect of α on Q^* and $TC(Q^*)$

Another important parameter in developing the model is the proportion of the scrap, α with respect to defective item produced. The effect of α on optimal batch quantity, Q^* is represented mathematically in equation (6.3).

$$\frac{\partial Q^*}{\partial \alpha} = \frac{\sqrt{DS(1-\alpha\beta)}[\beta H_1[(1-\alpha\beta)^2 P - D(2 + \beta - \beta^2 - 2\alpha\beta + \alpha^2\beta^2)] - 2C_i\beta(1-\alpha\beta)^2 P]}{\sqrt{2Pv}^{3/2}} \quad (6.3)$$

where

$$v = [H_1((P - D(1 + \beta + \beta^2)) - \alpha\beta(2P - 3D - 2\beta D - \alpha\beta P + \alpha\beta D)) + 2C_i\alpha\beta(1 - \alpha\beta)P].$$

The rate and direction of change of Q^* , with respect to α , depend on the values of the parameter used in numerical example for Case I of within cycle rework policy. The effect of α on the total cost, $TC(Q^*)$ can be represented mathematically by equation (6.4) as

$$\frac{\partial TC(Q^*)}{\partial \alpha} = CD\beta + C_r\beta Q^* + \frac{\beta Q^* H_1 [(1-\alpha\beta)^2 P - D(2 + \beta - \beta^2 - 2\alpha\beta + \alpha^2 \beta^2)]}{2(1-\alpha\beta)^2 P}. \quad (6.4)$$

Hence, using equation (6.3) and (6.4) and $\alpha \in [0.01, 0.7]$, the effect of α on Q^* and $TC(Q^*)$ is shown in Figure 6.3. According to the Figure 6.3 it can be observed that the rate of change in $TC(Q^*)$ with respect to α values is very small.

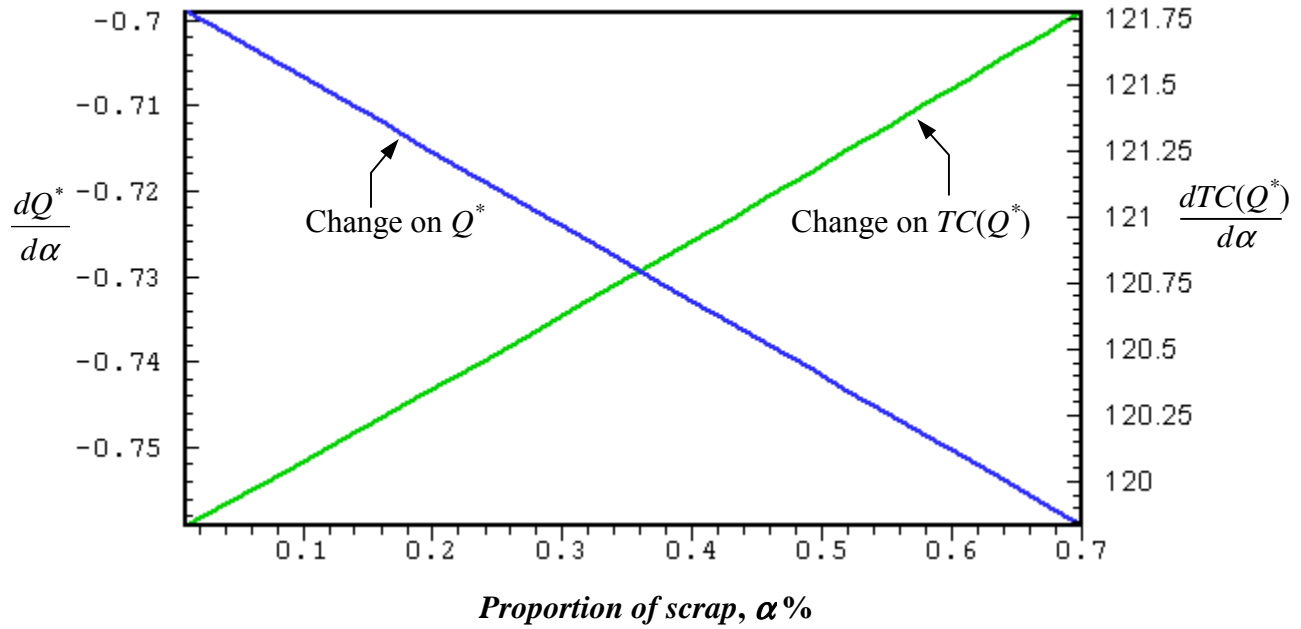


Figure 6.3: Effect of proportion of scrap on batch size and total cost.

The effect of scrap rates is studied by changing α values over the range from 0.05 to 0.7 and it is observed that change of optimum batch quantity Q^* is inversely proportional to the change of α for the Case I of first policy. In second policy, Q^* increases, but the total cost, $TC(Q^*)$ decreases somewhat. The values of α are varied from

0.2 to 0.8 and the changes in Q^* and $TC(Q^*)$ for all cases of both policies are shown in Table 6.3 and Table 6.4 respectively.

Table 6.3: Effect of α over $TC(Q^*)$ and Q^* for Case I and Case II.

α	CASE I				CASE II			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.2	37.39	3028.31	28.32	2819.28	37.62	3023.39	28.34	2818.49
0.3	37.32	3040.36	28.34	2818.50	37.67	3032.92	28.37	2817.46
0.4	37.25	3052.43	28.36	2817.71	37.72	3042.44	28.39	2816.51
0.5	37.17	3064.53	28.38	2816.91	37.77	3051.95	28.41	2815.64
0.6	37.10	3076.65	28.40	2816.10	37.82	3061.44	28.43	2814.87
0.8	36.95	3100.97	28.45	2814.46	37.92	3080.38	28.46	2813.57

Table 6.4: Effect of α over $TC(Q^*)$ and Q^* for Case III and Special Case.

α	CASE III				SPECIALCASE			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.2	37.61	3023.73	28.34	2817.64	37.62	3023.37	28.34	2818.55
0.3	37.64	3033.50	28.37	2816.17	37.67	3032.88	28.37	2817.54
0.4	37.68	3043.31	28.39	2814.79	37.72	3042.38	28.39	2816.61
0.5	37.71	3053.15	28.41	2813.48	37.77	3051.85	28.41	2815.77
0.6	37.74	3063.04	28.43	2812.26	37.82	3061.32	28.43	2815.01
0.8	37.80	3082.92	28.46	2810.07	37.93	3080.19	28.46	2813.73

6.3 Effect of Both α and β on Q^* and $TC(Q^*)$

When the two important parameters α and β are both changed, the effect on optimum batch quantity can be represented by equation (6.5) as

$$\frac{\partial^2 Q^*}{\partial \alpha \partial \beta} = \frac{3\sqrt{2DPS} [\beta H_1 [(1-\alpha\beta)^2 P - D(2 + \beta - \beta^2 - 2\alpha\beta + \alpha^2 \beta^2)] - 2C_i \beta (1-\alpha\beta)^2 P]}{(1-\alpha\beta)^{3/2} 4v^{5/2}} \times [H_1 \{(1-\alpha\beta)^2 \alpha P + D(1 + 2\beta - 2\alpha - 4\alpha\beta + 2\alpha^2 \beta - \alpha\beta^2 + 2\alpha^2 \beta^2 - \alpha^3 \beta^2)\}]$$

$$\begin{aligned}
& -2C_i\alpha(1-\alpha\beta)^2 P] + \frac{\sqrt{DS}(1-\alpha\beta)^{3/2}}{\sqrt{2P\nu^{3/2}}} [H_1 \{P(1-\alpha\beta)^3 \\
& - D(2+2\beta-2\alpha\beta-3\beta^2+\alpha\beta^3+3\alpha^2\beta^2-\alpha^3\beta^3)\} - 2C_i(1-\alpha\beta)^3 P], \quad (6.5)
\end{aligned}$$

where

$$\nu = [H_1((P-D(1+\beta+\beta^2))-\alpha\beta(2P-3D-2\beta D-\alpha\beta P+\alpha\beta D))+2C_i\alpha\beta(1-\alpha\beta)P].$$

Also the effect over total cost due to variation of α and β can be shown mathematically by equation (6.6) as

$$\begin{aligned}
\frac{\partial^2 TC(Q^*)}{\partial\alpha\partial\beta} &= CD + C_i Q^* - \frac{Q^* H_1}{2(1-\alpha\beta)^2 P} [(1-\alpha\beta)^3 P \\
& - D(2+2\beta-3\beta^2-2\alpha\beta+3\alpha^2\beta^2+\alpha\beta^3-3\alpha^3\beta^3+2\alpha^3\beta^4)]. \quad (6.6)
\end{aligned}$$

Using the above equations and $\alpha \in [0.01, 0.5]$, and $\beta \in [0.01, 0.5]$, the effect is shown in Figure 6.4 and Figure 6.5, respectively.

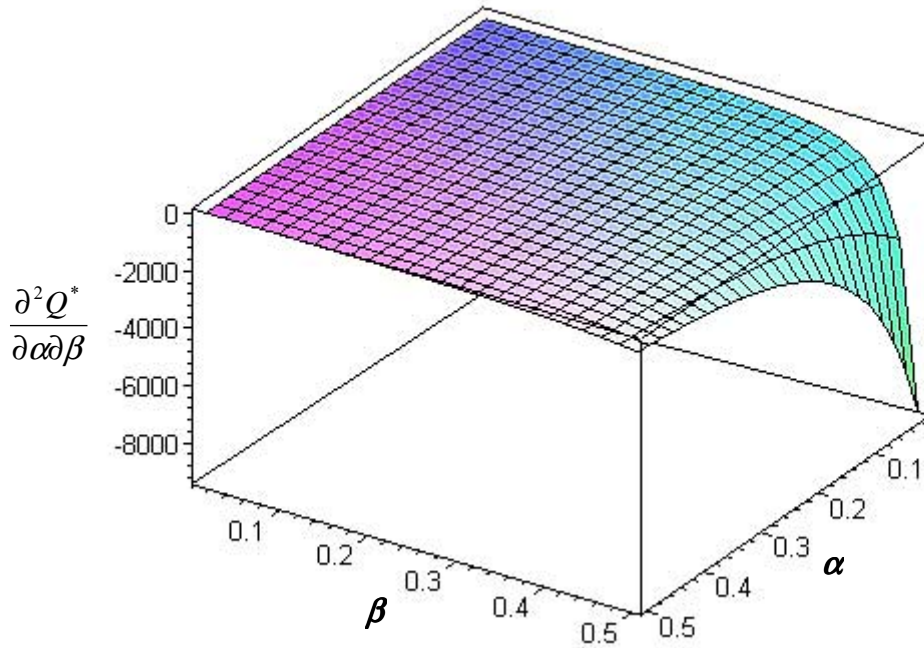


Figure 6.4: Effect of both proportion of scrap and proportion of defectives on batch size.

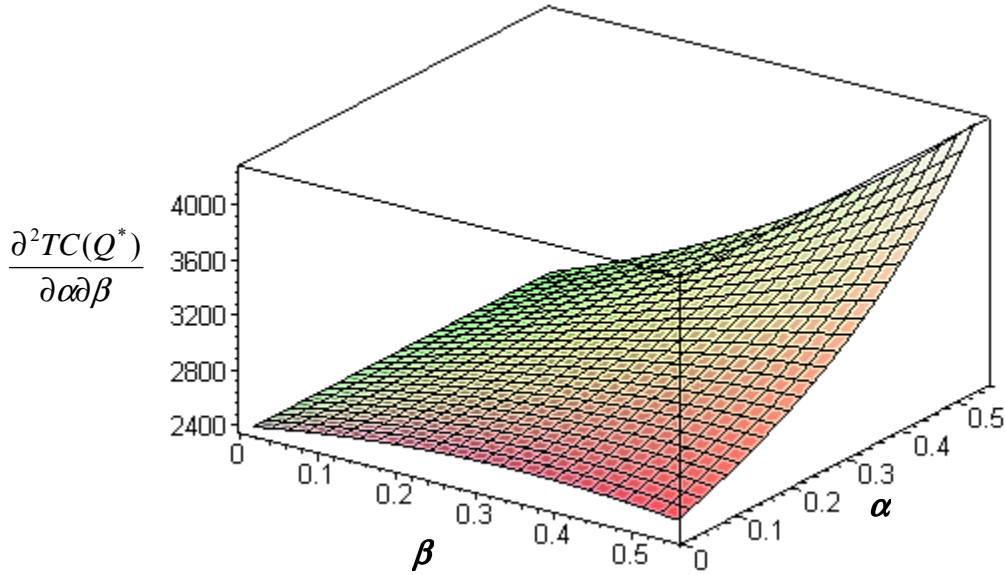


Figure 6.5: Effect of both proportion of scrap and proportion of defectives on total cost.

A study on the effects of changing both α and β over $TC(Q^*)$ and Q^* is represented in Table 6.5 and 6.6 for all cases, which shows that if both α and β increase simultaneously, the values of $TC(Q^*)$ and Q^* also increase. In this study, the same parametric values are used with the variation of α from 0.2 to 0.8 and β from 0.05 to 0.17 and all the values are as shown in the Table 6.5 and 6.6.

Table 6.5: Effect of both α and β on $TC(Q^*)$ and Q^* for Case I and Case II.

α	β	CASE I				CASE II			
		First Policy		Second Policy		First Policy		Second Policy	
		Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.2	0.05	37.39	3028.31	28.32	2819.28	37.62	3023.39	28.34	2818.49
0.3	0.07	37.76	3085.59	30.88	3426.77	38.27	3075.17	30.95	3424.55
0.4	0.08	37.86	3127.63	32.47	3729.55	38.65	3111.56	32.59	3726.03
0.5	0.10	38.11	3202.19	36.39	4336.05	39.41	3176.59	36.65	4329.54
0.6	0.15	38.81	3377.02	57.28	5836.18	41.47	3329.03	59.30	5813.28
0.8	0.17	38.28	3526.25	123.94	6289.63	42.70	3449.42	139.89	6240.63

Table 6.6: Effect of both α and β on $TC(Q^*)$ and Q^* for Case III and Special Case.

α	β	CASE III				SPECIALCASE			
		First Policy		Second Policy		First Policy		Second Policy	
		Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
0.2	0.05	37.61	3023.73	28.34	2817.64	37.62	3023.37	28.34	2817.64
0.3	0.07	38.21	3076.32	30.95	3421.80	38.27	3075.08	30.95	3421.80
0.4	0.08	38.53	3113.84	32.59	3720.96	38.66	3111.38	32.59	3720.96
0.5	0.10	39.14	3181.67	36.64	4318.38	39.43	3176.20	36.64	4318.38
0.6	0.15	40.52	3345.09	59.25	5764.40	41.54	3327.82	59.25	5764.40
0.8	0.17	40.58	3483.20	139.19	6042.72	42.87	3446.88	139.19	6042.72

6.4 Effect of S on Q^* and $TC(Q^*)$

In every manufacturing system, setup is an important cost factor. The variation of setup cost, S over optimum batch quantity Q^* can be represented by equation (6.7).

$$\frac{\partial Q^*}{\partial S} = \left[\frac{\sqrt{PD(1-\alpha\beta)}}{\sqrt{2S\nu}} \right], \quad (6.7)$$

where

$$\nu = [H((P - D(1 + \beta + \beta^2)) - \alpha\beta(2P - 3D - 2\beta D - \alpha\beta P + \alpha\beta D)) + 2C_i\alpha\beta(1 - \alpha\beta)P].$$

The effect of S of over Q^* is represented by Figure 6.6 using equation (6.7) and $S \in [50, 300]$. A study was done also with respect to S where the parametric values are considered as the same except the values of S , which are changed from 50 to 300 and the results are shown in Table 6.7 and 6.8. From the study it can be concluded that with the increase of setup cost, the optimum batch quantity Q^* and the total cost, $TC(Q^*)$ both increase.

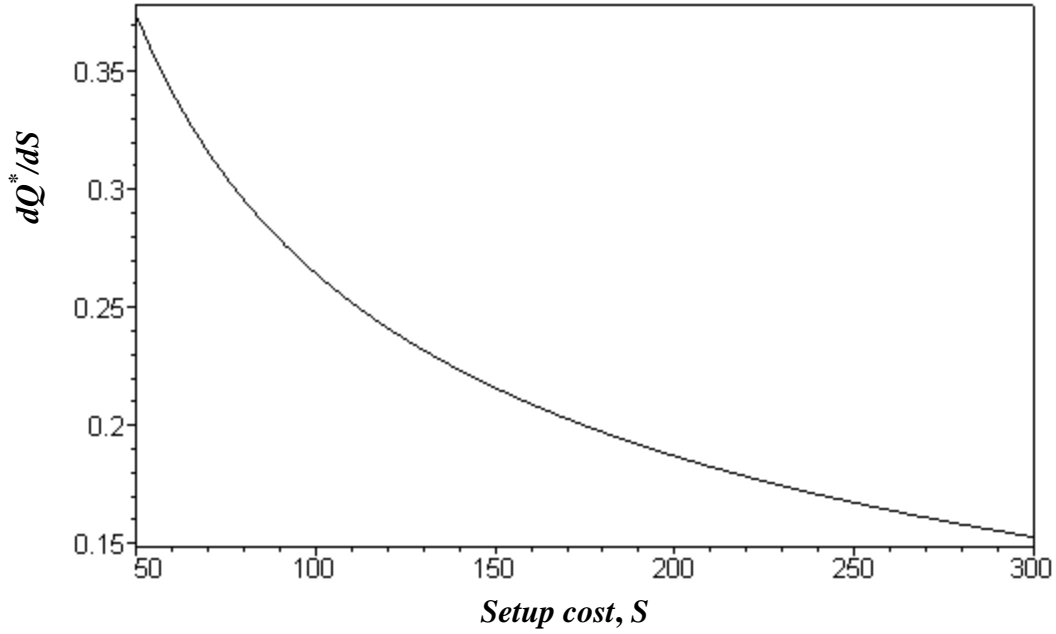


Figure 6.6: Effect of setup cost on batch size.

Table 6.7: Effect of S on $TC(Q^*)$ and Q^* for Case I and Case II.

S	CASE I				CASE II			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
50	37.39	3028.31	28.32	2819.28	37.62	3023.39	28.34	2818.49
100	52.88	3360.64	40.04	3258.16	53.21	3353.68	40.07	3257.05
150	64.76	3615.64	49.03	3594.99	65.17	3607.12	49.07	3593.63
200	74.78	3830.62	56.61	3878.97	75.25	3820.78	56.66	3877.40
250	83.61	4020.02	63.29	4129.18	84.13	4009.02	63.34	4127.42
300	91.59	4191.25	69.33	4355.39	92.16	4179.20	69.39	4353.46

Table 6.8: Effect of S on $TC(Q^*)$ and Q^* for Case III and Special Case.

S	CASE III				SPECIALCASE			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
50	37.61	3023.73	28.34	2817.64	37.62	3023.37	28.34	2818.55
100	53.19	3354.16	40.07	3255.84	53.21	3353.65	40.07	3257.13
150	65.14	3607.70	49.07	3592.15	65.17	3607.08	49.07	3593.73
200	75.21	3821.45	56.66	3875.69	75.25	3820.73	56.66	3877.51
250	84.09	4009.77	63.34	4125.51	84.13	4008.97	63.34	4127.55
300	92.12	4180.02	69.39	4351.37	92.16	4179.14	69.39	4353.60

6.5 Effect of C_t on Q^* and $TC(Q^*)$

In this research the scrap handling cost, C_t plays an important role in developing the model. The variation of C_t over optimum batch size, Q^* can be represented by

equation (6.8) as
$$\frac{\partial Q^*}{\partial C_t} = \left[\frac{-\sqrt{2\alpha^2 \beta^2 DS[(1-\alpha\beta)P]^{3/2}}}{\nu^{3/2}} \right], \quad (6.8)$$

where

$$\nu = [H_1((P - D(1 + \beta + \beta^2)) - \alpha\beta(2P - 3D - 2\beta D - \alpha\beta P + \alpha\beta D)) + 2C_t\alpha\beta(1 - \alpha\beta)P].$$

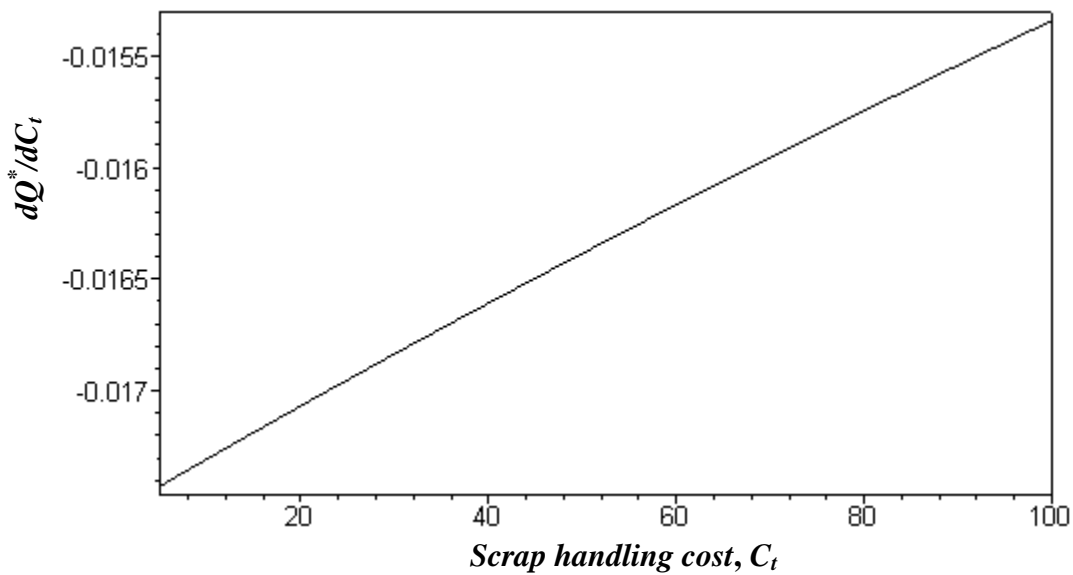


Figure 6.7: Effect of scrap handling cost on batch size.

If the value of C_t varies from 5 to 50, the change in Q^* can be calculated using the above equation and the parametric values used previously, and can be shown graphically in Figure 6.7.

The effect of scrap handling cost is studied by changing the values over the range from 5 to 50. It is observed that C_t is inversely proportional to Q^* and directly proportional to $TC(Q^*)$. The Tables 6.9 and 6.10 represents the changes over $TC(Q^*)$ and Q^* with respect to C_t .

Table 6.9: Effect of C_t on $TC(Q^*)$ and Q^* for Case I and Case II.

C_t	CASE I				CASE II			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
5	37.39	3028.31	28.32	2819.28	37.62	3023.39	28.34	2818.49
10	37.31	3030.18	28.28	2820.70	37.53	3025.27	28.31	2819.91
20	37.13	3033.90	28.21	2823.52	37.36	3029.01	28.23	2822.74
30	36.96	3037.60	28.13	2826.34	37.19	3032.74	28.16	2825.56
40	36.80	3041.29	28.06	2829.15	37.02	3036.45	28.08	2828.37
50	36.63	3044.96	27.99	2831.95	36.85	3040.15	28.01	2831.17

Table 6.10: Effect of C_t on $TC(Q^*)$ and Q^* for Case III and Special Case.

C_t	CASE III				SPECIAL CASE			
	First Policy		Second Policy		First Policy		Second Policy	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
5	37.61	3023.73	28.34	2817.64	37.62	3023.37	28.34	2818.55
10	37.52	3025.60	28.31	2819.06	37.54	3025.25	28.31	2819.97
20	37.34	3029.35	28.23	2821.89	37.36	3028.99	28.23	2822.79
30	37.17	3033.07	28.16	2824.71	37.19	3032.72	28.16	2825.61
40	37.00	3036.78	28.08	2827.52	37.02	3036.43	28.08	2828.42
50	36.83	3040.47	28.01	2830.33	36.85	3040.12	28.01	2831.23

6.6 Effect of H_1 on Q^* and $TC(Q^*)$

Generally, the inventory carrying cost, H_1 is an important factor in any inventory system. The change in H_1 over optimum batch size, Q^* can be represented by equation (6.9).

$$\frac{\partial Q^*}{\partial H_1} = \frac{-\sqrt{PDS(1-\alpha\beta)}[(P-D(1+\beta+\beta^2))-\alpha\beta(2P-3D-2\beta D-\alpha\beta P+\alpha\beta D)]}{\sqrt{2}\nu^{3/2}}, \quad (6.9)$$

where

$$\nu = [H_1((P-D(1+\beta+\beta^2))-\alpha\beta(2P-3D-2\beta D-\alpha\beta P+\alpha\beta D)) + 2C_r\alpha\beta(1-\alpha\beta)P].$$

If the value of H_1 varies from 50 to 300, the change in Q^* can be calculated using the above equation and the parametric values used previously. The effect of H_1 of over Q^* is represented by Figure 6.8 where $H_1 \in [50, \infty]$. Figure 6.8 shows that faster increases in Q^* can be observed from point [0,0] to [100, -0.168] with the increase of H_1 values for Case I and it can be shown as well for all cases.

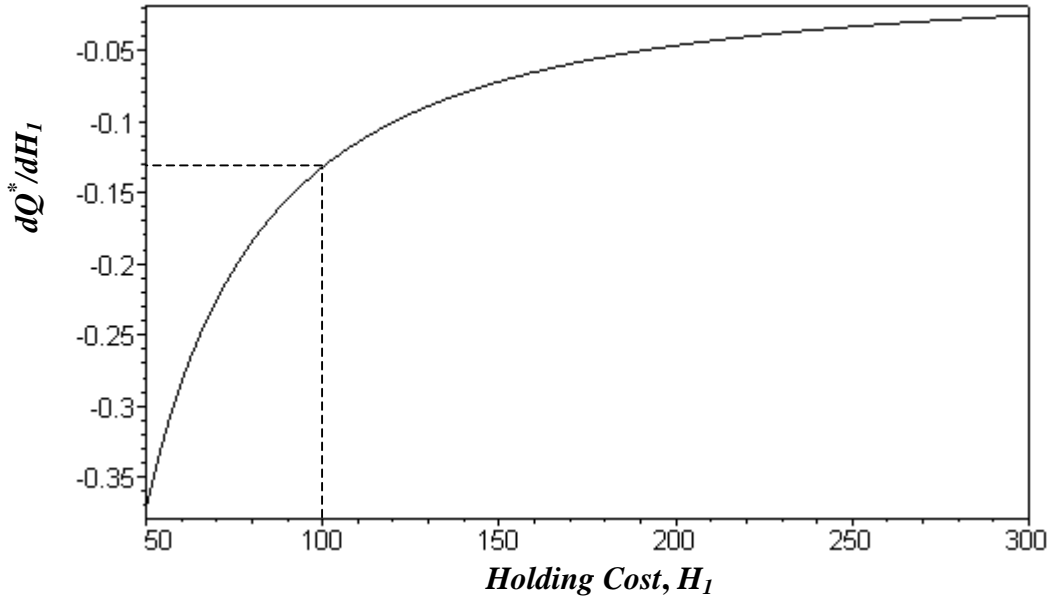


Figure 6.8: Effect of holding cost on $TC(Q^*)$ and Q^* for First Policy.

A study was done also with respect to H_l where the parametric values are considered, as the same except the values of H_l which are varied from 50 to 300 for the first policy and values of H_n from 118 to 300 for the second case. The results are shown in Table 6.11 and 6.12. From the study it can be concluded that, with the increase of inventory carrying cost, the optimum batch quantity Q^* decreases and the total cost $TC(Q^*)$ increases.

Table 6.11: Effect of H_l on $TC(Q^*)$ and Q^* for First Policy.

H_l	FIRST POLICY							
	Case I		Case II		Case III		Special Case	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
50	37.39	3028.31	37.62	3023.39	37.61	3023.73	37.62	3023.37
100	26.47	3359.32	26.64	3352.35	26.62	3352.83	26.64	3352.32
150	21.62	3613.48	21.76	3604.95	21.75	3605.53	21.76	3604.91
200	18.73	3827.81	18.85	3817.96	18.84	3818.63	18.85	3817.91
250	16.75	4016.67	16.86	4005.65	16.85	4006.40	16.86	4005.60
300	15.29	4187.43	15.39	4175.36	15.38	4176.18	15.39	4175.30

Table 6.12: Effect of H_n on $TC(Q^*)$ and Q^* for Second policy.

H_n	SECOND POLICY							
	Case I		Case II		Case III		Special Case	
	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$	Q^*	$TC(Q^*)$
118	28.32	2819.28	28.34	2818.49	28.34	2817.64	28.34	2818.55
130	26.72	2882.84	26.74	2882.02	26.74	2881.13	26.74	2882.08
150	24.56	2981.45	24.58	2980.58	24.58	2979.64	24.58	2980.65
200	20.86	3198.74	20.87	3197.76	20.87	3196.69	20.87	3197.83
250	18.44	3387.29	18.46	3386.21	18.46	3385.03	18.46	3386.29
300	16.71	3556.18	16.72	3555.00	16.72	3553.71	16.72	3555.08

6.7 Effect of δ on Q^* and $TC(Q^*)$ for Case II

In Case II, one of the parameters considered to model the inventory is the scrap production factor, δ that indicates the scrap production rate. The variation of scrap production rate over optimum batch quantity can be represented by equation (6.10).

$$\frac{\partial Q^*}{\partial \delta} = \frac{\sqrt{PDS(1-\alpha\beta)}[H_1\alpha\beta(\beta D + 2\alpha\beta D)]}{\sqrt{2\nu}^{3/2}}. \quad (6.10)$$

where

$$\nu = [H_1((P - D(1 + \beta + \beta^2)) - \alpha\beta(2P - 3D - 2\beta D - \alpha\beta P + \alpha\beta D)) + 2C_i\alpha\beta(1 - \alpha\beta)P].$$

The rate and direction of change of Q^* and $TC(Q^*)$, with respect to δ , depend on the values of the parameter used in the numerical example for Case I of within cycle rework policy. Hence, using equation (6.10) and $\delta \in [0, 1.0]$, the effect of δ on Q^* and $TC(Q^*)$ are shown in Figure 6.9.

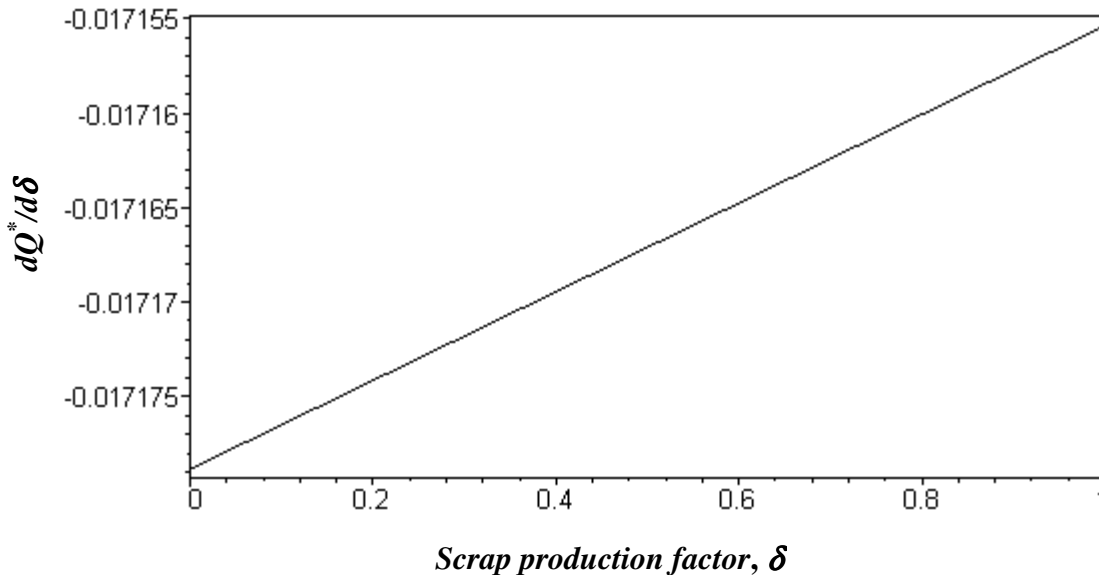


Figure 6.9: Effect of scrap production factor on batch size.

A study was done with respect to δ over Q^* where the parametric values are considered as the same. The values of δ which are varied from 0 to 1.0 for the Case II in both policies and it is found that the optimum batch quantity varies from 37.62 to 37.61 and the total cost from \$3023.37 to \$3023.73 for first policy. Also for the second case it was found that optimum total cost and batch quantity varied from \$2818.55 to \$ 2817.64 and from 28.34 to 28.35, respectively. According to this study, it can be concluded that with the increase of scrap production factor, the changes in optimum batch quantity Q^* and the total cost $TC(Q^*)$ are extremely small which can be ignored. Hence, the optimum batch quantity Q^* and the total cost, $TC(Q^*)$ is independent of scrap production factor δ for both policy.

Finally, according to the numerical examples, it can be concluded that the optimum batch quantity of the classical inventory model is less than the optimum batch quantity of this research, which means that this model associated with more cost than ideal inventory model and this can be used in the inventory models which involve practical conditions.

CHAPTER 7

RESEARCH SUMMARY AND CONCLUSIONS

This chapter highlights the various stages of the present research. A summary of the current research was described first followed by the conclusion and research significance. Finally, some propositions are made for future research on the inventory systems of a production process with rework and scrap.

7.1 Summary

In the past, many researchers have determined the optimal batch quantity models for single and multi-stage production systems to minimize the total cost of the system where some percentage of defective items are produced and reworked. In this research, the circumstances of practical situations are taken into consideration that describe the imperfection of rework process resulting scrap.

At first the drawbacks of previous research were found and described on the inventory system of a production process with rework option. Then a model is developed considering the omissions of the previous research such as rework with scrap. The models are developed for two types of rework process: (a) rework within the cycle, and (b) rework after N cycles. Those two rework processes also incur three types of scrap detection techniques: (i) scrap detected before rework, (ii) scrap detected during rework, and (iii) scrap detected after rework. The inventory models are developed and presented graphically and mathematically. After that, the total cost functions were developed for the inventory of the system and the convexity of the functions are proved for minimization techniques. After that, the optimal batch quantities are derived for all policies of rework with scrap detection techniques followed by numerical examples to illustrate the models.

The operational schedules for the two main cases of both policies were described mathematically and graphically after numerical examples.

At the end, a sensitivity analysis is conducted with respect to different parameters, which are used to develop the models. The variation of these parameters over the optimum batch quantity and the total cost function were represented graphically and numerically.

7.2 Conclusions

This research describes the inventory system of a single-stage production process with rework and scrap of the two policies. In the first policy, the defective items are reworked within the same cycle, and in the second policy the defective items are accumulated until a number of cycles are completed, after which the defectives parts are reprocessed. Both the rework policies consist of scrap production and detection. The closed-form solutions were developed to find the optimal batch quantities with various cases of scrap detection. Some numerical examples were carried out to illustrate the models. This research consisted of practical situations, which are involved in production processes. It is also observed that the total cost associated with the scrap detected during rework case is the lowest among all other cases.

According to the sensitivity analysis, it is observed that the total inventory cost in first policy is not too sensitive to lower proportions of defectives and scrap. When the defective proportion rises beyond 0.4, the optimum batch quantity increases rapidly as does the total inventory cost. For the second policy, the optimum batch quantity increases up to a certain level with the increase in the defective proportion and then it starts to

decrease. The total cost and the optimum batch quantity are independent of scrap production factor for both the policies.

7.3 Significance of Research

For more than a decade, the researchers have developed the optimal batch size models for inventory systems of production facilities under ideal conditions. Practically, the situations in the production processes may be imperfect due to various reasons resulting defectives production. Only a few researchers developed optimum batch quantity models for this type of production facility, but they assumed perfect reprocessing of the defectives to turn each defective into a finished product, which was unrealistic. Some of the defective items cannot be reworked or can be turned into defectives again. This research has incorporated the issues of (a) imperfect reworking and (b) scrap production during model development, and it also took into consideration the shortages due to scrap. Therefore, this research will make a significant contribution in solving the problems of the imperfect manufacturing systems.

7.4 Future Research

The present research addresses the inventory system of a production process with different policies of rework and scrap. This research can be extended as follows:

- Most of the production systems today are multi-stage systems, and in a multi-stage system the defective items and scrap can be produced in each stage. Again, the defectives and scrap proportion for a multi-stage system can be different in different stages. Taking these factors into consideration this research can be extended for a multi-stage production process.

- Traditionally, inspection procedures incurring cost is an important factor to identify the defectives and scrap, and remove them from the finished goods inventory. To better production, the placement and effectiveness of inspection procedures are required which is ignored for this research, so inspection cost can be included in developing the future models.
- The rework process is associated with high cost, risk of operation failure and lack of control, for that reason a further research can involve these practical conditions.
- The transporting situation of the lots in multi-stage production systems consists of time lag, delivery failure from one stage to another, etc. Also to minimize the transportation cost, the entire lot can be divided into sub lots, and deliver when needed, considering these situations an inventory model can be developed with rework and scrap.
- This research deals with the scrap detection methods separately. In practical point of view scrap can be detected from a single stage before, during and after rework operation in a same production cycle. In a single or multi-stage production system inventory model can be developed considering these circumstances.
- The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive and cheaper than the old one. On the other hand the demand of new product will increase. Thus, demand rate can be varied with time, so variable demand rate can be used to develop the model.

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APPENDIX A

EXAMPLE 1: CYCLE TIME VERIFICATION

To verify the model, a numerical example is shown which has the following data: $P = 1500$ units/year, $D = 1000$ units/year, percentage of defectives with respect to produced items is 5% ($\beta = 0.05$), percentage of scrap with respect to defectives is 20% ($\alpha = 0.20$). Also, to avoid shortages it is restricted that $P > \beta P + D$.

In order to test the geometry of the model presented in Figure 3.2, it is assumed that $t_1 = 0.60$ years. Therefore, $Q = Pt_1 = 1500 \times 0.60 = 900$ units/year and the number of defectives produced is $\beta Q = 900 \times 0.05 = 45$ units. Hence, $Q_1 = (1 - \beta - D/P)Q = (1 - 0.05 - 1000/1500) \times 900 = 255$ units and $Q_2 = (1 - D/P)Q = (1 - 1000/1500) \times 900 = 300$ units. So the number of defectives confirms to $(Q_2 - Q_1) = (300 - 255) = 45$ units.

Again, $t_2 = (1 - \alpha)\beta Q/P = (1 - 0.20) \times 0.05 \times 900/1500 = 0.024$ years and, from equation (3.1a),

$$t_3 = \frac{[1 - \alpha\beta - (1 + \beta - \alpha\beta)D/P]Q}{D} = \frac{[1 - 0.20 \times 0.05 - (1 + 0.05 - 0.20 \times 0.05)1000/1500]900}{1000}$$

$= 0.267$ years. Hence, $T_1 = t_1 + t_2 + t_3 = 0.60 + 0.024 + 0.267 = 0.891$ years. Using equation (3.1) it is also found that $T_1 = 0.891$ years which proves the model.

APPENDIX B

EXAMPLE 2: CYCLE TIME VERIFICATION

To confirm to the model of Case II, represented by Figure 3.3, the same numerical data used in Example 1 are considered. Also assume the scrap production factor $\delta = 0.0002$.

To estimate the value of Q , it is also assume that $t_1 = 0.60$ years as in Example 1. Hence, $Q = Pt_1 = 1500 \times 0.60 = 900$ units/year and the number of defectives produced is $\beta Q = 900 \times 0.05 = 45$ units during time t_2 . Therefore, $Q_1 = (1 - \beta - D/P)Q = 255$ units and $Q_2 = (1 - D/P)Q = 300$ units. So that number of defectives confirms to $(Q_2 - Q_1) = (300 - 255) = 45$.

Again, $t_2 = (1 - \alpha)\beta Q/P = (1 - 0.2) \times 0.05 \times 900/1500 = 0.024$ years, $t_3 = \alpha\beta\delta Q/P = 1.2 \times 10^{-6}$ and using equation (3.6) t_4 is found as

$$t_4 = \frac{900}{1000} \left(1 - 0.20 \times 0.05 - \frac{(1 + 0.05 - 0.20 \times 0.05 + 0.20 \times 0.05 \times 0.0002)1000}{1500} \right) = 0.2669988$$

years. Hence, $T_2 = t_1 + t_2 + t_3 + t_4 = 0.60 + 0.024 + 0.2669988 + 1.2 \times 10^{-6} = 0.891$ years.

Using equation (3.7) it is also found that $T_2 = 0.891$ years which confirms the model.

APPENDIX C

PROOF OF CONVEXITY OF $TC(Q)$ FOR WITHIN CYCLE REWORK

By differentiating equation (3.22) with respect to Q , it is found that,

$$\begin{aligned} \frac{\partial TC_i(Q)}{\partial Q} = & -\frac{DS}{Q^2} + C_i\alpha\beta + \frac{H_1}{2(1-\alpha\beta)P} [P - D(1 + \beta + \beta^2) \\ & - \alpha\beta(2P - 2D - 2\beta D - \alpha\beta P + \alpha\beta D)] + \frac{\alpha\beta DH_1}{2(1-\alpha\beta)P}, \end{aligned}$$

from where

$$\frac{\partial^2 TC_i(Q)}{\partial Q^2} = \frac{2DS}{Q^3}.$$

Now,

$$H(Q) = \left[\frac{2DS}{Q^3} \right].$$

Hence, the principal minor of the Hessian Matrix is

$$H_1 = \frac{2DS}{Q^3} > 0, \text{ for any value of } Q > 0.$$

According to the Hessian Matrix, equation (3.22) is a convex function. Similarly, it can be proved for equations (3.23), (3.24), and (3.25) that the total cost functions are convex functions and can be minimized by differentiating with respect to Q .

APPENDIX D

PROOF OF CONVEXITY OF $TC(Q)$ FOR REWORK AFTER N CYCLES

Applying the value of $N (= D/Q)$ in equation (4.32) it is found,

$$TC_I(Q) = \frac{DC_s t_s}{Q} + C_d t_d + C_i \alpha \beta Q + \frac{H_n Q}{2} \left[(1-\alpha) \beta^2 \left(1-\alpha - \frac{(1-2\alpha)D}{P} \right) + (1-\beta) \right. \\ \left. \times \left(1-\beta - \frac{D}{P} \right) \right] + \frac{\beta C_w}{1-\beta} \left[\frac{D(D/Q-1)t_s}{2} + \frac{D^2}{2P} \right] + \frac{C_p \beta D}{2(1-\beta)} \left[1-\beta - \frac{(1-3\beta+2\alpha\beta)Q}{D} \right]. \quad (A1)$$

By differentiating equation (A1) with respect to Q , it can be found that,

$$\frac{\partial TC_I(Q)}{\partial Q} = -\frac{DC_s t_s}{Q^2} + C_i \alpha \beta + \frac{H_n}{2} \left[(1-\beta) \left(1-\beta - \frac{D}{P} \right) + (1-\alpha) \beta^2 \left(1-\alpha - \frac{(1-2\alpha)D}{P} \right) \right] \\ - \frac{\beta C_w}{1-\beta} \left[\frac{D^2 t_s}{2Q^2} \right] - \frac{C_p \beta D}{2(1-\beta)} \left[\frac{(1-3\beta+2\alpha\beta)}{D} \right],$$

from which

$$\frac{\partial^2 TC_I(Q)}{\partial Q^2} = \frac{2DC_s t_s}{Q^3} + \frac{\beta C_w}{1-\beta} \left[\frac{D^2 t_s}{Q^3} \right].$$

Hence,

$$H(Q) = \left[\frac{2DC_s t_s}{Q^3} + \frac{\beta C_w}{1-\beta} \left[\frac{D^2 t_s}{Q^3} \right] \right].$$

The principal minor of the Hessian Matrix is $H_1 = \left[\frac{2DC_s t_s}{Q^3} + \frac{\beta C_w}{1-\beta} \left[\frac{D^2 t_s}{Q^3} \right] \right] > 0$, as β is a proportion so $(1 - \beta) > 0$ for any value of Q . Hence, equation (4.32) is a convex function and it can be minimized by differentiating with respect to Q or $N (= D/Q)$. Similarly, the convexity of equations (4.33), (4.34) and (3.35) can be proved using Hessian Matrix principle.

VITA

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